ENGR-4300
Spring 2010
Test 2

Name: ____________________________

Section: 1(MR 8:00) 2(MR 4:00) 3(TF 8:00)
(circle one)

Question I (20 points): ____________

Question II (20 points): ____________

Question III (20 points): ____________

Question IV (20 points): ____________

Question V (20 points): ____________

Total (100 points): _______________

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification. Also, remember that you may have more than one method available for finding the answer to a question. Use all information, even if it is only to check your results.

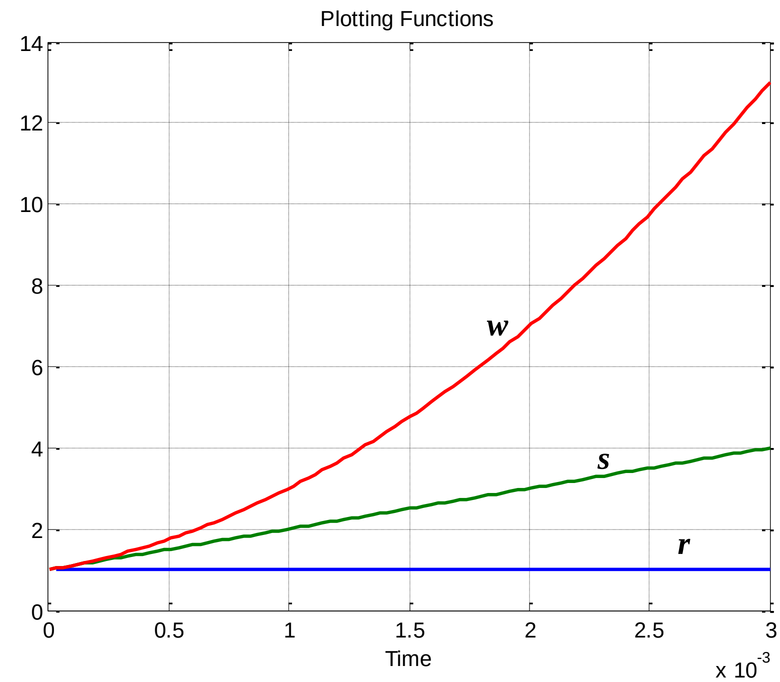
Question I – Mathematics (20 points)

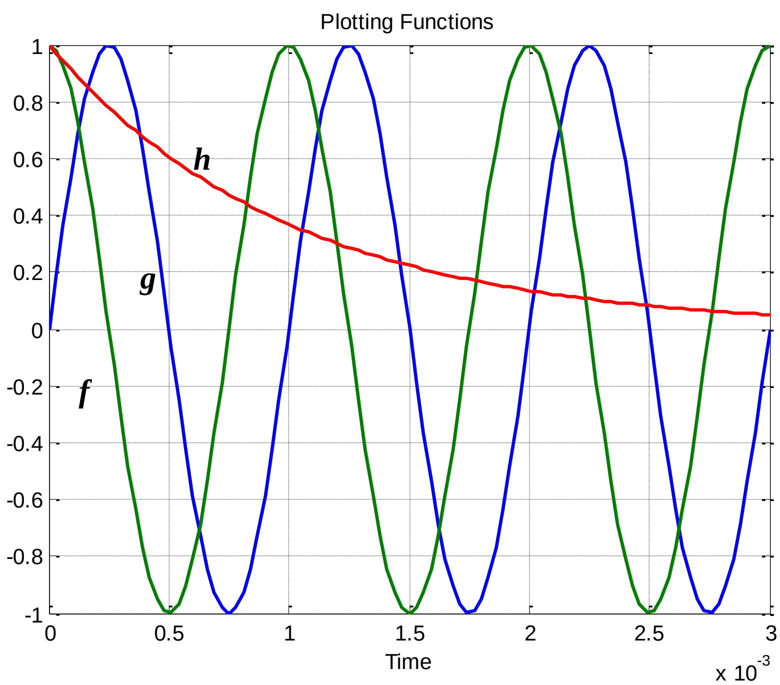
Given the following equations

$f = A\sin(\omega t)$ $r = a$ $g = B\cos(\omega t)$

$s = a + bt$ $h = Ce^{-\alpha t}$ $w = a + bt + ct^2$

1) (6 pts) Identify which plot corresponds to each of the functions above by labeling them with the function letter.





2) (4 pts) Evaluate the following integrals. *Neglecting constants. They can be included or not.*

$$\int_0^t f dt = \int_0^t (Asin\omega t) dt = \frac{A\cos\omega t}{-\omega}$$

$$\int_0^t r dt = \int_0^t a dt = at$$

$$\int_0^t h dt = \int_0^t Ce^{-\alpha t} dt = C\frac{e^{-\alpha t}}{-\alpha}$$

$$\int_0^t s dt = \int_0^t (a + bt) dt = at + \frac{1}{2}bt^2$$

3) (4 pts) Evaluate the following derivatives.

$$\frac{dg}{dt} = -\omega Bsin\omega t$$

$$\frac{dw}{dt} = 0 + b + 2ct$$

$$\frac{dh}{dt} = -\alpha Ce^{-\alpha t}$$

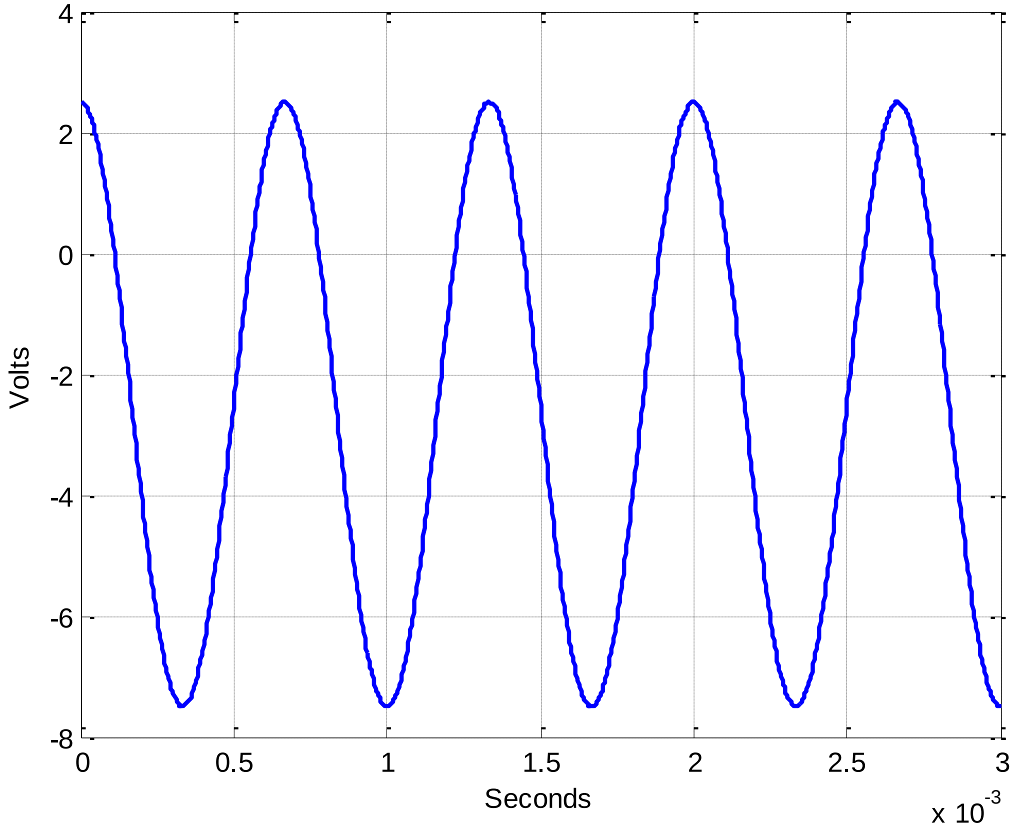
$$\frac{dr}{dt} = 0$$

4) (6 pts) From the given time scale for the plots, determine the frequency *f* of the sinusoidal functions and the decay constant α of the exponential function. *Hint: The answers should be very simple.*

$f = \frac{1}{T} = 1kHz$ since the period is 1ms

$\alpha = 1000$ since the exponential decays to 1/e of its original value in 1ms

Question II – Sinusoids and Damped Sinusoids (20 points)



1) (4 pts) Find the ***period and frequency*** of this signal. Include units.

There are 4.5 periods in 3ms. Thus, $T = \frac{.003}{4.5} = .67ms$ so that the frequency is 1.5kHz.

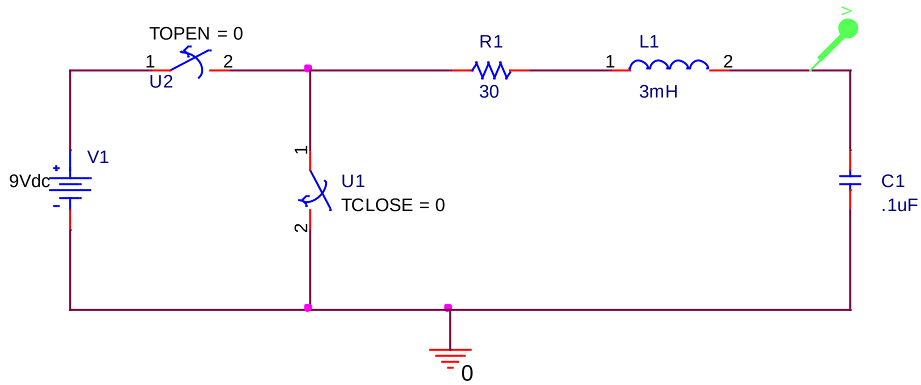
2) (2 pts) What is the value of the DC offset?

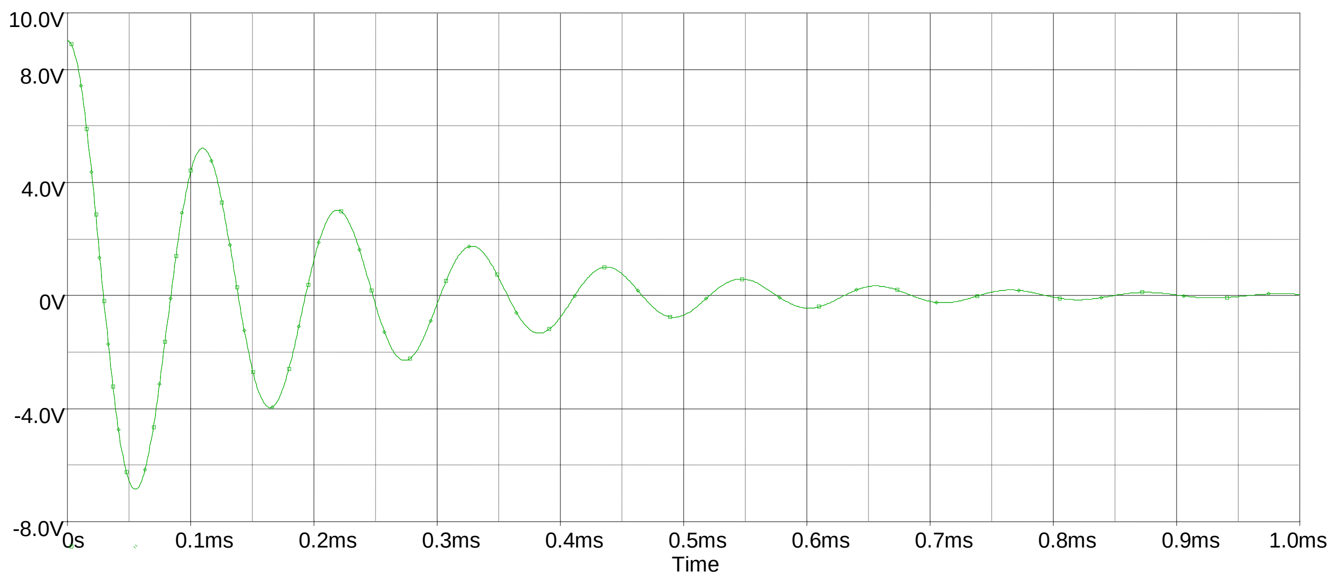
The max is +2.5V and the min is -7.5V so the middle is -2.5V which is the offset.

3) (4 pts) Write the mathematical expression for this signal and its offset. In general, this is given by $X = X_0 + X_1 \sin(\omega t + \Phi_0)$ which accounts for its frequency, phase shift, and offset. (Phase shift with respect to t=0)

$$V = -2.5 + 5\cos(2\pi 1500t)$$

The circuit below gives the simulation output shown.





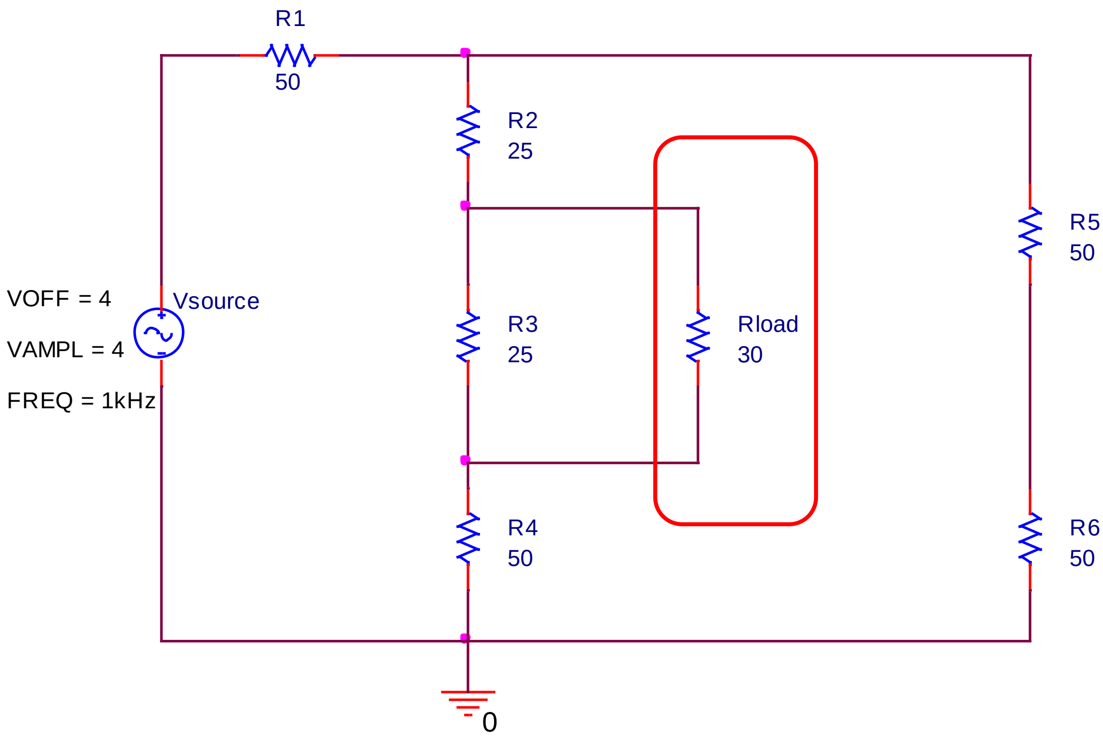
4) (6 pts) Find the damping constant α and frequency *f* and ω for this data. Note that you must write the frequency two different ways, i.e. as both *f* and ω. *Hint: be sure to use everything you know about this circuit when answering this question.*

The frequency can be determined from L and C, $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3(10^{-10})}} = 57735$ and *f* = *9189Hz*. This gives a period of a little more than .1ms, which is what it shows in the plot. For the damping constant, we can use the period to find the time. At 4T into the plot, or t = .435ms, the exponential has decayed from 9 to 1V or $-\alpha t = \ln\left(\frac{1}{9}\right)$ or $\alpha = \frac{2.2}{.000435} = 5050$ or thereabouts. Checking to see if $9e^{-5050(2T)} = 3$, which indeed works.

5) (4 pts) Write the mathematical expression for this data in the form of v(t)=A cos (ωt) $e^{-\alpha t}$

$$V(t) = 9e^{-5050\,t}cos(2\pi 9189t)$$

Question III – Thevenin Equivalents (20 points)

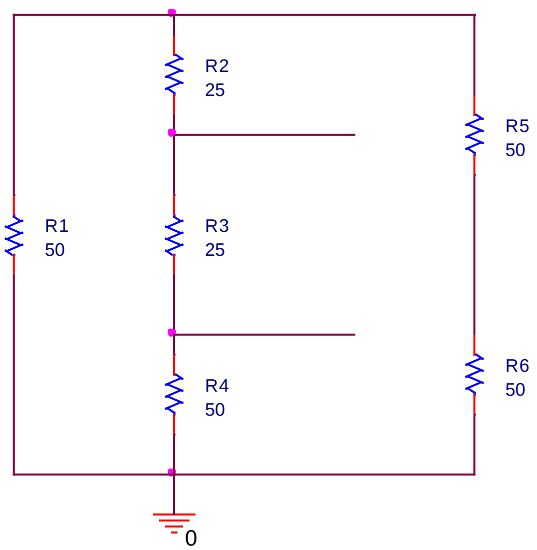


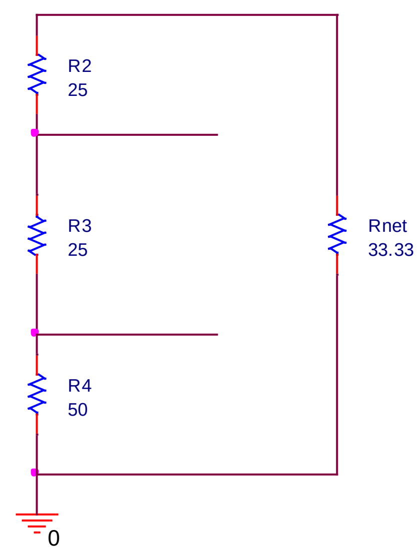
1) (6 pts) Find the Thevenin voltage (Vth) of the circuit driving the 30 Ohm load. The load resistor Rload is found in the rectangle above. Be sure your answer includes all information on the source.

The resistors in the right leg add to 100Ohms, as do the resistors in the center leg when the load is removed. Two 100Ohm resistors in parallel result in a 50Ohm resistor. Since the source impedance is also 50Ohms, 1/2 of the voltage appears across the three resistors in the center leg. The load is across a 25Ohm resistor so that ¼ of the source voltage is the Thevenin voltage or
$V(t) = 0.5 + 0.5cos(2\pi 1000t)$.

2) (6 pts) Find the Thevenin resistance.

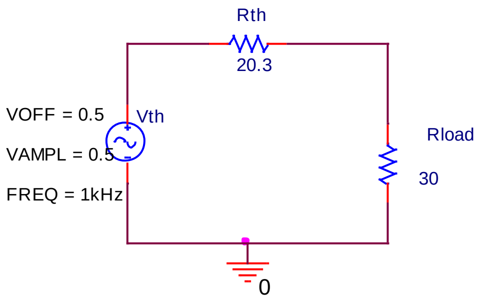
After shorting out the voltage source, we have the following circuit.



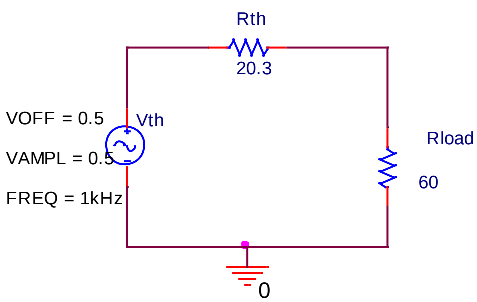


The right leg is 100Ohms again while the left leg is 50Ohms. These two legs are in parallel so that they add to 33.33Ohms, so the circuit ends up like the one at the right. Then the 25Ohm, the 50Ohm and the 33.33Ohm resistors are in series totaling 108.33Ohms. This is then in parallel with the 25Ohms netting a Thevenin resistance of 20.3Ohms.

3) (4 pts) Draw the Thevenin equivalent circuit with the load (30 Ohms).

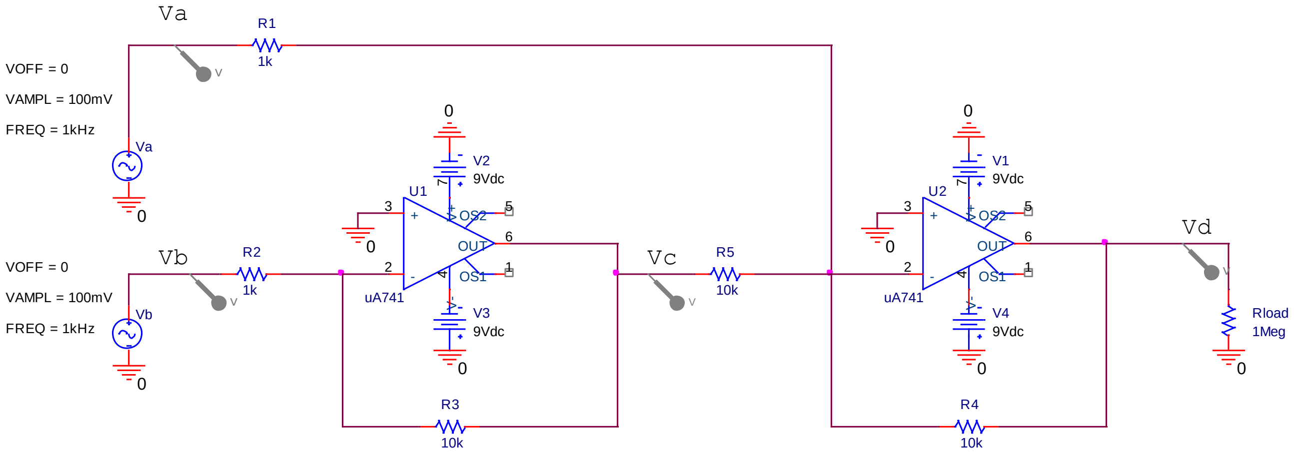


4) (4 pts) Change the load to 60 Ohms and then determine the voltage produced by this configuration.



Then $V(t) = (0.5 + 0.5 sin\omega t)\frac{60}{80.3} = .37 + .37 sin\omega t$ where $\omega = 2\pi 1000$

Question IV – Op-Amp Applications & Analysis (20 points)



1) (3pt) The circuit has 2 op-amps labeled as U1 and U2. State what the op-amp circuit is for each. Choices are: 1. Follower/Buffer, 2. Inverting Amp, 3. Non-inverting Amp, 4. Differentiator, 5. Integrator, 6. Adding (Mixing) Amp, 7. Difference (Differential) Amp, 8. Miller Integrator, 9. Practical Differentiator.

U1 Circuit: Inverting Amp U2 Circuit: Inverting Amp or Adding Amp

2) (8pt) What are the values relative to ground, of Va(t), Vb(t), Vc(t) and Vd(t). Write out the mathematical expressions for these voltages.

a) Voltage at point Va(t): $0.1 sin\omega t$ with $\omega = 2\pi 1000$

b) Voltage at point Vb(t): $0.1 sin\omega t$

c) Voltage at point Vc(t): $-1 sin\omega t$

d) Voltage at point Vd(t): 0

3) (3pt) What is the function of this complete circuit?

Choices are: 1. Follower/Buffer, 2. Inverting Amp, 3. Non-inverting Amp, 4. Differentiator, 5. Integrator, 6. Adding (Mixing) Amp, 7. Difference (Differential) Amp, 8. Miller Integrator, 9. Practical Differentiator. Difference Amp

4) (2pt) Using the Golden Rules of Op-Amps, determine the output voltage Vd in terms of Vb for the case where Va is zero Volts.

$$Vd = Vb\left(-\frac{10k}{1k}\right)\left(-\frac{10k}{10k}\right) = 10Vb$$

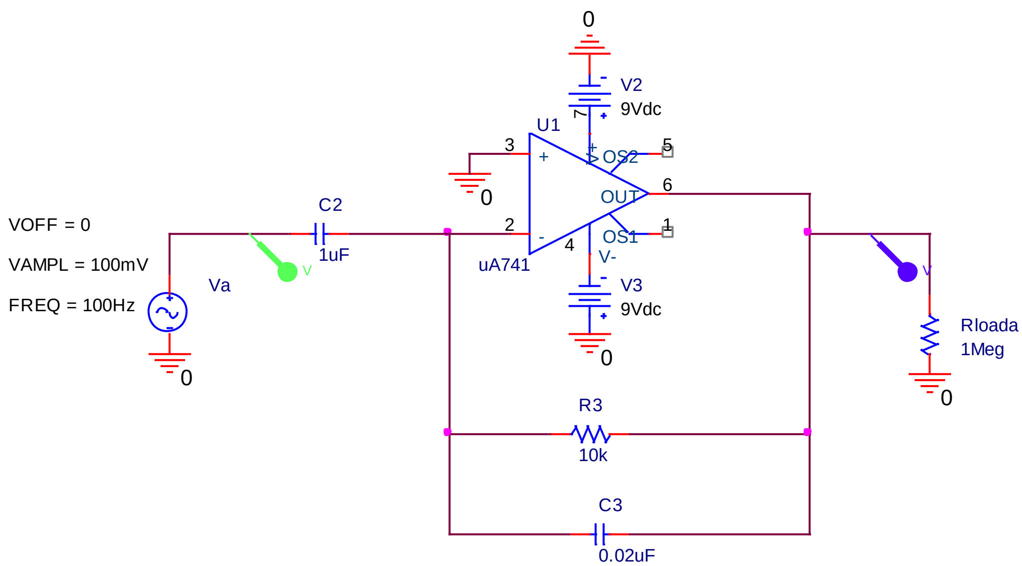
5) (2pt) Using the Golden Rules of Op-Amps, determine the output voltage Vd in terms of Va for the case where Vb is zero Volts.

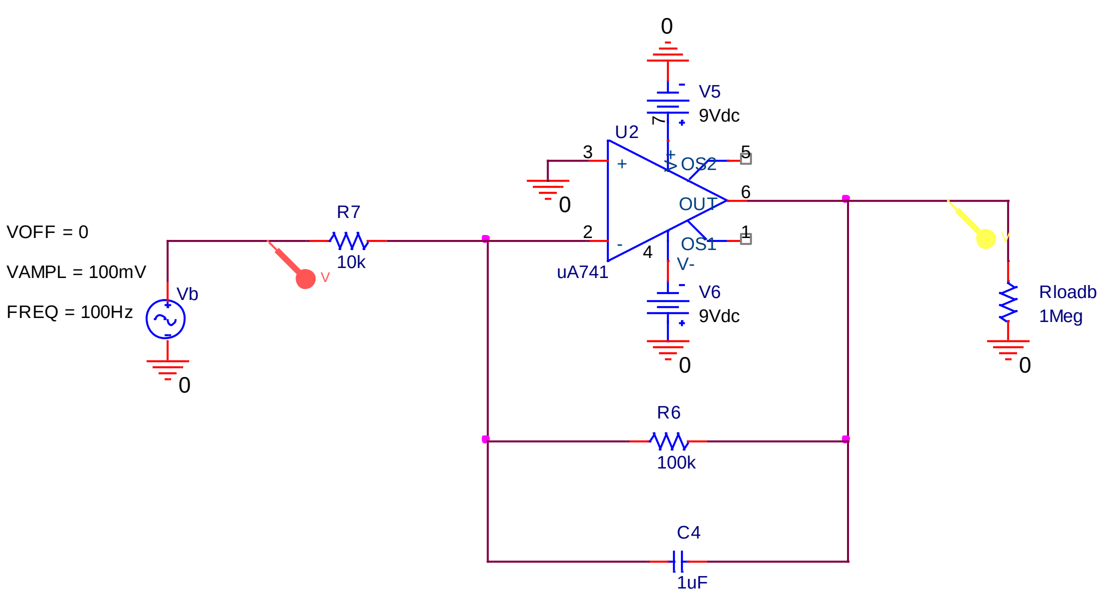
$$Vd = Va\left(-\frac{10k}{1k}\right) = -10Vb$$

6) (2pt) Using the Golden Rules of Op-Amps, determine the output voltage Vd in terms of Va for the case where Va = Vb.

$$Vd = Vb\left(-\frac{10k}{1k}\right)\left(-\frac{10k}{10k}\right) + Va\left(-\frac{10k}{1k}\right) = 0$$

Question V – Op-Amp Integrators and Differentiators (20 points)





1) (4pt) Find the transfer function H(jω) = Vout(jω)/V1(jω) for these two circuits. (Substitute in the values provided for the components after you find the general form.)

$H_{top}(j\omega) = \frac{V_{out}}{V_{in}} = -\frac{R_3||Z_{C3}}{Z_{C2}}$ and $H_{bottom}(j\omega) = \frac{V_{out}}{V_{in}} = -\frac{R_6||Z_{C4}}{R_7}$ with $R_3||Z_{C_3} = \frac{R_3\frac{1}{j\omega C_3}}{R_3+\frac{1}{j\omega C_3}}$ and $R_6||Z_{C_4} = \frac{R_6\frac{1}{j\omega C_4}}{R_6+\frac{1}{j\omega C_4}}$ so $H_{top}(j\omega) = -\frac{j\omega R_3 C_2}{1+j\omega R_3 C_3} = -\frac{j\omega\, 0.02}{1+j\omega\, 0.0002}$ and $H_{bottom}(j\omega) = -\frac{R_6/R_7}{1+j\omega R_6 C_4} = -\frac{10}{1+j\omega\, 0.02}$

2) (2pt) What function is each circuit designed to perform at high frequencies?

Top inverting op amp, bottom integrator

3) (2pt) What function is each circuit designed to perform at low frequencies?

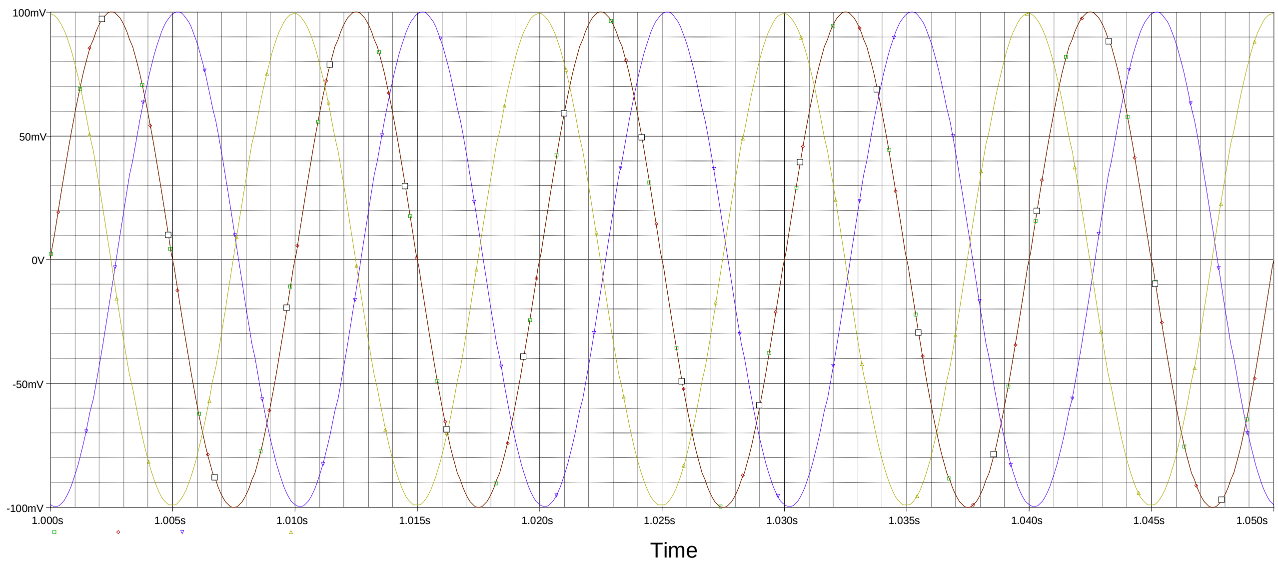
Top differentiator , bottom inverting op amp

4) (2pt) What is the corner frequency (*in Hz*) for each circuit where it transitions from its low frequency performance to its high frequency performance?

$\omega_c = \frac{1}{RC} = \frac{1}{0.0002}$ and $\omega_c = \frac{1}{RC} = \frac{1}{0.02}$ for top and bottom, respectively. In Hertz then the top has a corner frequency of 796Hz and 8Hz.

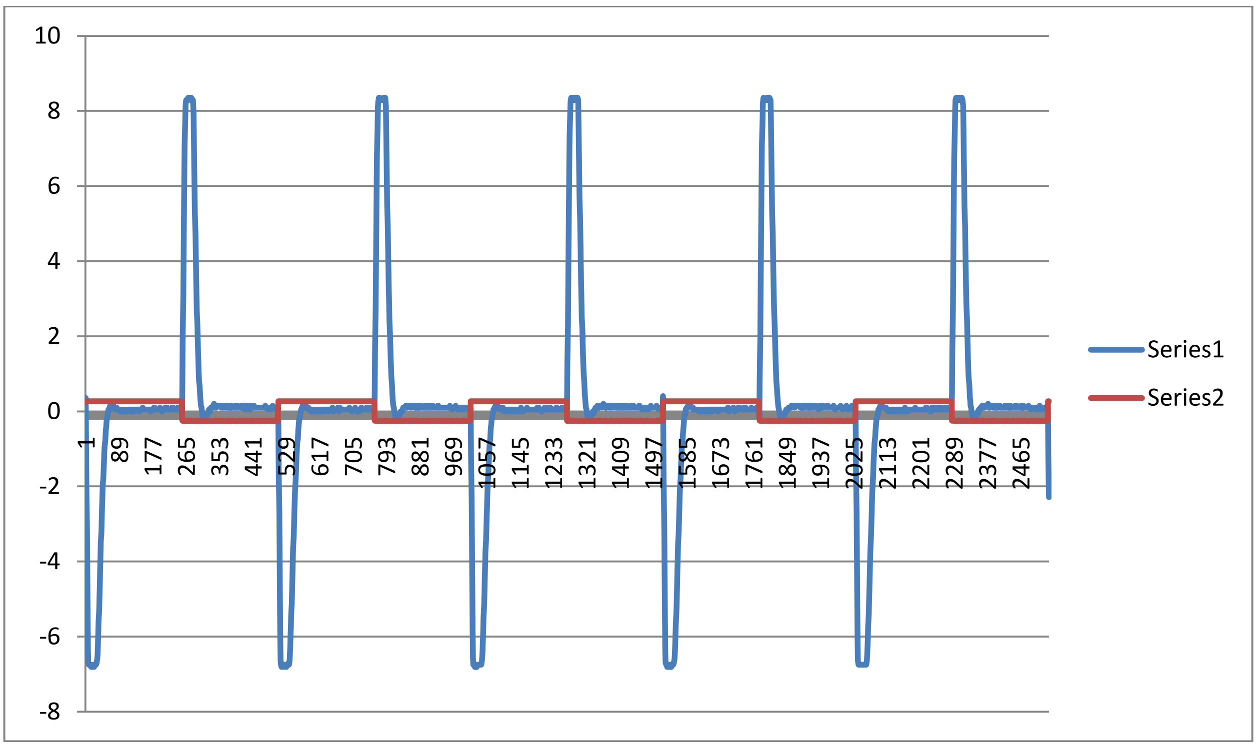
5) (5pt) The plot below shows the input signal and the output signals for both circuits. Two signals have been scaled so that all three are the same size. The first curve is multiplied by 6.25 while the third curve is divided by 6.25. (That is, the original size of the first curve was much smaller while the size of the third curve was much greater.) Identify which curve is which by labeling them as input, output for the top circuit (with source Va) and output for the bottom circuit (with source Vb). Explain your answer. Be sure that your labeling is clear.

integral ***source*** ***derivative***



The derivative is much bigger than the source while the integral is much smaller.

6) (5pt) The plot below shows the output for one of these circuits taken experimentally with the Mobile Studio IOBoard. The horizontal scale indicates the data point, not the time. Identify which of the two circuits above was used to obtain this data. Explain your answer. Also, label which curve is the input and which is the output. *Remember that this is data from a real circuit so you cannot expect it to perform ideally.*



This is clearly the differentiator working on a square wave. The spikes at the beginning and ending of each half cycle are trying to be infinite but are clipped to the saturation voltage. The input is the red square wave and the output is the blue more jagged data.