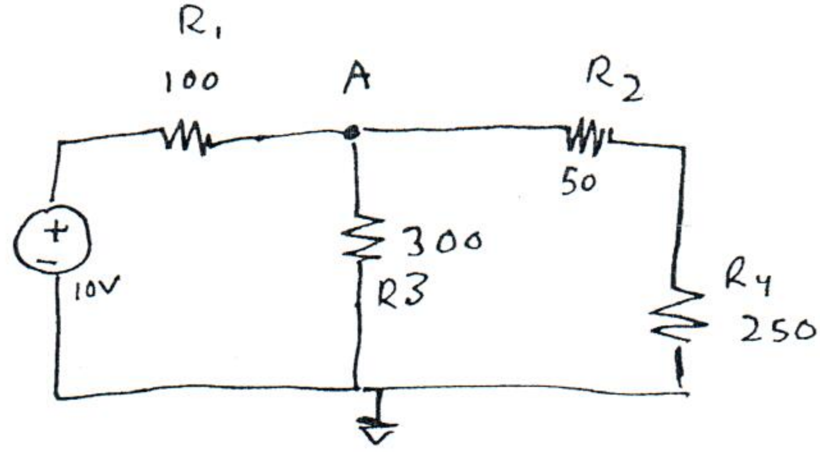
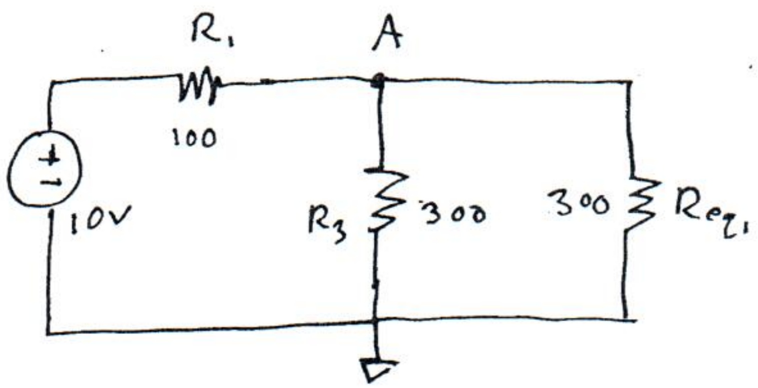
Quiz I

soln.

① a.)

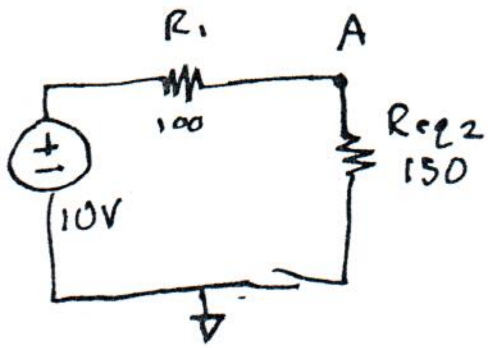


Resistors R_2 and R_4 are in series and can be combined into a single equivalent branch resistance:



R_3 and R_{eq_1} are in parallel and can be combined into R_{eq_2}.

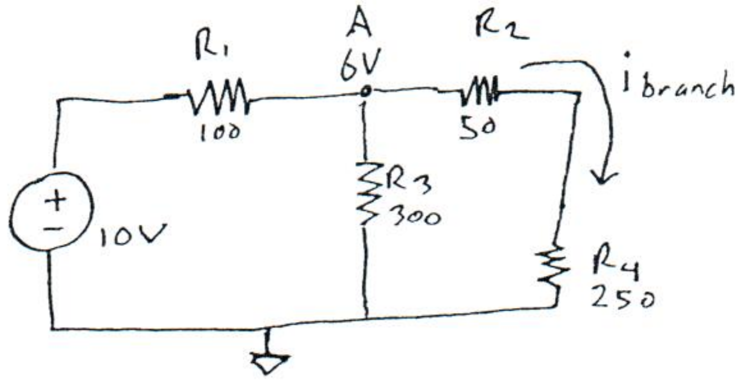
$$R_{eq_2} = \frac{R_3 \times R_{eq_1}}{R_3 + R_{eq_1}} = \frac{(300)^2}{600} = 150\Omega$$



$$V_A = 10V \cdot \frac{(150\,\Omega)}{(100\,\Omega + 150\,\Omega)}$$

$$V_A = 6V$$

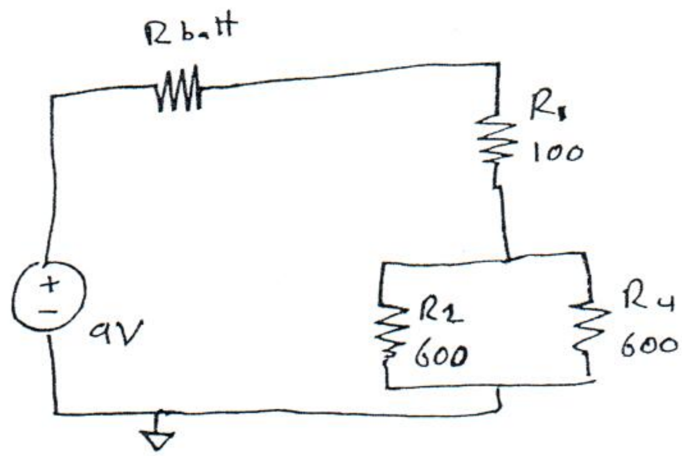
b.)



The circuit branch between point A and ground that has R_2 and R_4 on it has $R_{tot} = R_2 + R_4 = 300\Omega$.

$$i_{branch} = \frac{V_A}{R_{tot}} = \frac{6\,V}{300\,\Omega} = 20\,mA$$

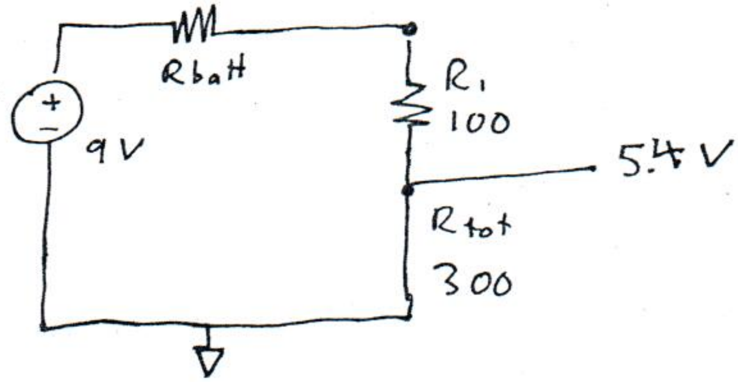
c.)



d.) The equivalent resistance of R_2 and R_4 together is

$$R_{tot} = \frac{R_2 R_4}{R_2 + R_4} = \frac{600^2}{1200} = 300$$

therefore we draw the equivalent circuit:



$$V_{out} = 5.4V = 9V \cdot \frac{300\Omega}{300\Omega + (100\Omega + R_{batt})}$$

$$300\Omega + 100\Omega + R_{batt} = \frac{9V}{5.4V} \cdot 300\Omega$$

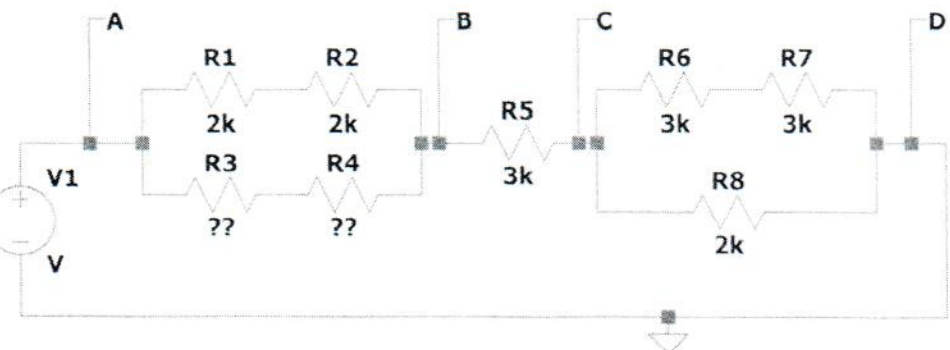
$$R_{batt} = \frac{9}{5.4} \cdot 300\Omega - 100\Omega - 300\Omega = 100\Omega$$

e.) This is more than usual; typical values are a few ohms or tenths of an ohm.

f.) A probe with lower input resistance would effectively place a third resistor in parallel with R_2 and R_4, lowering R_{tot} and therefore lowering V_{out}.

II. Resistor Combinations, concepts and miscellaneous (20 points) *Note: Page 2 of this quiz has background information. The crib sheet may also be useful.*

The following circuit consists of 8 resistors and there are 4 voltage markers at points A, B, C, and D. V1 is a dc voltage source which can be created using W1 of the instrumentation board. **Note:** several of the following questions are independent of each other, but not all.



a. (4 pt) It is desired to have the effective resistance of R3 combined with R4 to be 8kΩ. Using the table of standard 5% resistor values pick resistors for R3 and R4. State the resistance of each. These have 5% tolerance, what is the 4-band color code for each resistor? You should note that 8k isn't a standard value.

Four solution: 6.8k / 1.2k; 6.2k // 1.8k; 5.6k // 2.4k; 4.7k // 3.3k

R3 value: 6.2 Color bands: Blue, red, red, gold

R4 value: 1.8 Color bands: Brown, Violet, red, gold

can switch which is R3 or R4

b. (3 pts) If the voltage is measured to be 1V at point D (C), what is the current through R7, R8, and R5? Be sure to include units on all answers.

$I(R7) = \frac{1V}{R6+R7} = \frac{1}{6k} = 0.17mA$, 167µA

$I(R8) = \frac{1}{2k} = 0.5mA$

$I(R5) = I(R7) + I(R8) = 0.67mA$

I(R7) 0.17mA

I(R8) 0.5mA

I(R5) 0.67mA

c. (2 pts) What is the equivalent resistance between points A and B, Call this R_{AB}.

$R_{AB} = (R1+R2) // (R3+R4) = \frac{4k \cdot 8k}{4k+8k} = \frac{32}{12}k = 2.67k$

R_{AB} 2.67kΩ

d. (2 pts) What is the equivalent resistance between points C and D, Call this R_{CD}.

$R_{CD} = \underbrace{(R6+R7)}_{6k} // R8 = \frac{6k \cdot 2k}{6k+2k} = \frac{12}{8}k = 1.5k$

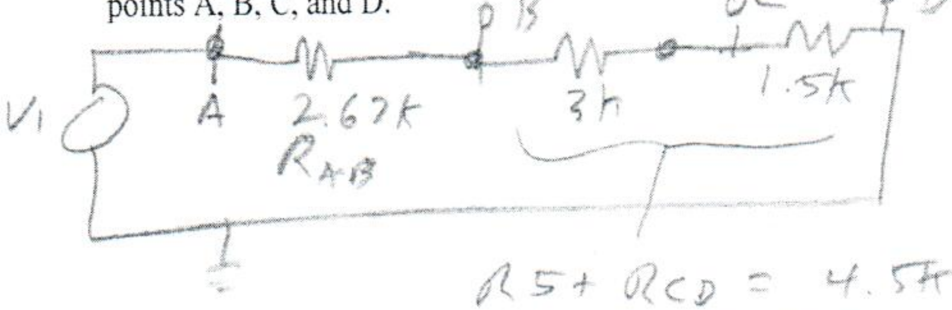
R_{CD} 1.5kΩ

R total 8000

R1	colors	R2	colors	Rtotal
6800	blue, gray, red, gold	1200	brown, red, red, gold	8000
6200	blue, red, red, gold	1800	brown, violet, red, gold	8000
5600	green, blue, red, gold	2400	red, yellow, red, gold	8000
4700	yellow, violet, red, gold	3300	orange, orange, red, gold	8000

Prop II a.

e. (2 pts) Redraw the circuit with 3 resistors, R_{AB}, R5, R_{CD}. Label the resistor values and mark points A, B, C, and D.

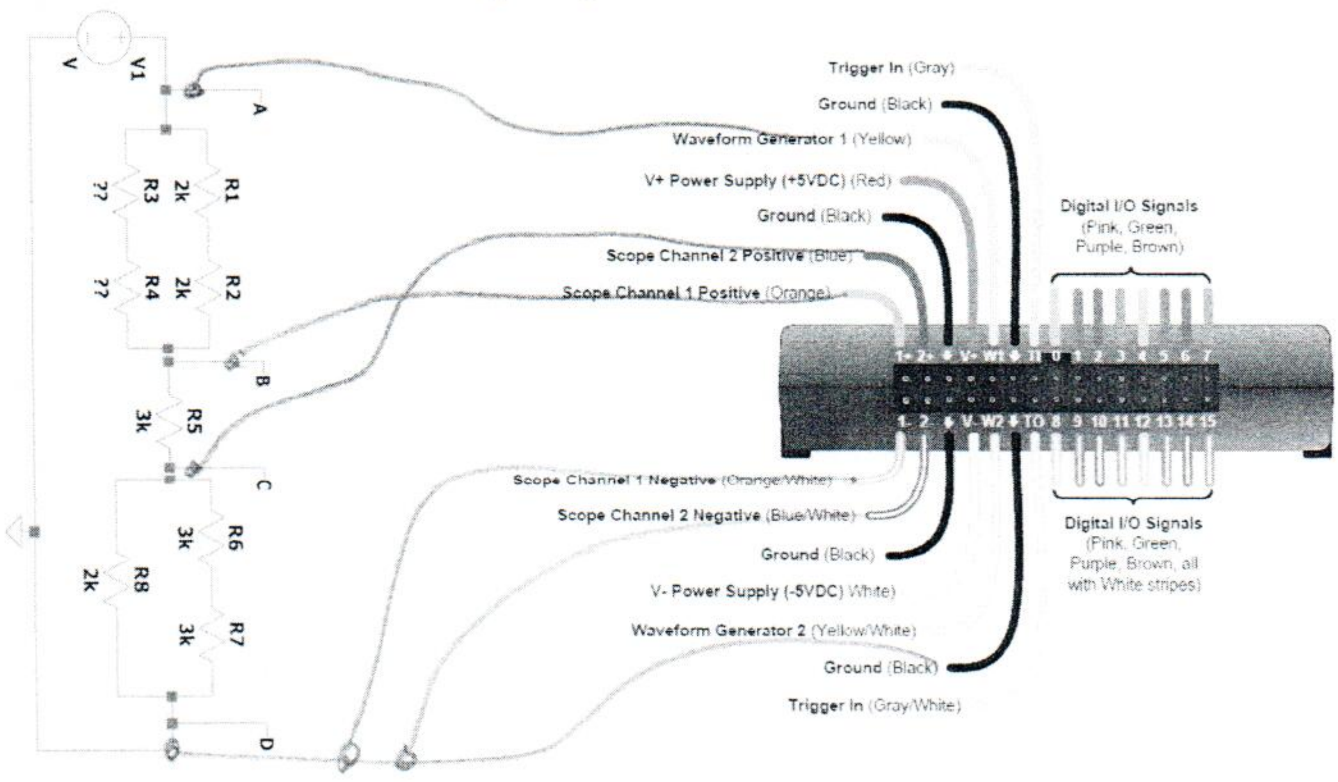


$R5 + R_{CD} = 4.5k$

f. (2 pts) Using the figure you drew for part e., if V1=5V what is the voltage at point B?

$$V_B = V_1 \cdot \frac{R5 + R_{CD}}{R_{AB} + R5 + R_{CD}} = 5 \cdot \frac{4.5}{2.67 + 3 + 1.5} = 3.1V$$

g. (5 pts) Wire this circuit by drawing lines on the figure below. Use the wave gen/signal gen to provide a signal for V1, which is point A. Use channel 1 to measure the signal at point B and use channel 2 to measure the signal at point C.



Draw lines between the 2 figures to indicate wires. Both the M2K and the AD2 have the same wiring and colors. W1 is called Signal Generator on the M2K and Waveform Generator on the AD2.

(3) a.)

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_{eq}}{Z_{eq} + R}$$

$$Z_{eq} = \frac{Z_C Z_L}{Z_C + Z_L} = \frac{\frac{1}{j\omega C} \cdot j\omega L}{j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{j\omega L}{1 - \omega^2 LC}$$

$$H(j\omega) = \frac{\dfrac{j\omega L}{1-\omega^2 LC}}{\dfrac{j\omega L}{1-\omega^2 LC} + R} = \frac{j\omega L}{R(1-\omega^2 LC) + j\omega L}$$

b.) At low frequency,

$$H(j\omega) = \frac{j\omega L}{R - \cancel{R\omega^2 LC} + \cancel{j\omega L}} = \frac{j\omega L}{R}$$

$$|H(j\omega)| = \frac{\omega L}{R} \qquad \angle H(j\omega) = 90^\circ$$

At high frequency,

$$H(j\omega) = \frac{j\omega L}{\cancel{R} - R\omega^2 LC + \cancel{j\omega L}} = \frac{-j}{\omega RC}$$

$$|H(j\omega)| = \frac{1}{\omega RC}$$

$$\angle H(j\omega) = -90^\circ$$

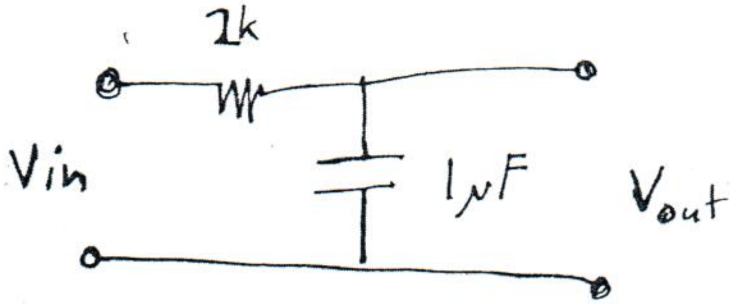
c.)

The magnitude of the transfer function is low at low frequencies due to the low impedance of the inductor and also low at high frequencies due to the low impedance of the capacitor. It will reach its maximum amplitude at an intermediate frequency. Therefore it is a band pass filter.

d.)

Z1	Z2	Low-pass	High-pass
R	C	Y	N
C	R	N	Y
R	R	Y	Y
R	L	N	Y
L	R	Y	N

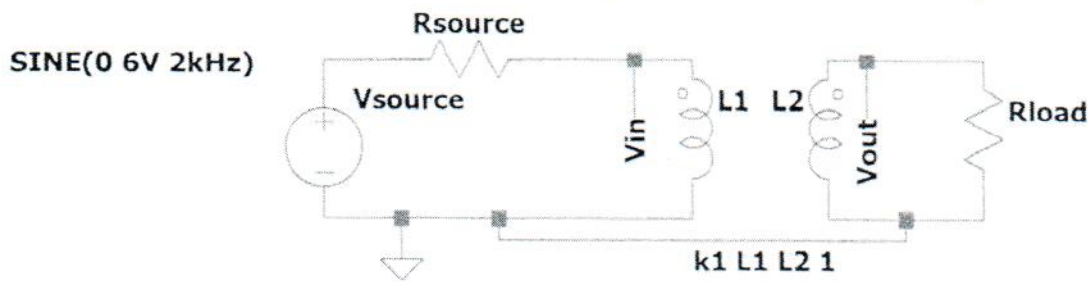
e.)



$$\omega_c = \frac{1}{RC} = \frac{1}{(2\times10^3)(1\times10^{-6})} = 500 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{500}{2\pi} = 79.6 \text{ Hz}$$

IV – Phasors and Transformers (20 points)



1) Assume L1 and L2 form an ideal transformer with full coupling. The transformer has these specifications:
   a=1/3, L1= 300mH

a. Determine the value of L2 that will match the allow the result in the transformer matching these specs: (2pts)

$a = \sqrt{\frac{L_2}{L_1}}$ $L_2 = a^2 L1 = \left(\frac{1}{a}\right)(300m\,H) = 33.3\,mH$

L2= 33 mH

b. Determine the ratios Vout/Vin, and Iout/Iin (4 pts)

$V_{out}/V_{in} = a = 1/3$

$I_{out}/I_{in} = \frac{1}{a} = 3$

Vout/Vin= 1/3 or 0.33

Iout/Iin= 3

c. Find the value of Rload that results in Rin = 3kΩ (Rin is Vin/Iin) (3 pts)

$R_{in} = \frac{R_{load}}{a^2}$ $R_{load} = a^2 R_{in} = \left(\frac{1}{3}\right)^2 (3k)$

$= 333\,\Omega$

Rload= 333 Ω

d. Given Rsource = 500Ω and $Vsource = 6\text{Sin}(2\pi * 2000t)$ What is the time domain value of Vin? Give the answer in the form of: **v(t)=V₁Sin(ωt+θ₁)** (2 pts)

Vin = 5.14 sin(2π * 2000t) ✓

$V_{in} = \frac{R_{in}}{R_{in} + R_{source}} \cdot V_{source}$

$\frac{R_{in}}{R_{in} + R_{src}} = \frac{3k}{3k + 500} = 0.86$

$V_{in} = (0.86)(6\sin(2\pi * 2000t))$

$= 5.14\sin(2\pi * 2000t)\,V$

$= 5.14\sin(2\pi * 2000t + 0°)\,V$

$= 5.14\sin(1.26\times10^4 t)\,V$

any form

e. The ideal transformer model assumes that self-inductance L1 and L2 are infinite, $|j\omega L1|$ and $|j\omega L2|$ approach infinity. On the practical side the transformer is close to ideal if $|j\omega L1| > 10 * R_{source}$ and $|j\omega L2| > 10 * R_{load}$. Just looking at L1, will this transformer be close to the ideal? If not, how could the transformer be changed to approach ideal but yet have the same value for a? (3pts) $|j\omega L1| = (2\pi)(2000)(0.3) = 3770\Omega$

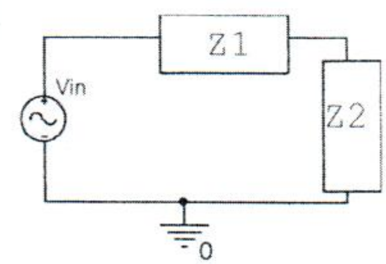
(No) $10 * R_{source} = 500\Omega$ $3770 \not> 10 * R_{source}$ (NOT)   Is it near ideal? Yes or No

If not say in words how the transformer could be changed to approach ideal but without changing the value of a. Increase L1 by at least a factor of $\frac{5000}{3770} * L1$, then change L2 by same factor to keep from changing a

sufficient to say to increase L1 to meet the criteria + then increase L2 by same percentage.

2. Phasors: This circuit shown has 2 complex impedances, Z1 and Z2, connected as shown.

Given: $Vin = 6V\angle 0^o$ and the voltage across Z2 is measured to be $Vz2 = 4V\angle -45^o$ (This format is a magnitude and a phase angle.)



a. Write Vin and Vz2 in Cartesian form. (2pts)

$V_{in} = 6 + 0j = 6V$

$V_{Z2} = 4\cos(-45^\circ) + j4\sin(-45^\circ) = 2.8 - j2.8\ V$

b. Determine Vz1, the voltage across Z1 in Cartesian and polar form (~~2pts~~ 3pts)

KVL $V_{in} = V_{Z1} + V_{Z2}$ $6V = (\underbrace{a + jb}_{V_{Z1}} + 2.8 - j2.8)\ V$

$a = 6 - 2.8 = 3.2$

$b = 0 - (-2.8) = 2.8$ $V_{Z1} = 3.2 + j2.8 = 4.25\angle 41^\circ$

$\angle = \tan^{-1}\left(\frac{2.8}{3.2}\right)$

3. Give the names of 2 of the people teaching this course. This can be first names or last names and can be the professors, teaching assistants, or undergraduate student assistants. Spelling doesn't count. Using their Discord name is also valid. (1pt)

Many correct answers.