ENGR-2300

Electronic Instrumentation

Quiz 1

Spring 2023

Print Name ________________ RIN ________________

Section ____

I have read, understood, and abided by the Collaboration and Academic Dishonesty statement in the course syllabus. The work presented here was solely performed by me.

Signature: ______________________________

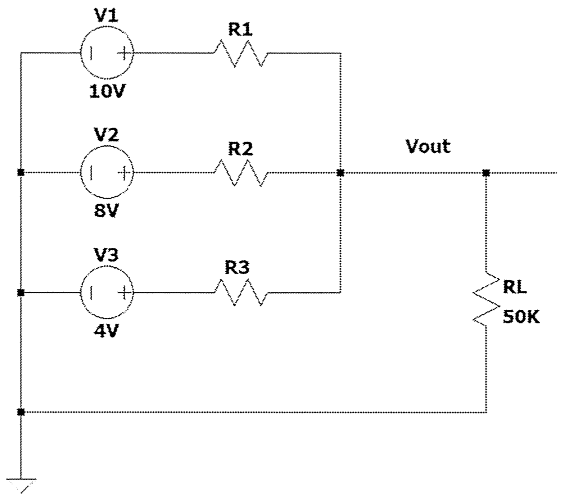
Date: ________

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification. Unless otherwise stated in a problem, provide 3 significant digits in answers. Read the entire quiz before answering any questions. Also, it may be easier to answer parts of questions out of order.

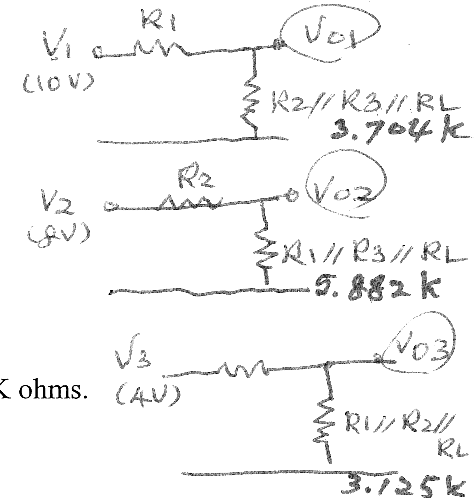
I. Circuit Analysis (16 points)

DC Voltage Divider Questions

The following circuit has three voltage sources:



Calculate sources one by one



1. (3 pts) Find Vout given the resistances R1 = 10K, R2 = 5K, and R3 = 20K ohms.

$$V_{o1} = 10V \cdot \frac{3.704}{10 + 3.703} = 0.541\ V$$

$$V_{o2} = 8V \cdot \frac{5.882}{5 + 5.882} = 4.324\ V$$

$$V_{o3} = 4 \cdot \frac{3.125}{20 + 3.125} = 2.703\ V$$

$$V_{out} = V_{o1} + V_{o2} + V_{o3} = 7.568\ V$$

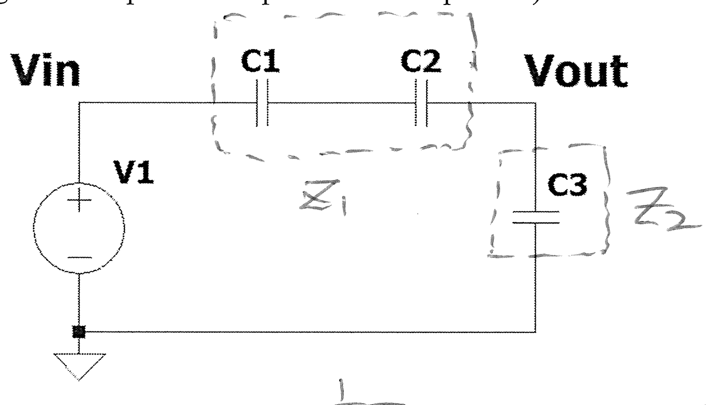
2. (2 pts) Find the current through R1, R2, R3 and RL.

$$I_{R1} = \frac{V_1 - V_{R1}}{R_1} = \frac{10 - 7.568}{10\ K} = 0.243\ mA$$

$$I_{R2} = \frac{V_2 - V_{R2}}{R_2} = \frac{8 - 7.568}{5\ K} = 0.086\ mA$$

$$I_{R3} = \frac{V_3 - V_{R3}}{R_3} = \frac{4 - 7.568}{20\ K} = (-)0.178\ mA$$

3. (4 pts) The following circuit has a voltage dividing function. Derive the transfer function equation (Vout/Vin) in terms of V1, C1, C2 and C3. (Hint: even though Vin is a DC source, still approach this question using the concept of the impedance of a capacitor.)



series connection

$$Z_1 = C_1 + C_2 = \frac{C_1 C_2}{C_1 + C_2}$$

$$H = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{1}{j\omega C_3}}{\frac{1}{j\omega\left(\frac{C_1 C_2}{C_1 + C_2}\right)} + \frac{1}{j\omega C_3}} \begin{matrix} \times j\omega \\ \times j\omega \end{matrix}$$

$$= \frac{1/C_3}{1/\left(\frac{C_1 C_2}{C_1 + C_2}\right) + \frac{1}{C_3}} \begin{matrix} \times C_3 \\ \times C_3 \end{matrix} = \frac{1}{\frac{C_3(C_1 + C_2)}{C_1 C_2} + 1}$$

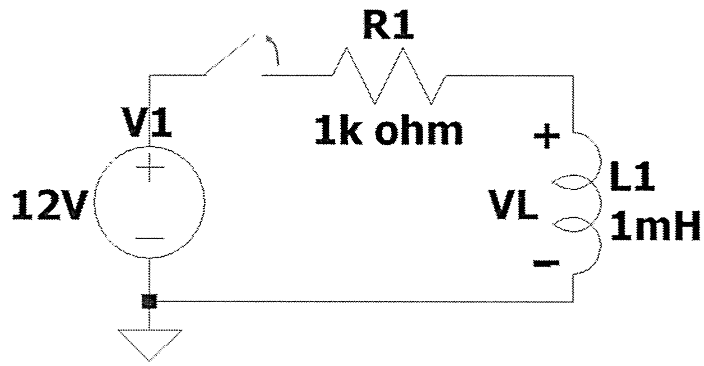
$$= \frac{1}{\frac{C_1 C_3 + C_2 C_3 + C_1 C_2}{C_1 \cdot C_2}} = \boxed{\frac{C_1 \cdot C_2}{C_1 C_3 + C_2 C_3 + C_1 C_2}}$$

4. (1 pts) What is the value of Vout for the case of V1 = 12V, C1 = 5uF, C2 = 2uF and C3 = 7uF?

$$V_{out} = 12V \cdot \left(\frac{5 \cdot 2}{5 \cdot 7 + 2 \cdot 7 + 5 \cdot 2}\right)$$

$$= 12V \cdot \frac{10}{59} = \boxed{2.034V}$$

The following circuit has a 12V DC voltage source connected to a resistor (1k ohm) and an inductor (1mH). It has been switched ON for a while and switches OFF at t=0.



5. (3 pts) What is the current through R1 (1K ohm) before the circuit switches OFF?

$$V = IR \Rightarrow I = \frac{V}{R}$$

$$\frac{12V}{1K} = \boxed{12\ mA}$$

6. (3 pts) At t=0, the circuit is switched OFF, and it takes 10us to settle down to zero current. What is VL (voltage across the inductor)?

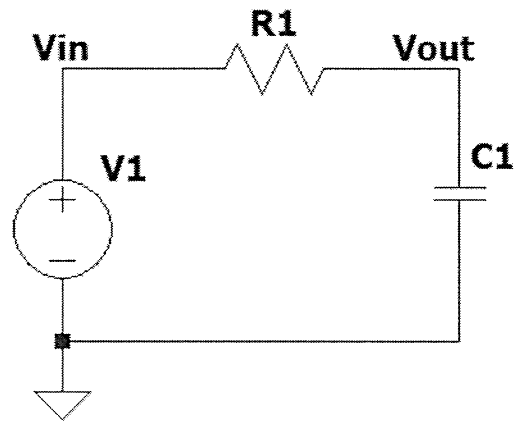
$$I_f - I_{int} = 0 - 12mA = -12mA$$

$$V = L \cdot \frac{dI}{dt} = (1mH) \cdot \frac{-12mA}{10\mu s}$$

$$= \boxed{-1.2\ V}$$

II. Filters (24 points)

RC Circuit Questions



1. (2 pts) V1 is an AC source with varying frequency (ω). Find the transfer function (Vout/Vin) in terms of ω, R1 and C1.

$Z_1 = R_1$

$Z_2 = \frac{1}{j\omega C_1}$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} \times \left(\frac{j\omega C_1}{j\omega C_1}\right)$$

$$= \frac{1}{j\omega R_1 C_1 + 1}$$

2. (1 pt) What is the simplified transfer function equation at two extreme frequencies (low: ω → 0, and high: ω → ∞)?

$$H(j\omega) = \frac{1}{j\omega R_1 C_1 + 1}$$

@ $\omega \to 0$ $\quad H(j\omega) = 1$

@ $\omega \to \infty$ $\quad H(j\omega) = 0$

3. (1 pt) Find the corner frequency of this RC circuit for the case: R1 = 100 ohm, C1 = 0.1uF.

$$f_{corner} = \frac{1}{2\pi R_1 C_1}, \quad R_1 = 100\,\Omega, \quad C_1 = 0.1\,\mu F$$

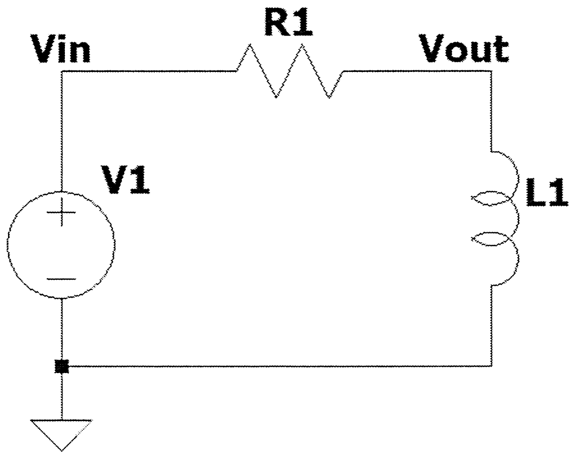
$$= \frac{1}{2\pi \cdot (100)(10^{-7})} = 15.195\ KHz$$

4. (1 pt) Is this a high pass or low pass filter?

see question #2

Low pass filter

RL Circuit Questions



1. (2 pts) V1 input is AC with varying frequency (ω). Find the transfer function (Vout/Vin) in terms of ω, R1 and L1.

$Z_1 = R_1$

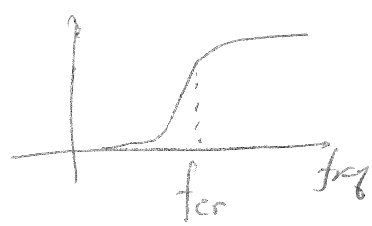
$Z_2 = j\omega L_1$

$$H(j\omega) = \frac{j\omega L_1}{R_1 + j\omega L_1}$$

2. (1 pt) What is the simplified transfer function equation at two extreme frequencies (low: ω → 0, and high: ω → ∞).

@ $\omega \to 0$ $H(j\omega) = 0$

@ $\omega \to \infty$ $H(j\omega) = 1$



3. (2 pts) Find the corner frequency of this RL circuit for the case: R1= 100 ohm, L1 = 0.4H.

$$f_{corner} = \frac{R_1}{2\pi \cdot L_1} = 40\ Hz$$

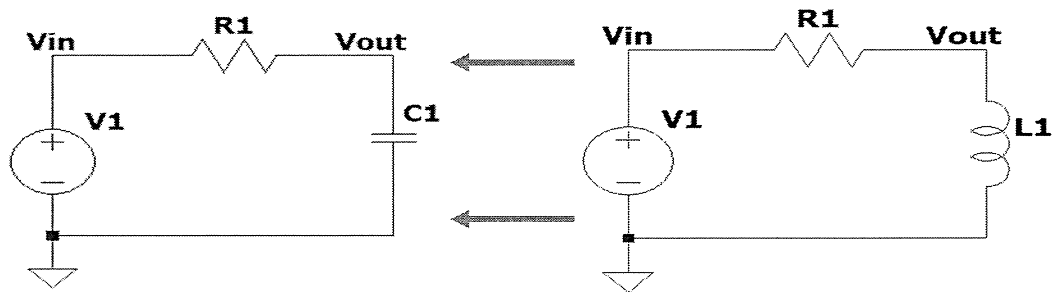
4. (1 pt) Is this a high pass or low pass filter?

~~freq~~ from soln #2

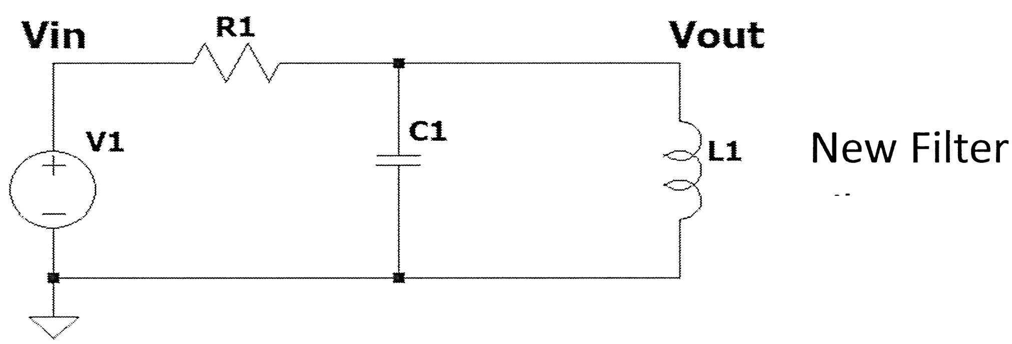
High pass filter

RLC Circuit Questions

Now we want to build a new filter by cascading the filters above.



Cascade two filters into one by sharing Vin source & R1



1. (4 pts) V1 input is the same: AC with varying frequency (ω). Find the transfer function (Vout/Vin) of the new filter in terms of ω, R1, C1, and L1.

$Z_1 = R_1, \quad Z_2 = \frac{1}{j\omega C_1} \parallel j\omega L_1$

$$= \frac{\frac{j\omega L_1}{j\omega C_1}}{\frac{1}{j\omega C_1} + j\omega L_1} \times \left(\frac{j\omega C_1}{j\omega C_1}\right)$$

$$= \frac{j\omega L_1}{1 - \omega^2 L_1 C_1}$$

$$H(j\omega) = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{j\omega L_1}{1-\omega^2 L_1 C_1}}{R_1 + \frac{j\omega L_1}{1-\omega^2 L_1 C_1}} \times \left(\frac{1-\omega^2 L_1 C_1}{1-\omega^2 L_1 C_1}\right)$$

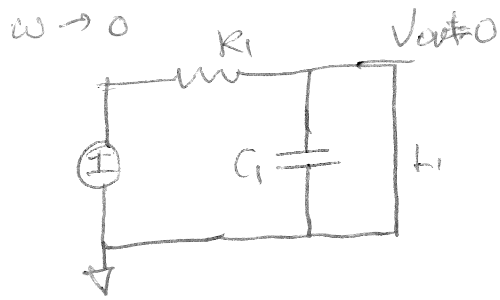
$$= \frac{j\omega L_1}{R_1(1-\omega^2 L_1 C_1) + j\omega L_1}$$

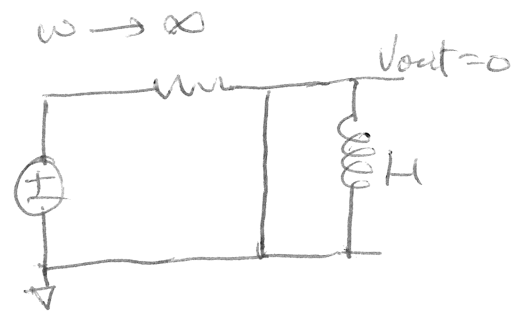
2. (1 pt) What is the simplified transfer function equation at two extreme frequencies (low: ω → 0, and high: ω → ∞).

@ $\omega \to 0$ $H(j\omega) = 0$

@ $\omega \to \infty$ $H(j\omega) \to 0$

3. (1 pt) Draw simplified circuit diagrams for the two extreme cases (low: ω → 0, and high: ω → ∞).





4. (2 pts) Derive the equation for the resonant frequency of this filter. Show your reasoning and assumptions in derivation process. (Hint: at what ω does the magnitude of the transfer function have a maximum? $|H_{max}| = 1$)

Want $H(j\omega) \to$ max

$$H(j\omega) = \frac{j\omega L_1}{R_1(1-\omega^2 L_1 C_1) + j\omega L_1}$$

condition:

$$1 - \omega^2 L_1 C_1 = 0$$

$$\omega^2 L_1 C_1 = 1$$

$$\omega^2 = \frac{1}{L_1 C_1}$$

$$\omega = \frac{1}{\sqrt{L_1 C_1}}$$

@ resonance freq.

$$H(j\omega) = \frac{j\omega L_1}{j\omega L_1}$$

5. (2 pts) What is the phase angle of the transfer function at the resonant frequency? Show the calculation process.

@ resonance freq.

$$H(j\omega) = \frac{j\omega L}{j\omega L} = 1$$

$$\angle H(j\omega) = 0^\circ$$

6. (1 pt) Now we have two corner frequencies. If the values of R, C and L have the same values as in previous questions (R1= 100 ohm, L1=0.4H and C1=0.1uF), what kind of filter is this?

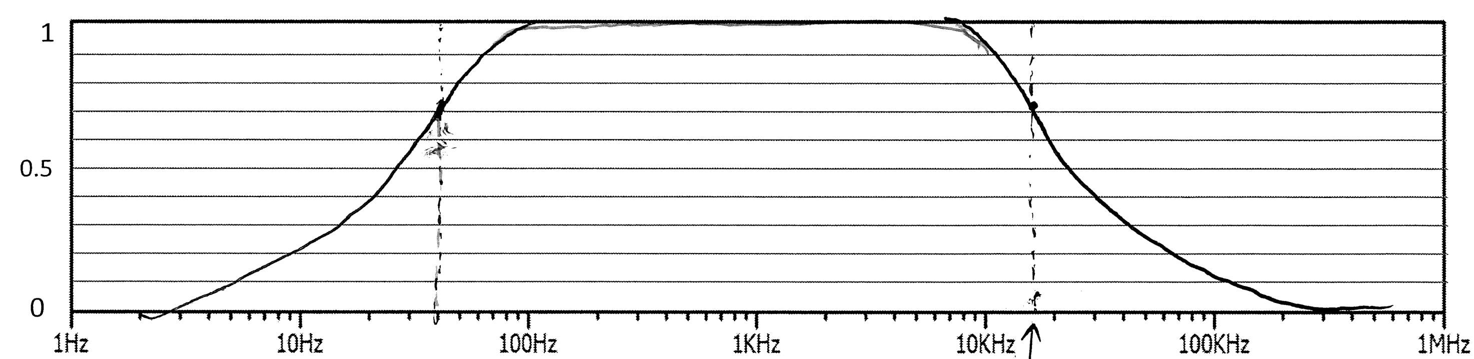
   a. Low pass filter

   b. High pass filter

   c. Band pass filter — correct Ans.

   d. ~~Band reject filter~~

7. (2 pts) Draw the transfer function in a linear plot, specifying the two corner frequencies on the x-axis (frequency axis). A rough drawing is OK: H(ω) max is 1 on the y-axis. Labeling of the corner frequency and the relative amplitude on a linear scale on the y-axis is important.

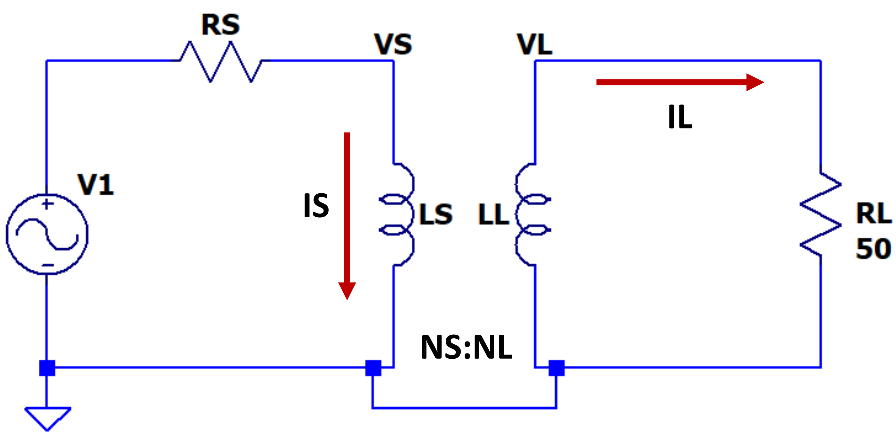


$f_{c1} = 40\,Hz$ $f_{c2} = 15.915\,kHz$

III. Inductors, Transformers, and Phasors (20 points)

Inductors and Transformers

You are given a transformer connected to a source and load circuit, as shown below. You know nothing about the transformer except that the wire is wound around a ferrite core.



1. (4 pts) With an AC voltage applied to the circuit, you measure VS = 4V and IS = 320mA. If $Z_{in} = \frac{VS}{IS}$, what is the value of a?

$$Z_{in} = \frac{Z_L}{a^2} \rightarrow a = \sqrt{\frac{Z_L}{Z_{in}}} = \sqrt{Z_L \frac{IS}{VS}} = \sqrt{50\Omega \cdot \frac{320 \times 10^{-3}\text{A}}{4\text{V}}} = \mathbf{2}$$

[4 pts: 2 pts for correct approach/equation (Zin); 1 pt for using VS and IS; 1 pt for correct calculation]

2. (2 pts) If you remove RL such that the load is now an open circuit, no current will flow through the load circuit and no voltage will form across the load side's inductor. In this case you can measure the impedance of LS, the inductor on the source side. At a frequency of f = 10kHz, you measure the impedance of the source side's inductor to be Z_{LS} = j188.5Ω. What is the inductance LS?

$$Z_{LS} = j\omega LS = j \cdot 2\pi \cdot f \cdot LS \rightarrow LS = \frac{Z_{LS}}{j2\pi f} = \frac{j188.5}{j2\pi \cdot 10^4 \text{ kHz}} = 3\text{mH}$$

[2 pts: 1 pt for correct approach/equation; 1 pt for correct calculation]

3. (3 pts) If the toroidal, ferrite core has an inductance factor of L_f = 10.6 µH/turns^2, how many wire turns are in inductor LS (round up to the nearest whole number)?

$$LS = L_f NS^2 \rightarrow NS = \sqrt{\frac{LS}{L_f}} = \sqrt{\frac{3 \times 10^{-3} H}{10.6 \times 10^{-6} H/turns^2}} = 17 \text{ turns}$$

[3 pts: 2 pts for correct approach/equation; 1 pt for correct calculation]

4. (2 pts) How many wire turns are in inductor LL (round up to the nearest whole number)?

$$NL = a \cdot NS = 2 \cdot 17 = 34 \text{ turns}$$

[2 pts: 1 pt for correct approach/equation; 1 pt for correct calculation]

5. (3 pts) How much current is flowing through RL, given VS = 4V as in question 1?

$$IL = \frac{VL}{RL} = \frac{NL}{NS} \cdot \frac{VS}{RL} = a \cdot \frac{VS}{RL} = 2 \cdot \frac{4V}{50\Omega} = 160 \text{ mA}$$

[4 pts: 2 pts for correct approach/equations (Ohm's law); 1 pt for correct use of *a*; 1 pt for correct calculation]

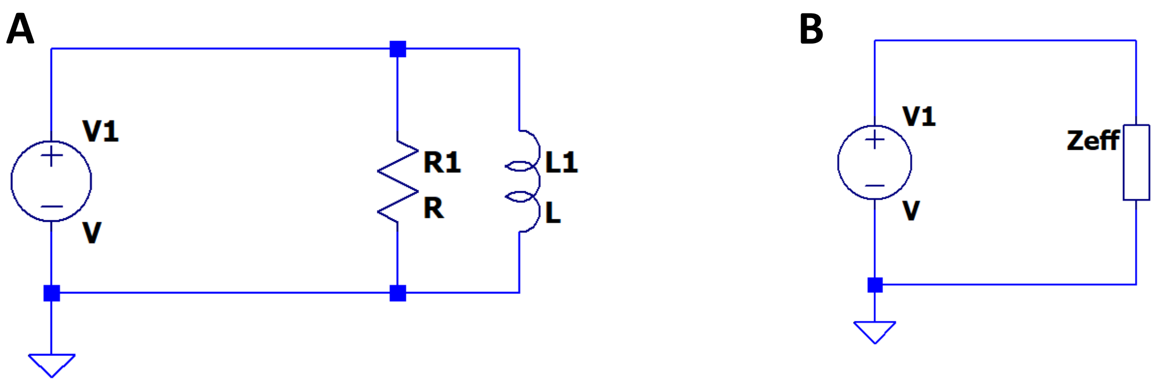
6. (2 pts) True or false: for every transformer, there exists a minimum frequency below which the transformer will not behave as the equations predict.

True

[2 pts: correct answer]

Phasors

7. (4 pts) If you combine the impedances in circuit A below into an effective impedance *Zeff*, as shown in circuit B, what is the magnitude of *Zeff*?



$$Z_{eff} = Z_{C1} || Z_{L1} = \frac{R \cdot (j\omega L_1)}{R + j\omega L_1} = \frac{j\omega L_1}{1 + \frac{j\omega L_1}{R}}$$

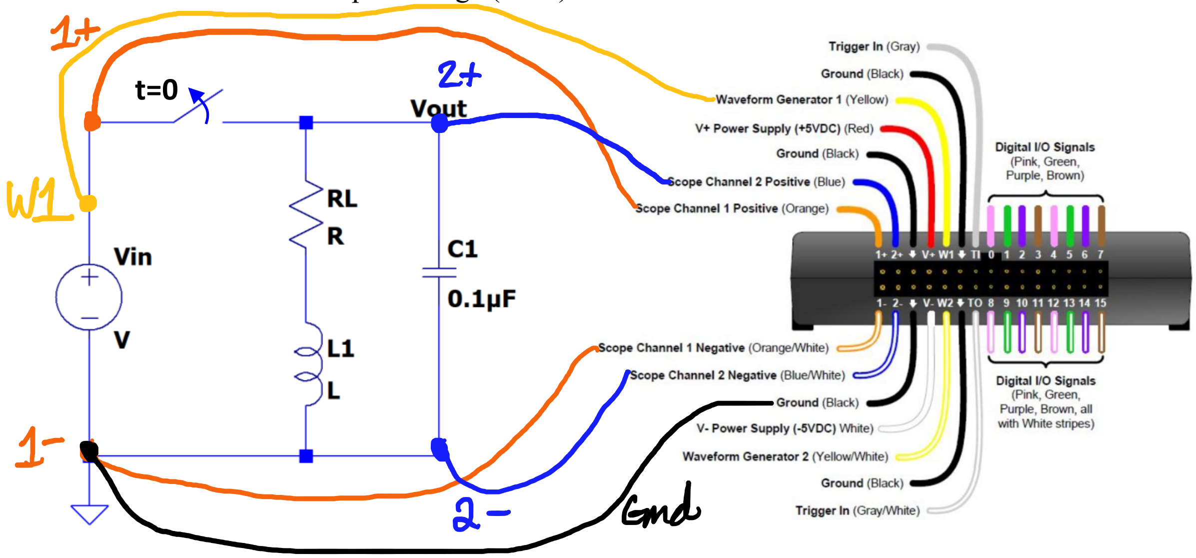
$$|Z_{eff}| = \sqrt{\left(\frac{j\omega L_1}{1 + \frac{j\omega L_1}{R}}\right) \cdot \left(\frac{-j\omega L_1}{1 - \frac{j\omega L_1}{R}}\right)} = \frac{\omega L_1}{\sqrt{1 + \frac{\omega^2 L_1}{R}}}$$

[4 pts: 2 pts for correct Zeff; 2 pts for correct magnitude]

IV. Instrumentation Fundamentals and Concepts (20 points)

1. (6 pts) In experiment 3, you estimated the inductance of your homemade inductor by measuring the resonant frequency of a circuit similar to the one below. By drawing lines from the channels to locations in the circuit, show how you would:

   a. Source the voltage Vin from the instrumentation board
   b. Measure the input voltage (Vin)
   c. Measure the output voltage (Vout)



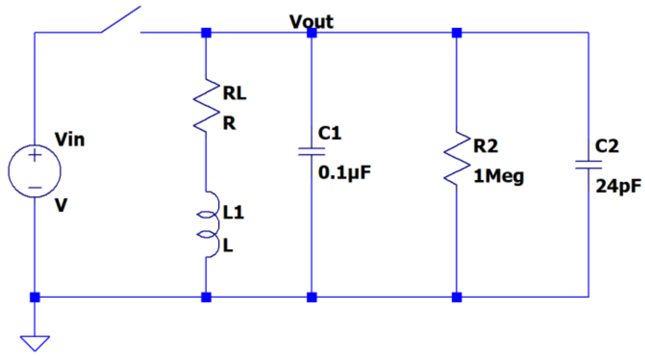
[6 pts: 2 pts each for correct locations of scope channel 1 and 2 wires; 1 pt for correct waveform generator wire location; 1 pt for ground location]

2. (2 pts) If you measured an oscillation frequency of $f_0 = 5$ kHz, what would be your estimate for the inductance of inductor L1 (to the nearest 0.01 mH)?

$$f_0 = \frac{1}{2\pi\sqrt{L_1 C_1}} \rightarrow L_1 = \frac{1}{(2\pi)^2 f_0^2 C_1} = \frac{1}{(2\pi)^2 \cdot (5 \times 10^3 \text{ Hz})^2 \cdot 0.1 \times 10^{-6} \text{ F}} = 10.13 \text{ mH}$$

[2 pts: 1 pt for correct equation/approach; 1 pt for correct calculation]

3. (2 pts) If the non-idealities of the instrumentation board are taken into account, the input impedance of the board must also be modeled, resulting in the following circuit.



In this case, the value of the input resistance is R2 = 1MΩ and the input capacitance is C2 = 24pF. If you needed to find these values specifically for the M2K or the Analog Discovery 2, where would you find them?

You would need to look at the datasheet for the M2K or Analog Discovery 2.

[2 pts for correct answer]

4. (2 pts) To determine the effect of the input capacitance on your inductance estimate, combine C1 and C2 in the circuit in IV.3 into a single, effective capacitance and recalculate L1, using the same resonant frequency as in IV.2 (to the nearest 0.01 mH).

Since capacitances in parallel combine like resistors in series (they simply add), we get a circuit that has one capacitor where C1 is, but with the value $C_{eff} = C1 + C2$, so

$$L_1 = \frac{1}{(2\pi)^2 f_0^2 (C_1 + C_2)} = \frac{1}{(2\pi)^2 \cdot (5 \times 10^3 \text{ Hz})^2 \cdot (0.1 \times 10^{-6} \text{ F} + 24 \times 10^{-12} \text{ F})} = 10.13 \text{ mH}$$

[2 pts: 1 pt for correct Ceff; 1 pt for correct calculation]

5. (2 pts) Given that the input capacitance is fixed at 24pF for a given instrumentation board, under what circumstances for C1 would you need to be concerned that the input capacitance of the board is significantly affecting your measurement?

If C1 approaches the value of C2 (on the order of 10s of picofarads), the input capacitance is no longer negligible compared with the capacitance of interest in the circuit.

[2 pts for valid answer]

Miscellaneous Concepts

6. (2 pts) Suppose that you calculate that the output voltage from a circuit should be $V_{out} = 500(1 + j)$ mV. What is the amplitude of the voltage that you expect to measure?

The voltage amplitude that would be measured is the magnitude of this voltage:

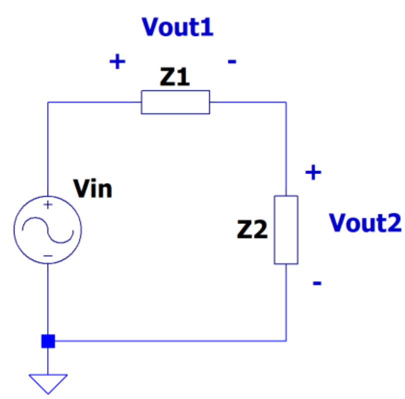
$$V_{meas} = |V_{meas}| = \sqrt{(500(1+j))(500(1-j))} = 500 \cdot \sqrt{2}\text{ mV} = 707.1\text{ mV}$$

[2 pts: 1 pt for correct approach; 1 pt for correct calculation]

7. (1 pt) Why doesn't it matter where the two inductor coils of a transformer are located on a ferrite core, as opposed to an air core?

Because the ferrite core confines or "conducts" the B field within it, the B field does not decay appreciably from one side of the core to the other (as it would in air), so no matter where the 2nd set of wire wrappings are on the core, it will pick up the B field generated on the primary side without much loss in its concentration.

8. (1 pt) Circle one: you run an AC sweep of the circuit to the right and determine that the circuit acts as a high-pass filter when you measure the voltage across Z2 (Vout2). What kind of filter would you have if you instead measured across Z1 (Vout1)?



a) high-pass filter

b) low-pass filter

c) band-pass filter

d) band-reject filter

e) Not enough information to determine the type of filter

9. (1 pt) Describe how you would measure the current flowing through a resistor using the M2K or Analog Discovery 2.

To measure a current, you need to measure the voltage across the resistor, then use a math channel to divide the voltage by the resistance of that resistor.

10. (1 pt) Circle one: what effect would replacing the ferrite core of an inductor with an air core have on the inductance of that inductor?

    a) decrease the inductance

    b) leave the inductance unchanged

    c) increase the inductance

    d) the inductance would be zero