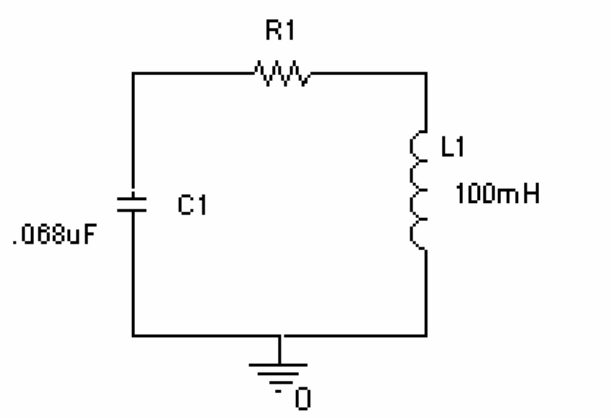
Questions on sine waves involving decaying sinusoids.

Fall 2004

3. Sinusoids (25 points)



The circuit above was created with Pspice. Note C1=0.068μF and L1=100mH.

a) On the following page are three plots. Each was created by assigning a different value to R1: R1=1K ohms, R1=100 ohms, and R1=0.1 ohms (essentially 0). Indicate which plot corresponds to which resistance: (A, B, or C) *(6 points)*

1K ohms: _____

100 ohms: _____

0.1 ohms _____

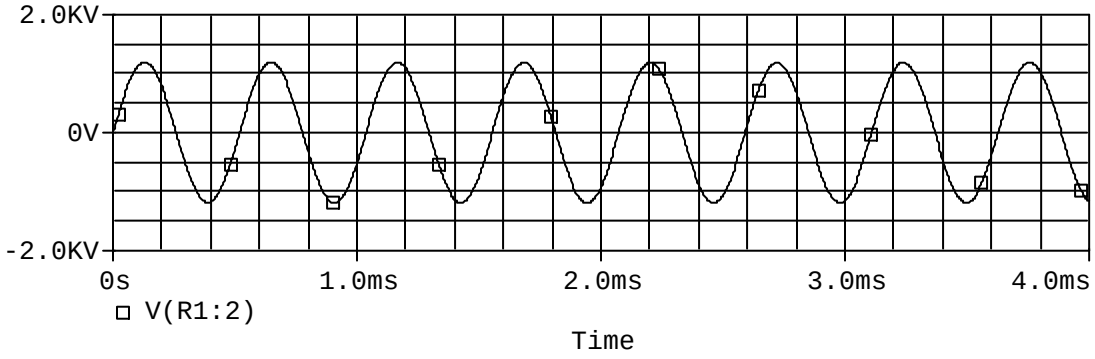
b) Find the resonant frequency:

i) The equation for resonant frequency is $\omega=1/(LC)^{1/2}$. Using the values of L and C given above, calculate the resonant frequency for this circuit. *(3 points)*

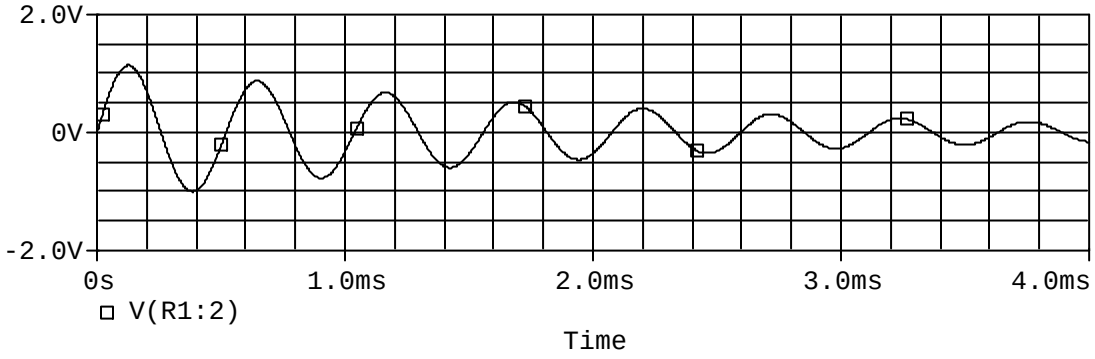
ii) Determine the resonant frequency of the circuit from the plots on the following page. *(3 points)*

iii) Demonstrate that the theoretical resonant frequency and the actual resonant frequency are consistent. *(2 points)*

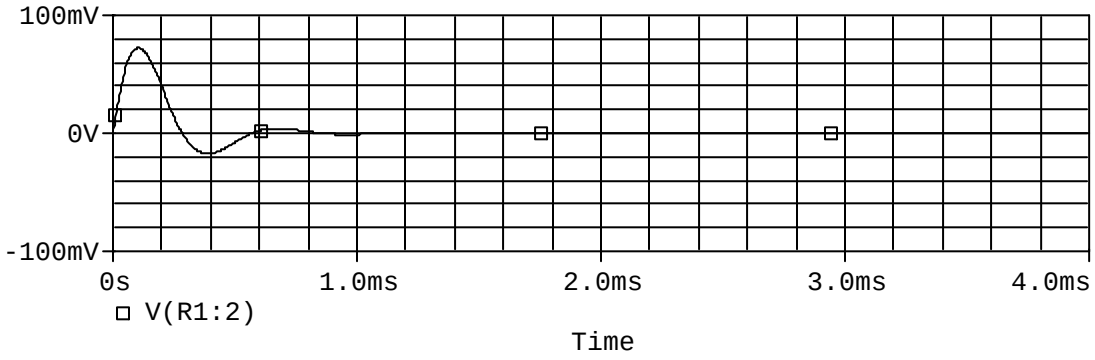
A.



B.



C.



c) Find the equations which model circuit behavior.

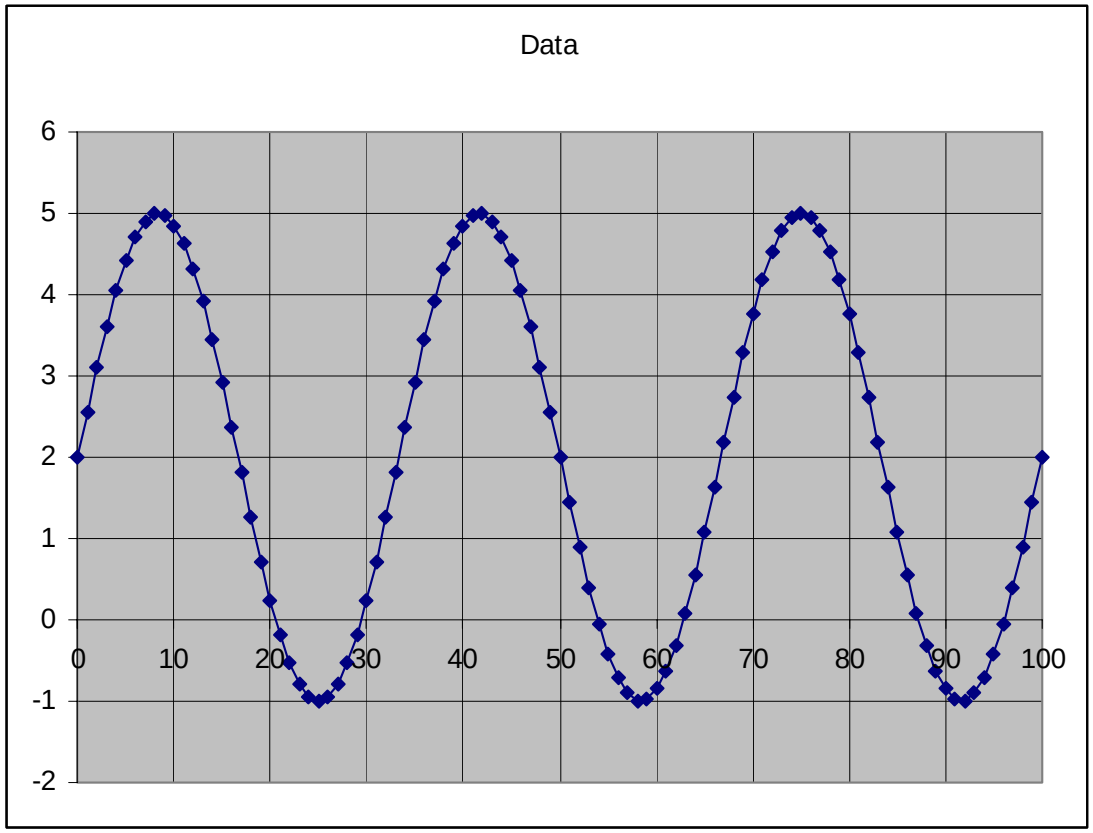
i) Write an equation for plot A in the form $v(t)=A\sin(\omega t+\phi)$ *(5 points)*

ii) Find the decay constant for plot B. *(6 points)*

Fall 2004 solution
(none available)

Spring 2004
4) Sinusoids (20 points)

The following data was created using Excel.



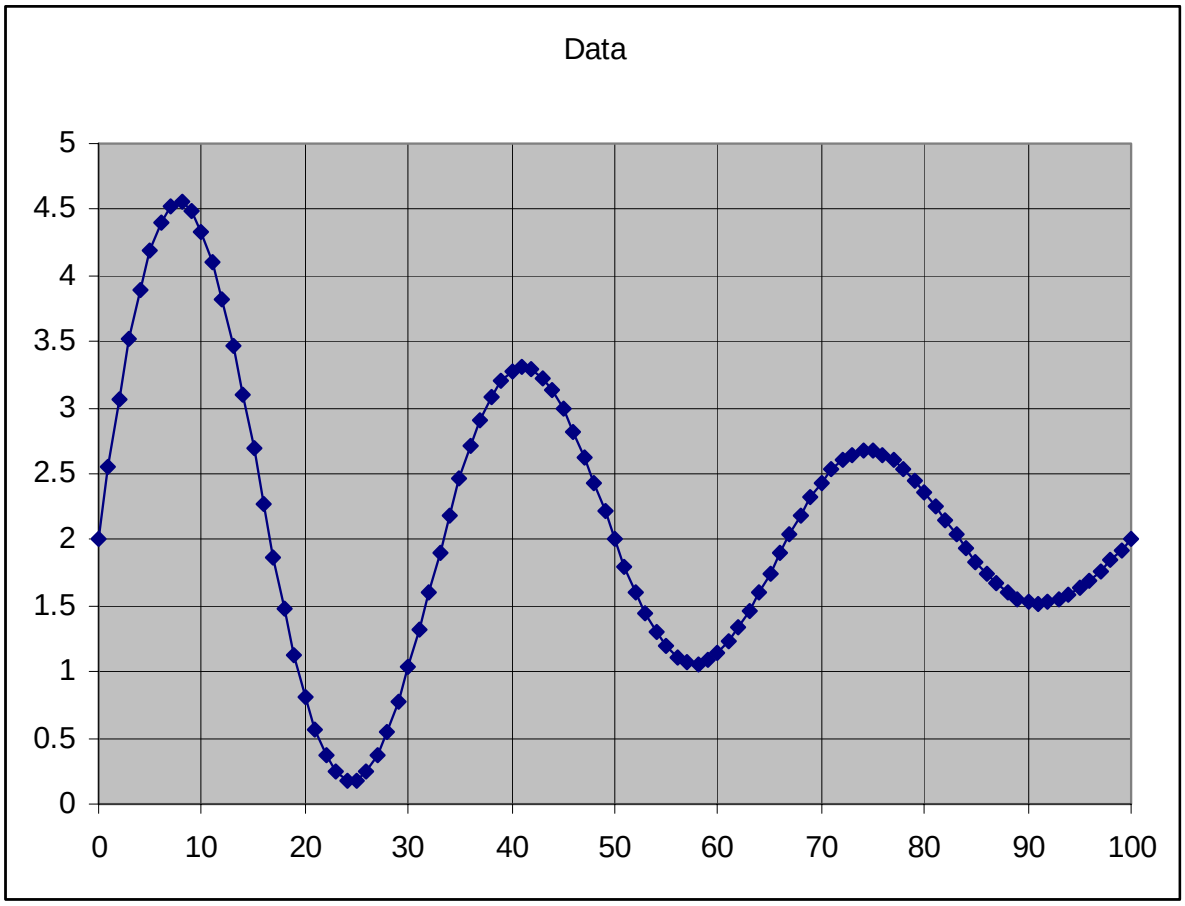
a. Assuming the horizontal scale is in seconds, find the frequency and period of this signal. Include units. *(4 points)*

b. What is the DC offset of this signal? Include units. *(2 points)*

c. What is the phase of the signal? Include units. *(2 points)*

d. Write the mathematical expression for this signal and its offset. In general, this is given by $X = X_o + X_1 \sin(\omega t + \phi_o)$ which accounts for its frequency and phase shift. *(2 points)*

The same data with damping looks like:



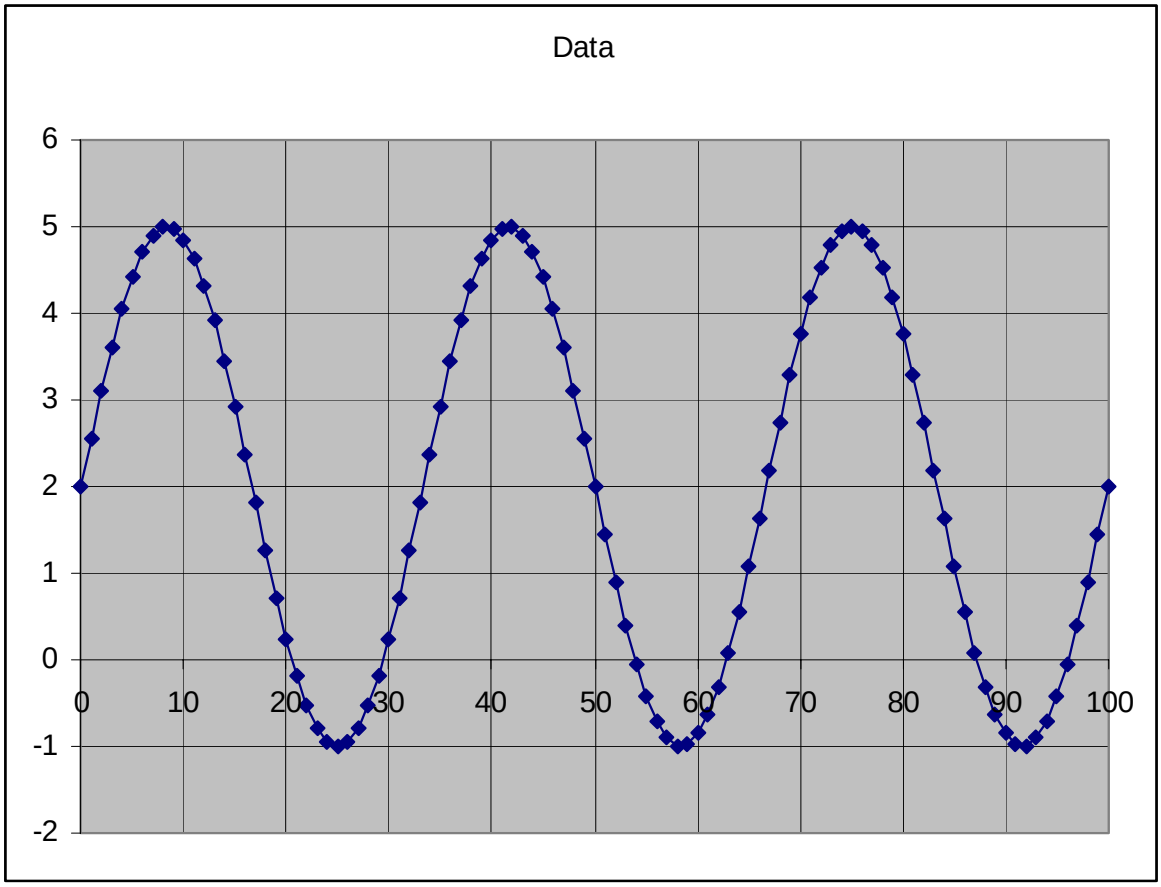
e. Find the damping constant α for this data. Include units. *(6 points)*

f. Write the mathematical expression for this data $X = X_o + X_1 \sin(\omega t + \phi_o)e^{-\alpha t}$
Your units must be consistent. *(4 points)*

Spring 2004 solution

4) Sinusoids (20 points) [Both A and B]

The following data was created using Excel.



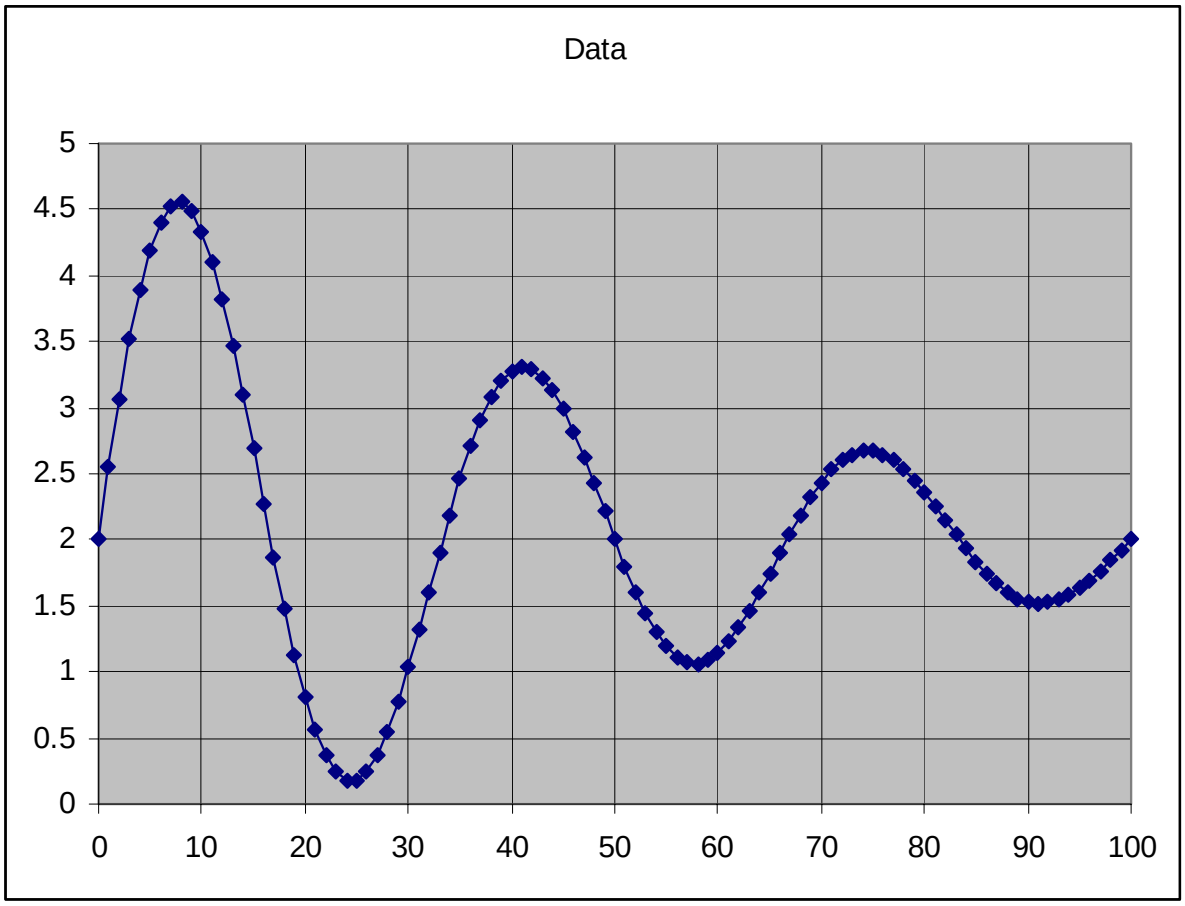
g. Assuming the horizontal scale is in seconds, find the frequency and period of this signal. Include units. *(4 points)*
T=33s – 0 ***T=33s***
f = 1/33s ***f = 0.03 Hz***

h. What is the DC offset of this signal? Include units. *(2 points)*
VDC = 2V

i. What is the phase of the signal? Include units. *(2 points)*
phase = 0 degrees or 0 radians

j. Write the mathematical expression for this signal and its offset. In general, this is given by $X = X_o + X_1 \sin(\omega t + \phi_o)$ which accounts for its frequency and phase shift. *(2 points)*
$\omega = 2\pi f = 2*3.24*0.03 = 0.1885$ *rad/sec* ***X = 2V + 3V sin (0.1885 t)***

The same data with damping looks like:



k. Find the damping constant α for this data. Include units. *(6 points)*

VDC = 2V Therefore, let V=2V be the zero point. V0=(7s, 4.6-2) = (7,2.6)
V1=(74s,2.7-2) =(74,0.7)
$V1 = V0\, e^{-\alpha(t1-t0)}$ $0.7 = 2.6\, e^{-\alpha(74-7)}$ $-\alpha(67)=\ln(0.7/2.6)$ ***$\alpha=0.0196/s$***

l. Write the mathematical expression for this data $X = X_o + X_1 \sin(\omega t + \phi_o)e^{-\alpha t}$
Your units must be consistent. *(4 points)*

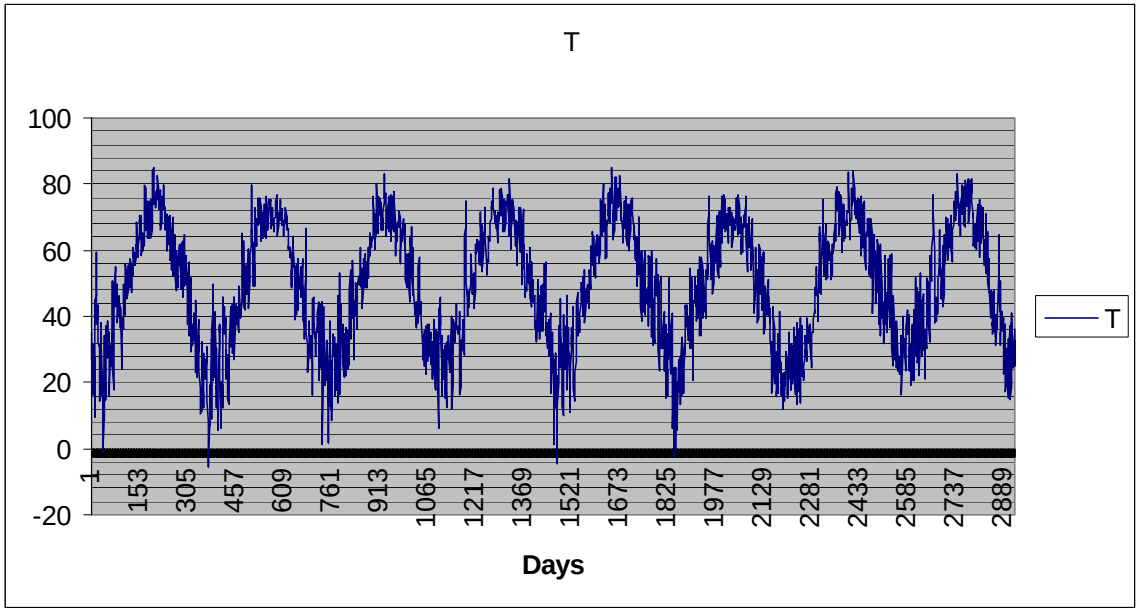
From part d we know that:
VDC = 2V Amplitude = 3V f=0.03Hz $w=2\pi f=0.1885$ rad/sec

$X = 2V + 3V \sin(0.1885\, t)\, e^{-0.0196\, t}$

Fall 2003

3. Sinusoids (20 points)

The following data was obtained for some kind of a measurement.



The horizontal scale is in days. The vertical scale is unknown at the moment.

a) Since this data looks approximately sinusoidal, determine its period and frequency. You may make any reasonable assumption.

b) The data is not exactly sinusoidal since it is very noisy and also the maximums and minimums are not the same. In addition, there is a finite offset (DC) value. What is the approximate value of the offset?

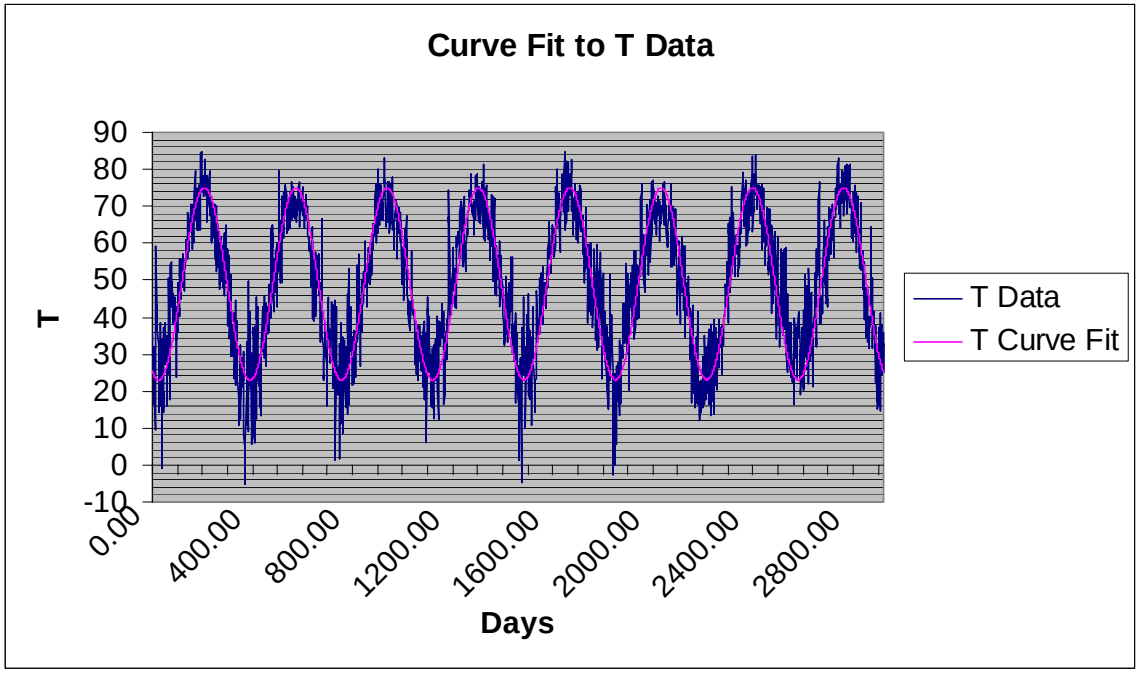
c) Write a mathematical expression for the signal and its offset.

d) Based on your answer to part (a) and the values given for the data, what do you suppose is being measured?

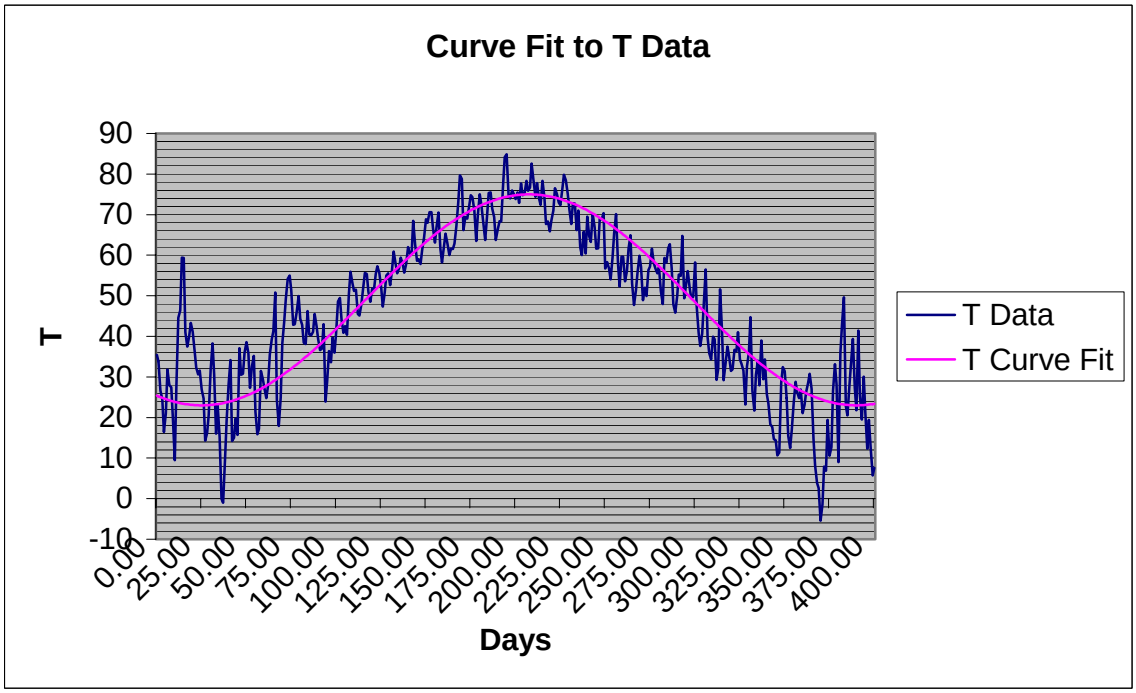
Fall 2003 solution

3. Sinusoids (20 points)

The following data was obtained for some kind of a measurement.



The horizontal scale is in days. The vertical scale is unknown at the moment.
With the horizontal scale expanded,



a) Since this data looks approximately sinusoidal, determine its period and frequency. You may make any reasonable assumption. *(6 points)*

T = (400-35) = 365 days
F = 1/T = 0.00274 cycles/day
Anything in a reasonable range is acceptable.

b) The data is not exactly sinusoidal since it is very noisy and also the maximums and minimums are not the same. In addition, there is a finite offset (DC) value. What is the approximate value of the offset? *(3 points)*

peak-to-peak voltage = 76-24=52 amplitude = 52/2 = 26
Offset is the center of the wave, which is 24 to bottom+26 or about 50.
50 (units unknown)
Anything in a reasonable range is acceptable.

c) Write a mathematical expression for the signal and its offset. (Note that there is also a phase shift in this signal.) $T=T_0 + T_1\sin(\omega t+\phi_0)$ where ϕ_0 is the phase term, T_0 is the offset and T1 is the amplitude of the sine function. *(9 points)*

We still need to find the radial frequency and the phase shift.
$\omega=2\pi f = 2\pi(0.00274)=0.0172$ *radians/day*
$\phi_0=-\omega t_0=-(0.0172)(130)=-2.24$ *radians*

T = 50 + 26 sin(0.0172t – 2.24)

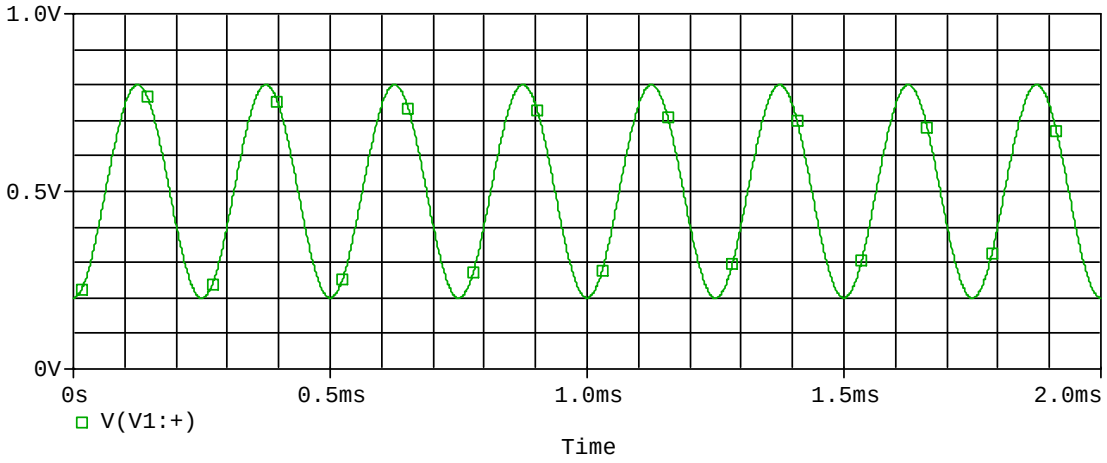
a) Based on your answer to part (a-c) and the values given for the data, what do you suppose is being measured? *(2 points)*

It looks to me like temperature in degrees Fahrenheit over several years. Other ideas that seemed reasonable were also accepted.

Fall 2002

3. Sine Waves (20 points)

a) sinusoid



i) Consider trace in the plot above and give the following values (6 points):
(Do not forget the units.)

frequency:

amplitude:

rms value:

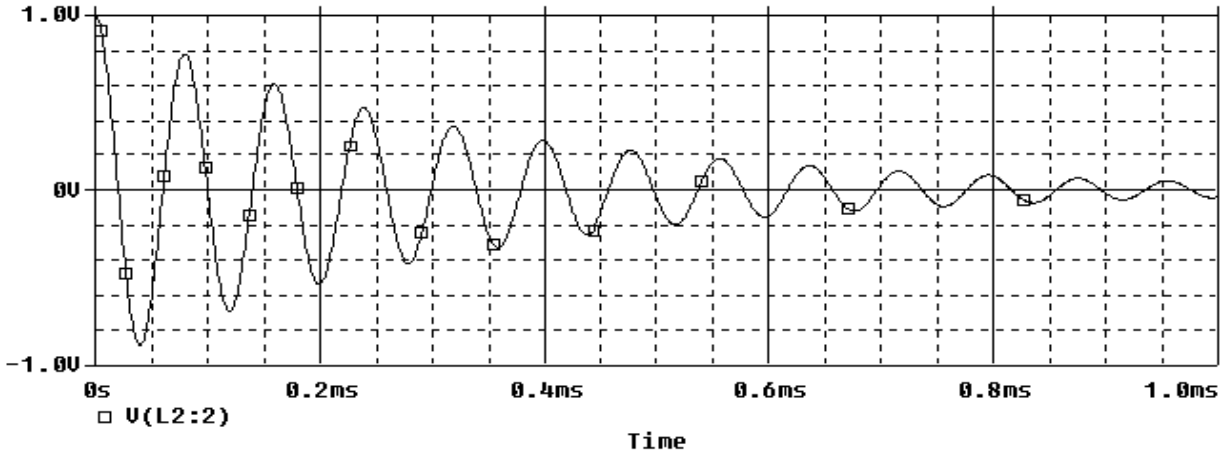
peak-to-peak voltage:

phase shift:

offset voltage:

ii) Write down the mathematical expression for trace a) in the form
$v(t) = V_{dc} + A \sin(\omega t + \varphi)$. (4 points)

b) decaying sinusoid



i) Consider the plot above and give the following values (6 points):
(Do not forget the units.)

frequency:

amplitude (at t=0 s):

decay constant α:

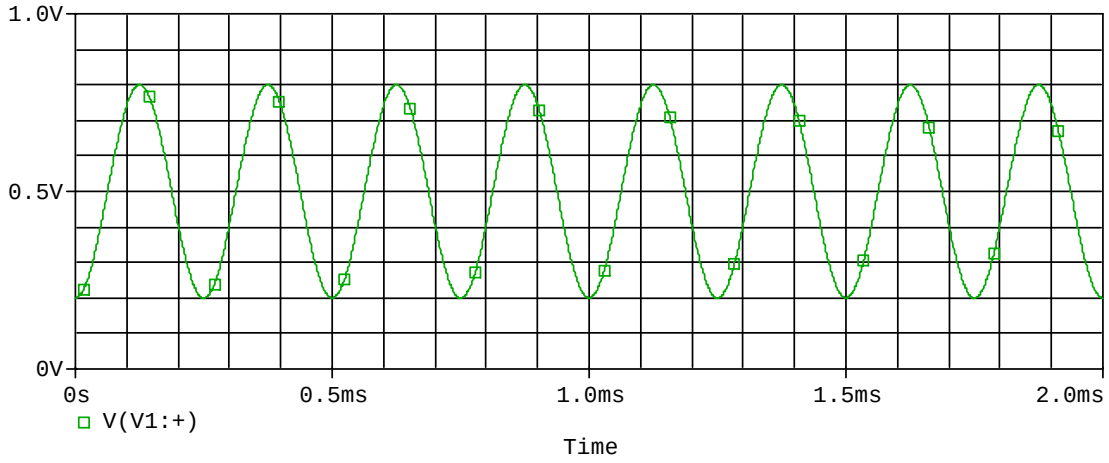
ii) Write down the mathematical expression for the trace b) in the form
$v(t) = A\, e^{-\alpha t} \sin(\omega t)$. [Ignore the phase shift] (4 points)

Extra credit: What is the phase shift for the decaying sinusoid in b)?

Fall 2002 solution

3. Sine Waves (20 points)

a) sinusoid



i) Consider trace in the plot above and give the following values (6 points):
(Do not forget the units.)

frequency: *period = 0.25ms frequency = 1/.25m =* ***4 KHz***

amplitude: *A=Vp-p/2 = (.8-.2)/2 =* ***0.3 volts***

rms value: $V_{rms} = \sqrt{V_{off}^2 + A^2/2} = \sqrt{.5^2 + .3^2/2} = \sqrt{.295}$ = **0.543 volts**

peak-to-peak voltage: *Vp-p =* ***0.6 volts***

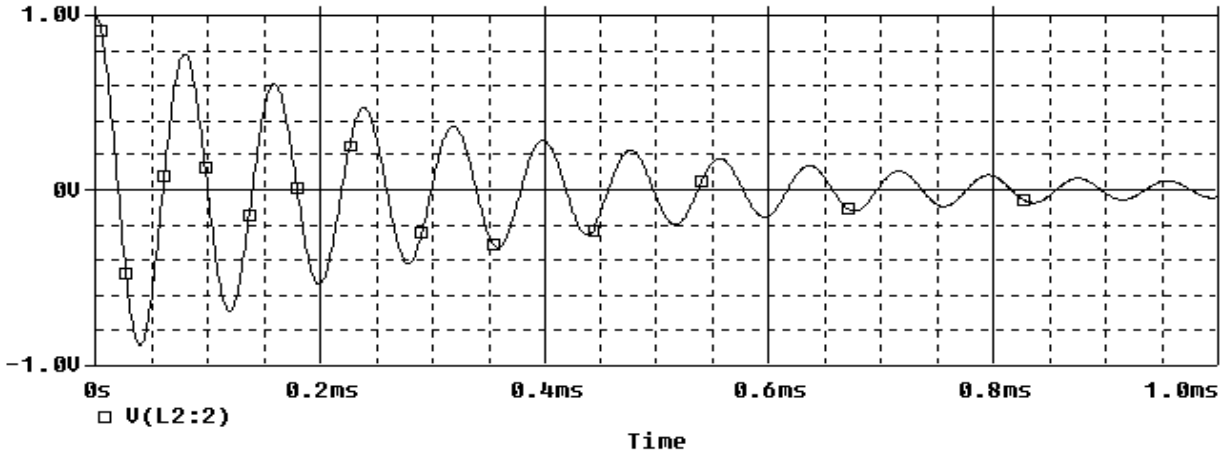
phase shift: φ = -2π*1/4 = **-π/2 rad/sec** or
$\varphi = -\omega t_0 = -2\pi f(.06ms) = -0.48\pi$ *rad/sec = -1.51 rad/sec*

offset voltage: offset = ***0.5 volts***

ii) Write down the mathematical expression for trace a) in the form
$v(t) = V_{dc} + A \sin(\omega t + \varphi)$. (4 points)

v(t) = 0.5 + 0.3 sin(8Kπt – π/2)

b) decaying sinusoid



i) Consider the plot above and give the following values (6 points):
(Do not forget the units.)

frequency: *5/0.4ms =* ***12.5 KHz***

amplitude (at t=0 s): ***1.0 volt***

decay constant α: *P0=(0ms,1volt) P1=(0.79ms, 0.08volts)*

$0.08 = 1\ e^{-\alpha(t1-t0)}$

$ln(0.08) = -\alpha(0.79)$

α = 3.2 per millisecond

α = 3197 per second

ii) Write down the mathematical expression for the trace b) in the form
$v(t) = A\ e^{-\alpha t} \sin(\omega t)$. [Ignore the phase shift] (4 points)

$v(t) = 1.0\ e^{-3197t} \sin(25Kt)$ for seconds or
$v(t) = 1.0\ e^{-3.2t} \sin(25t)$ for milliseconds

Extra credit: What is the phase shift for the decaying sinusoid in b)?

$+\pi/2$ rad/sec *$(+2\pi*1/4cycle)$ or $-\omega t_0 = -2\pi(12.5K)(-.02m) = 0.5\pi$ rad/sec*