## Intro to ECSE

## Quiz 2

Fall 2022

| 1. | $/ 15$ |
| :---: | :---: |
| 2. | $/ 10$ |
| 3. | $/ 20$ |
| Total | $/ 45$ |

Name

Notes:
SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification. Use the backs of pages if there is not enough room on the front.

For partial credit on some questions, you may want to re-draw circuit diagrams as you simplify the circuits.

Many problems can be solved using more than one method. check your answers by using a second method.

At least skim through the entire quiz before you begin and then start with the problems you know best. The proctor will only answer clarification questions where wording is unclear or where there may be errors/typos. No other questions will be responded to.

## Problem 1 (15 Points): Nodal Analysis



Figure 1
1.1) (2 pts) How many voltage nodes are in this circuit? Label them on the circuit schematic.

There are 5 nodes (1 pt)
Labeled correctly on circuit (1 pt)
1.2) (4 pts) Which of these node voltages are already known (if any)? Write their numerical values on the circuit schematic.
$\mathrm{Va}=2 \mathrm{~V} \quad(1 \mathrm{pt})$
$\mathrm{Vd}=3 \mathrm{~V} \quad(1 \mathrm{pt})$
$\mathrm{Ve}=0 \mathrm{~V}$ (1 pt)
Labeled correctly on circuit (1 pt)
1.3) (2 pts) How many linearly-independent equations are needed to solve for all unknowns in this circuit? Why?
\# of equations = \# of nodes - \# of voltage sources - 1 = 2 (1 pt)
Explanation (1 pt): There are 5 total nodes. After accounting for voltage sources (2), which each provide a known nodal voltage equal to the individual source voltage, and ground, which provides a known nodal voltage of OV , only two unknowns are left to solve for, which requires two equations.
1.4) (4 pts) Write down the KCL equation for each node in terms of the nodal voltages you labeled on the schematic in 1.1 and 1.2.

At node $\mathrm{b}: \frac{V b-2}{5 k}+\frac{V b-V c}{5 k}+\frac{V b}{2 k}=0 \quad$ (2 pts)

At node c: $\frac{V c-V b}{5 k}+\frac{V c-3}{10 k}+\frac{V c}{1 k}+\frac{V c-2}{10 k}=0 \quad$ (2 pts)
1.5) (3 pts) Write the equations from 1.4 in matrix form.

In standard form:
At node b: $V b\left(\frac{1}{5 k}+\frac{1}{5 k}+\frac{1}{2 k}\right)+V c\left(-\frac{1}{5 k}\right)=\frac{2}{5 k}$

At node c: $V b\left(-\frac{1}{5 k}\right)+V c\left(\frac{1}{5 k}+\frac{1}{10 k}+\frac{1}{10 k}+\frac{1}{1 k}\right)=\frac{2}{10 k}+\frac{3}{10 k}$

In matrix form:

$$
\left[\begin{array}{ccc}
(1 \mathrm{pt}) & (1 \mathrm{pt}) \\
\frac{1}{5 k}+\frac{1}{5 k}+\frac{1}{2 k} & -\frac{1}{5 k} \\
-\frac{1}{5 k} & \frac{1}{5 k}+\frac{2}{10 k}+\frac{1}{1 k}
\end{array}\right]\left[\begin{array}{l}
V b \\
V C
\end{array}\right]=\left[\begin{array}{c}
\frac{2}{5 k} \\
\frac{5}{10 k}
\end{array}\right]
$$

## Problem 2 (10 pts): Linear and Non-Linear Circuit Elements (Conceptual Questions)

2.1) ( 2 pts) How can you determine if a circuit element is linear or non-linear?

Explanation (2 pts)
A linear element has an I-V characteristic that is a single straight line for all I and V
OR
A linear element has a single, constant resistance for all I and V
OR
A linear element has a constant resistance that is independent of I and V
2.2) (4 pts) Identify two different linear circuit elements and draw their IV characteristics.

Choose 2 ( 1 pt each): resistor OR voltage source OR current source
Correct IV characteristic for each chosen circuit element above (1 pt each)




An op-amp circuit is also a valid answer as long as it's a transfer characteristic (Vout vs Vin) and it is clearly stated/labeled that it must operate in the linear regime.
2.3) (2 pts) The transfer characteristic (Vout vs. Vin) below belongs to a non-inverting amplifier with a gain of 2. Label the linear region(s) and saturation region(s) of the transfer characteristic.


Figure 2
Saturation (1 pt) and linear (1 pt) regions correctly labeled on transfer characteristic.
2.4) ( 2 pts) A diode is a non-linear circuit element. What does that tell us about its resistance, as opposed to that of linear circuit elements?

Explanation (2 pts): A diode's resistance changes as a function of $I$ and $V$, so it does not have a constant resistance across all I and V , unlike linear circuit elements.

## Problem 3 (20 Pts): Operational Amplifiers

Shown below is a simple circuit for amplifying an audio signal. Assume that U1, U2, and U3 are ideal opamps. For all parts of this problem, the positive (Vs+) and negative (Vs-) supply voltages are +5 V and 5 V . V1 is a sinusoidal wave with an amplitude of 1 V .


Figure 3
3.1) (3 Pts) What kind of op-amp circuit is stage U1? Write its transfer function both in terms of resistor names (if applicable) and numerically.

Stage U1 is a voltage follower (1 pt)
The transfer function for a voltage follower is $\mathrm{H} 1=1$ (2 pts)
3.2) ( 1 Pt ) What is the output voltage after stage $\mathrm{U} 1(\mathrm{VoU1})$ ?

$$
\mathrm{VoU1}=\mathrm{H} 1 * \mathrm{~V} 1=1 * 1 \mathrm{~V}=1 \mathrm{~V} \quad(1 \mathrm{pt})
$$

3.3) (3 Pts) What kind of op-amp circuit is stage U2? Write its transfer function both in terms of resistor names (if applicable) and numerically.

Stage U2 is a non-inverting amplifier ( 1 pt )
The transfer function for a non-inverting amplifier is: $H 2=\left(1+\frac{R f}{R g}\right)(1 \mathrm{pt})$
$H 2=\left(1+\frac{3 k}{1 k}\right)=4 \quad(1 \mathrm{pt})$
3.4) (1 Pt) What is the output voltage after stage U2 (VoU2)?

$$
\mathrm{VoU} 2=\mathrm{H} 2 * \mathrm{VoU1}=4 * 1 \mathrm{~V}=4 \mathrm{~V} \quad(1 \mathrm{pt})
$$

3.5) (3 Pts) What kind of op-amp circuit is stage U3? Write its transfer function both in terms of resistor names (if applicable) and numerically.

Stage U3 is a voltage follower ( 1 pt )
The transfer function for a voltage follower is H3 = 1 (2 pts)
3.6) ( 1 Pt ) What is Vout?

$$
\text { Vout }=H 3 * \text { VoU2 }=1 * 4 V=4 V \quad(1 p t)
$$

3.7) (2 Pts) What is the overall transfer function $\mathrm{H}_{\text {total }}$ of the entire circuit in terms of resistor names?

$$
\begin{aligned}
H_{\text {total }} & =\mathrm{H} 1 * \mathrm{H} 2 * \mathrm{H} 3 \quad(1 \mathrm{pt}) \\
& =(1) *(1+\mathrm{Rf} / \mathrm{Rg}) *(1)=(1+\mathrm{Rf} / \mathrm{Rg}) \quad(1 \mathrm{pt})
\end{aligned}
$$

3.8) (3 Pts) Given that Vin = 1 V , what is the maximum gain that stage U 2 can have and still ensure that the op-amp circuit does not saturate? If $\operatorname{Rg}=1 \mathrm{k} \Omega$, what new value of $\operatorname{Rf}$ would provide that gain?

$$
\begin{aligned}
& \text { Vo, } \max =5 \mathrm{~V} \quad \text { (1 pt) } \\
& V_{o, \max }=V_{\text {in }}\left(1+\frac{R f}{R g}\right) \quad \text { (1 pt) } \\
& 5 V=1 V\left(1+\frac{R f}{1 k \Omega}\right) \rightarrow R f=4 k \Omega \quad \text { (1 pt) }
\end{aligned}
$$

3.9) (3 Pts) Audio systems commonly have an LED indicator that lights up when the audio signal is nearing the maximum voltage that the circuit can output without distorting the signal.

Suppose you are given the following circuit schematic, which compares the input signal ( $\mathrm{V}_{\text {audio }}$ ) to a reference voltage ( $\mathrm{V}_{\text {ref }}$ ), and are asked to choose resistor values such that the LED (D1) lights up when $V_{\text {audio }}$ is greater than $V_{\text {ref }}$ and is off when $V_{\text {audio }}$ is less than $V_{\text {ref. }}$


| Available Resistors |  |
| :---: | :---: |
| $1.5 \mathrm{k} \Omega$ | $2.2 \mathrm{k} \Omega$ |
| $10 \mathrm{k} \Omega$ | $20 \mathrm{k} \Omega$ |

Figure 4
Which values for R1 and R2 from the available resistors above would provide a reference voltage of $V_{\text {ref }}=4.5 \mathrm{~V}$ for the circuit? You may use each resistor value only once.
$V_{r e f}=V_{1} \frac{R_{2}}{R_{1}+R_{2}}(\mathbf{1} \mathbf{p t})$
$4.5 \mathrm{~V}=5 \mathrm{~V} \frac{R_{2}}{R_{1}+R_{2}} \rightarrow \frac{9}{10}=\frac{R_{2}}{R_{1}+R_{2}} \rightarrow 9 R_{1}=R_{2}$
Of the possible resistor values, the only two that satisfy the above relationship are $\mathrm{R} 2=2.2 \mathrm{k} \Omega$ and R1 $=20 \mathrm{k} \Omega$. ( 2 pts )

Checking the result: $V_{\text {ref }}=5 V \frac{20 k \Omega}{2.2 k \Omega+20 k \Omega}=(5 \mathrm{~V})(0.901)=4.50 \mathrm{~V}$

