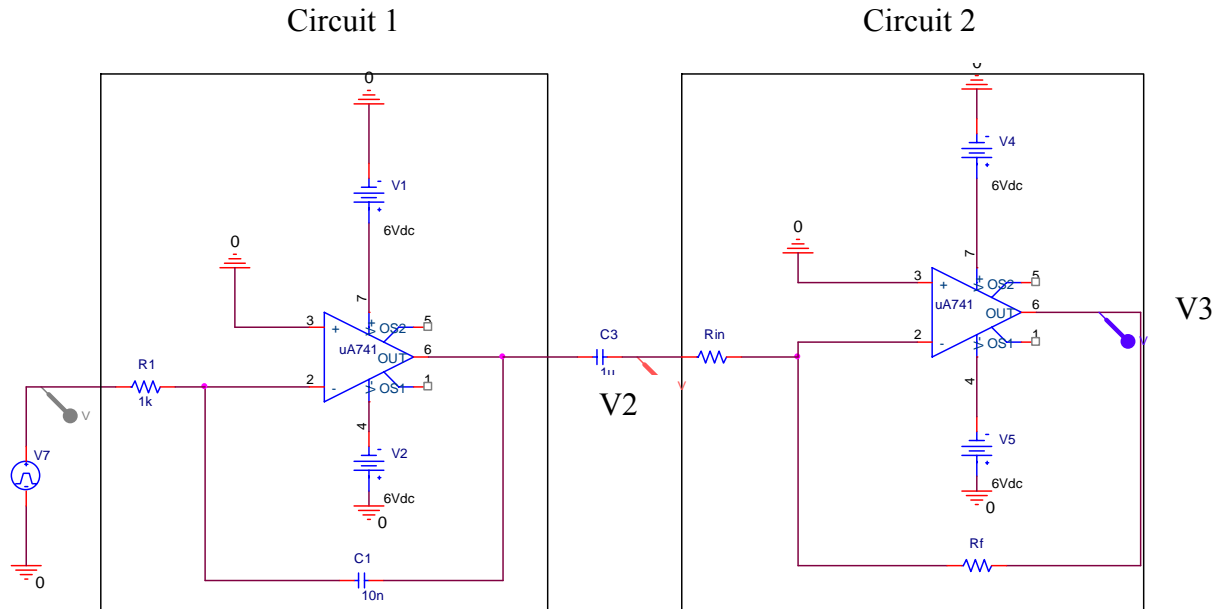


Fall 2004

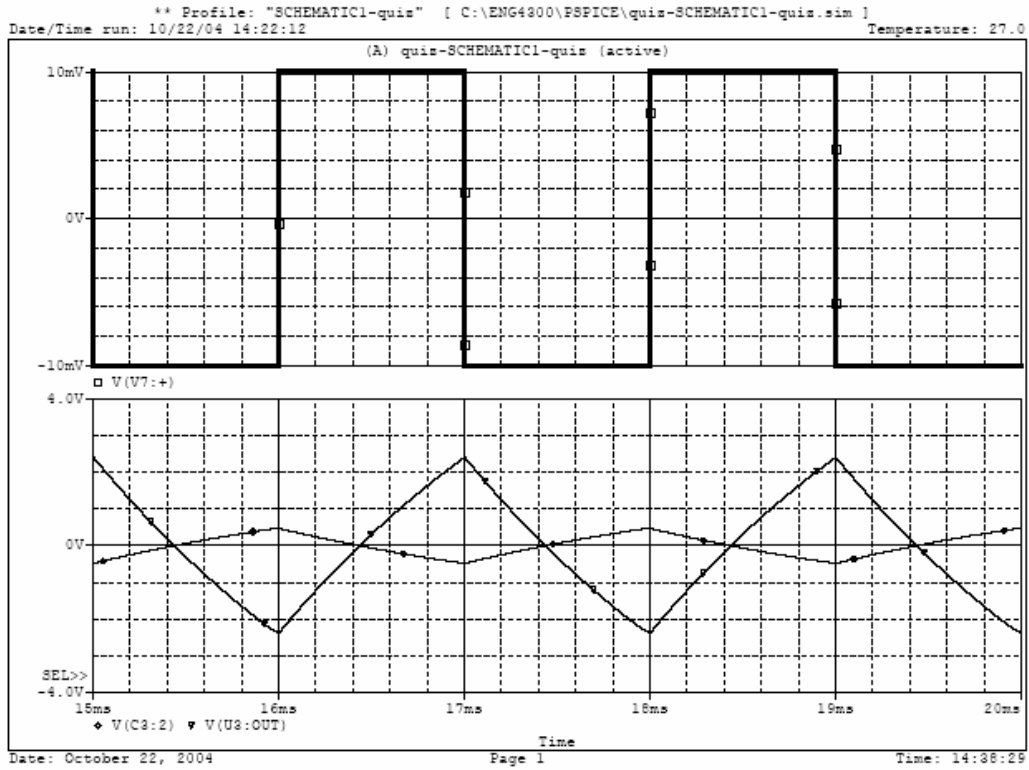
Question 4 -- (Op Amps) (25 points)

Below is a Capture schematic of two Op-amp circuits that you should recognize



1. What is the function of circuit 1? (2 points)
2. What is the function of the capacitor, C3, between the circuits? (2 points)
3. What is the function of circuit 2? (2 points)
4. Find an expression for V3 in terms of Rf, Rin, and V2. (4 points)

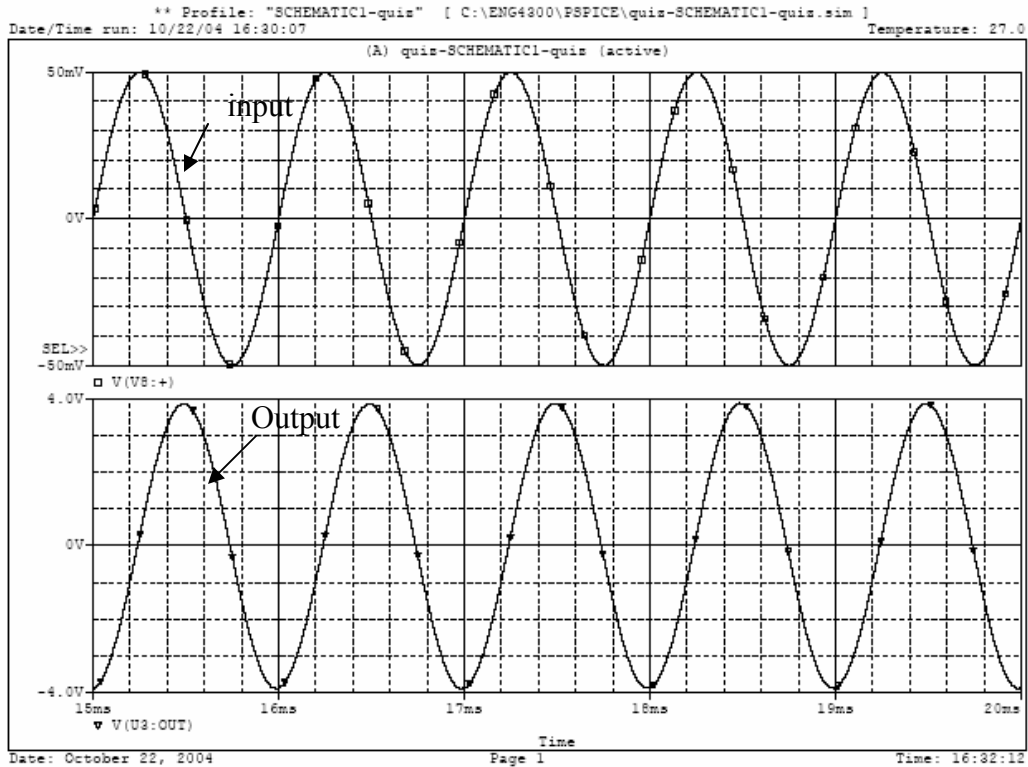
The plot shown below is the input square wave applied to the schematic composed of circuits 1 and 2.



5. Mark on the plot which one is V2 and which one is V3. (4 points)

6. Which value for  $R_f$  was used to produce this plot if  $R_{in}$  is 2K ohms? (3 points)

If the input to the complete system formed of the two circuit is a sine wave  $v(t) = A \sin(\omega t + \phi)$ , the simulation results for both the input and output will be as shown in the figure below.



7. What is the general expression for the input signal? Please give the numerical values for  $A$  and  $\omega$  and  $\phi$ ? (3 points)

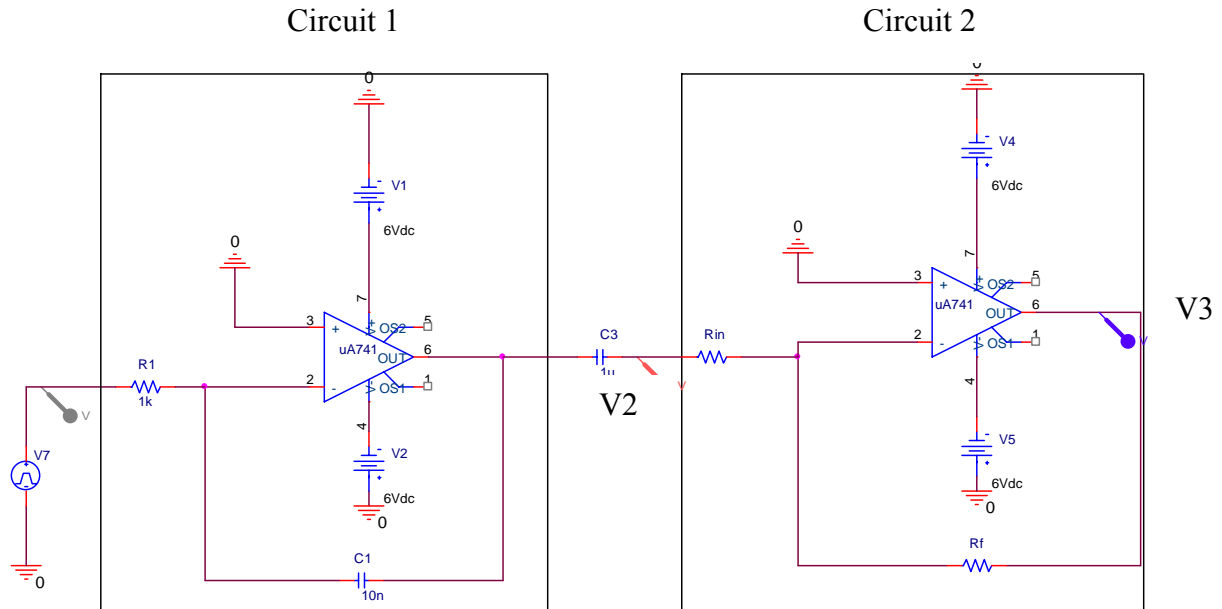
8. What is the general expression for the output signal? Please give the numerical values for  $A$  and  $\omega$  and  $\phi$  (3 points)

9. Explain the phase difference between  $V_{in}$  and  $V_{out}$ . (2 points)

**Fall 2004 solution**

**Question 4 -- (Op Amps) (25 points)**

Below is a Capture schematic of two Op-amp circuits that you should recognize



2. What is the function of circuit 1? (2 points)

*Ideal Integrator (integrates the signal)*

2. What is the function of the capacitor, C3, between the circuits? (2 points)

*DC bias blocker (removes the DC offset)*

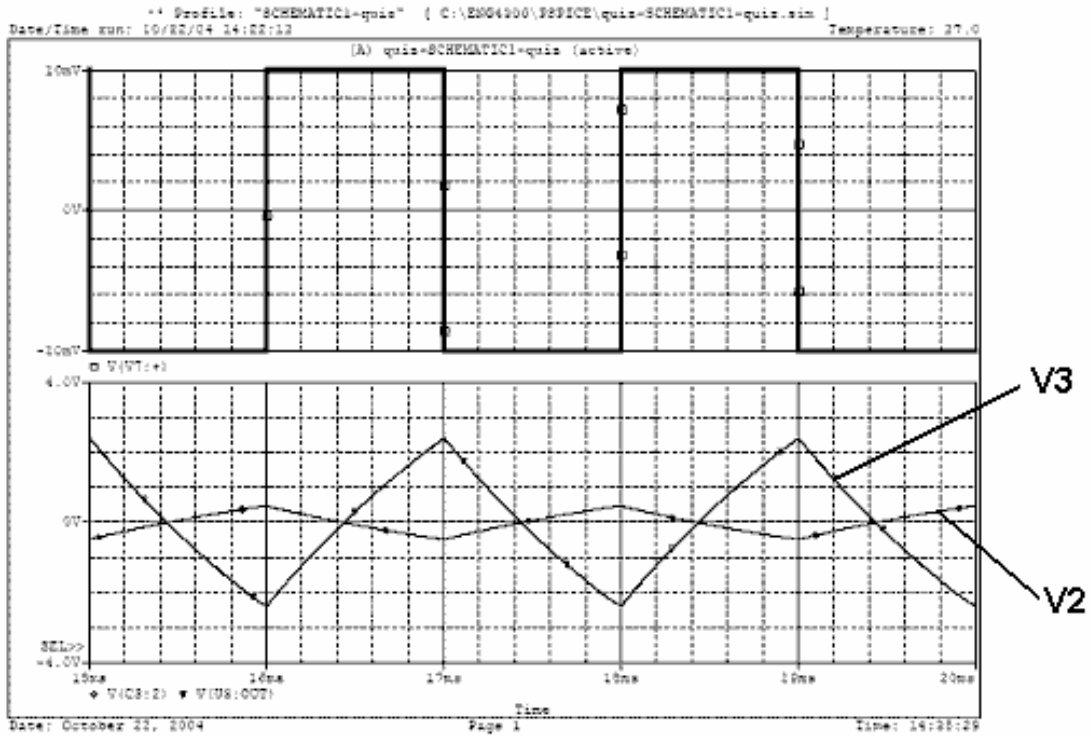
3. What is the function of circuit 2? (2 points)

*Inverting Op-Amp (inverts and amplifies the signal)*

4. Find an expression for V3 in terms of Rf, Rin, and V2. (4 points)

$$V3 = -(Rf/Rin)V2$$

The plot shown below is the input square wave applied to the schematic composed of circuits 1 and 2.



5. Mark on the plot which one is V2 and which one is V3. (4 points)

Note: When the input is  $-10\text{mV}$ , the mathematical integration should slope down. However, the circuit integrates and inverts, so V2 is the smaller wave which slopes up. V3 is the amplified version of V2. Circuit 2 is an inverting amplifier. The output should be larger and inverted.

A-6. Which value for  $R_f$  was used to produce this plot if  $R_{in}$  is  $2\text{K}$  ohms? (3 points)

V2 has an amplitude of  $-0.5\text{V}$  and V3 has an amplitude of  $2.3$  volts

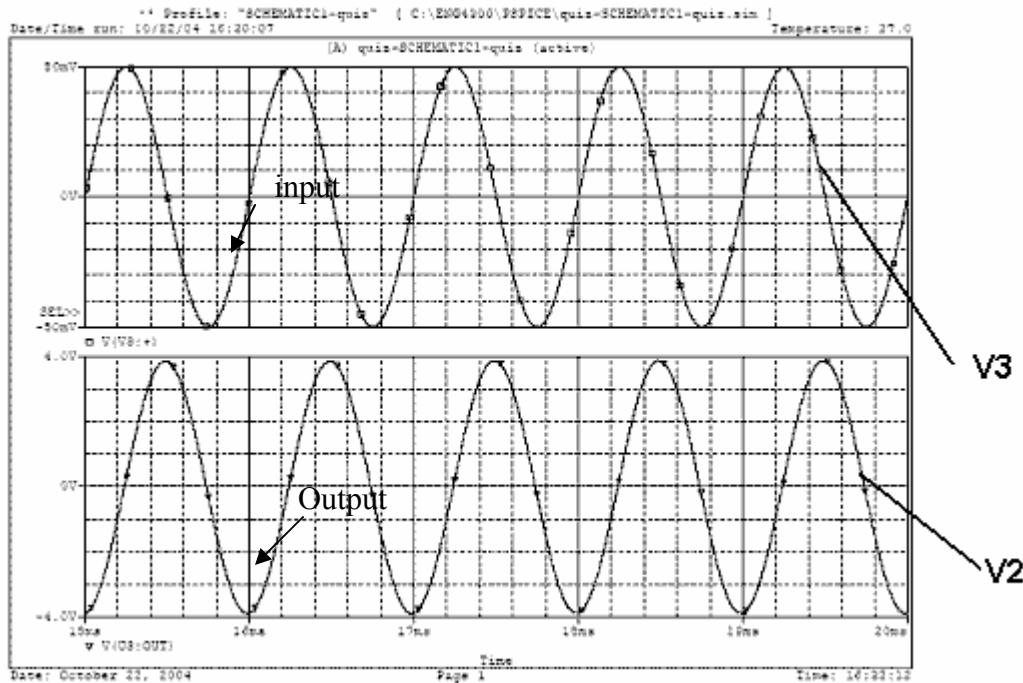
$$2.3 = (-R_f/2\text{K})/(-0.5) \quad R_f = 9.2\text{K ohms}$$

B-6. Which value for  $R_f$  was used to produce this plot if  $R_{in}$  is  $3\text{K}$  ohms? (3 points)

V2 has an amplitude of  $-0.5\text{V}$  and V3 has an amplitude of  $2.3$  volts

$$2.3 = (-R_f/3\text{K})/(-0.5) \quad R_f = 13.8\text{K ohms}$$

If the input to the complete system formed of the two circuit is a sine wave  $v(t) = A \sin(\omega t + \phi)$ , the simulation results for both the input and output will be as shown in the figure below.



7. What is the general expression for the input signal? Please give the numerical values for  $A$  and  $\omega$  and  $\phi$ ? (3 points)

*Input:  $A = 50mV$   $T = 1ms$   $f = 1K$   $\omega = 2\pi K$*

$$V_{in}(t) = 50mV \sin(2\pi K t)$$

8. What is the general expression for the output signal? Please give the numerical values for  $A$  and  $\omega$  and  $\phi$  (3 points)

*Output:  $A = 4V$   $\phi = -2\pi(1/4) = -\pi/2$   $T = 1ms$   $f = 1K$   $\omega = 2\pi K$*

$$V_{out}(t) = 4V \sin(2\pi K t - \pi/2)$$

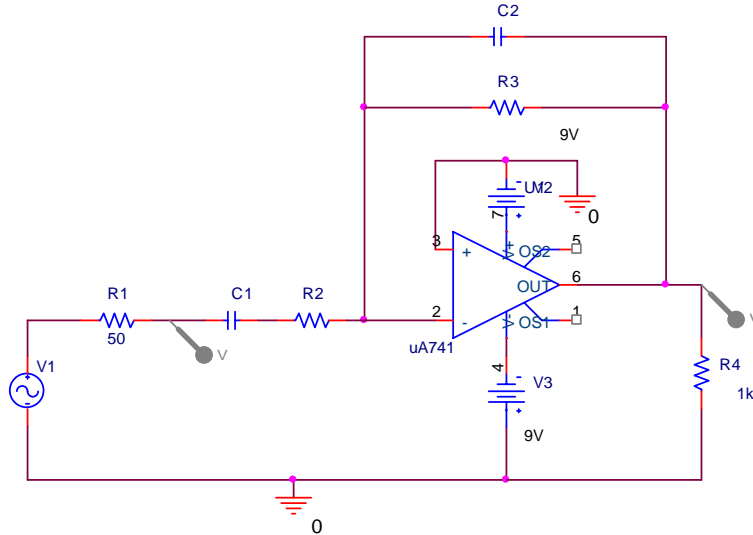
9. Explain the phase difference between  $V_{in}$  and  $V_{out}$ . (2 points)

*The first circuit is an integrator and the input is a sine wave. If  $\int \sin(t) dt = -\cos(t)$  then the output should have a phase shift of  $-\pi/2$ . However, this circuit also inverts, so  $-\pi/2 + \pi = +\pi/2$ . The second circuit inverts the signal again, so we get  $+\pi/2 + \pi = -\pi/2$ . This is the  $-\pi/2$  phase shift we see in the output.*

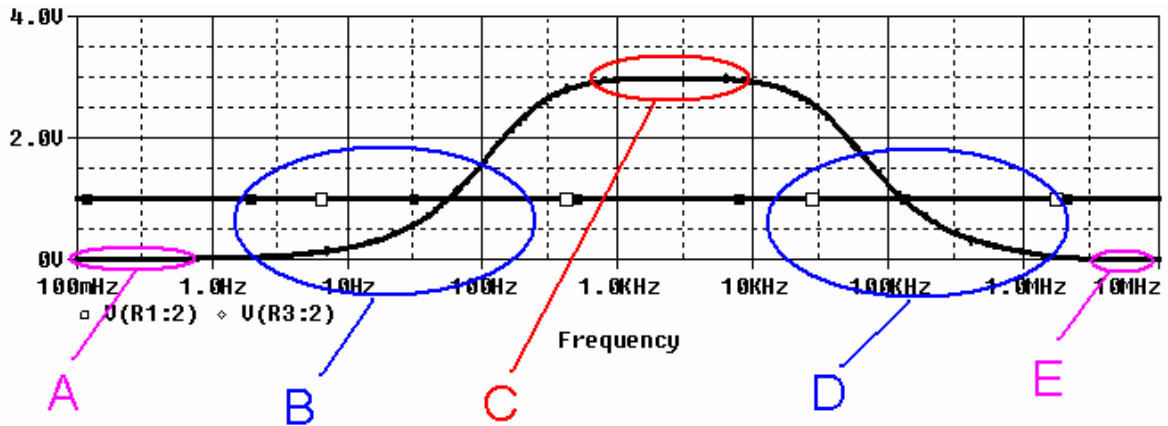
Spring 2004

4. Op-Amps (25 points)

Here is a combined differentiator/integrator similar to the one you implemented in experiment 8. Let  $C1=0.01\mu\text{F}$ ,  $R2=100\text{K}$  ohms,  $C2=0.01\text{ nF}$ , and  $R3=300\text{K}$  ohms.



a. Below is an AC sweep for the above circuit.



i) identify the input and the output traces. (2 points)

ii) If you built this circuit in the studio, in which of the circled regions would the output look like the following? (2 points each) [Total=8 points]

a reasonable integration of the input?

a reasonable differentiation of the input?

an amplified inversion of the input?

disappear into the noise?

b. What are the general equations for the following: (Give specific values based on the components in the circuit.)

i. The circuit when it is acting as an ideal integrator (*3 points*)

ii. The circuit when it is acting as an ideal differentiator (*3 points*)

iii. The circuit when it is acting as an inverting amplifier (*3 points*)

cA. Sketch the AC sweep of an integrator that integrates between 1K and 4K hertz. Give a ballpark estimate of the corner frequency. Mark 1K, 4K and the corner frequency on the sketch. Justify your decisions. (*6 points*)

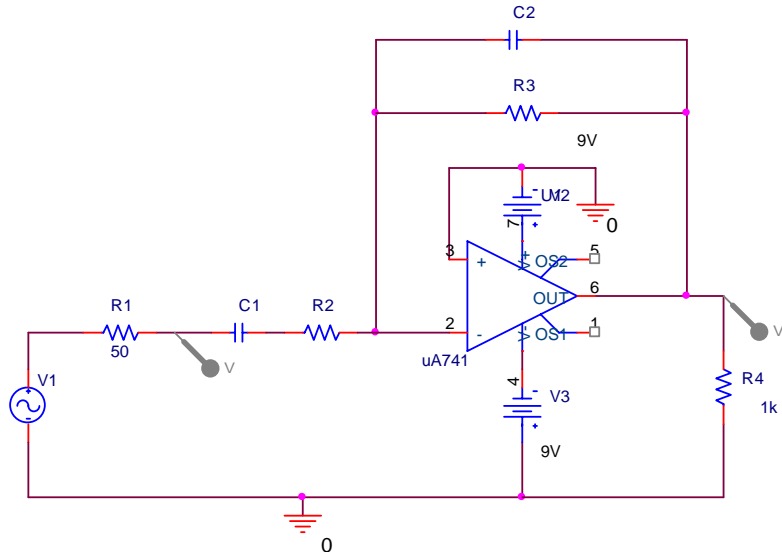
cB. Sketch the AC sweep of a differentiator that differentiates between 1K and 4K hertz. Give a ballpark estimate of the corner frequency. Mark 1K, 4K and the corner frequency on the sketch. Justify your decisions. (*6 points*)



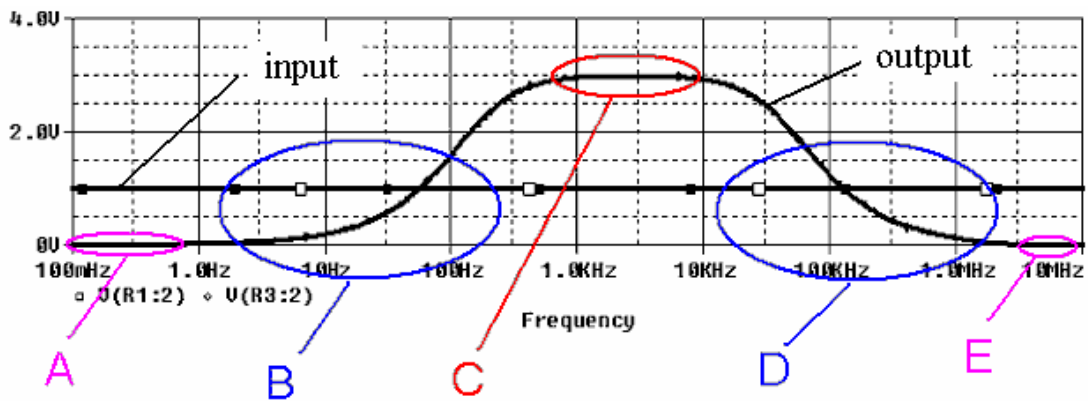
Spring 2004 solution

4. Op-Amps (25 points) (Test A Only)

Here is a combined differentiator/integrator similar to the one you implemented in experiment 8. Let  $C1=0.01\mu\text{F}$ ,  $R2=100\text{K}$  ohms,  $C2=0.01\text{ nF}$ , and  $R3=300\text{K}$  ohms.



b. Below is an AC sweep for the above circuit



i) Identify the input and the output traces. (2 points) (Test A)

ii) If you built this circuit in the studio, in which of the circled regions would the output look like the following? (2 points each) [Total=8 points]

a reasonable integration of the input? **D**

a reasonable differentiation of the input? **B**

an amplified inversion of the input? **C**

disappear into the noise? **A,E**

b. What are the general equations for the following: (Give specific values based on the components in the circuit.)

i. The circuit when it is acting as an ideal integrator (3 points)

$$v_{out} = \frac{-1}{R_{in}C_f} \int v_{in} dt = \frac{-1}{(100K)(0.01n)} \int v_{in} dt = -1Meg \int v_{in} dt$$

(Note how the large gain compensates for  $1/\omega$  in the integration.  $\omega$  is large, its inverse is small, so a large gain is needed to make the signal recognizable.)

ii. The circuit when it is acting as an ideal differentiator (3 points)

$$v_{out} = -R_f C_{in} \frac{dv_{in}}{dt} = -(300K)(0.01\mu) \frac{dv_{in}}{dt} = -3m \frac{dv_{in}}{dt}$$

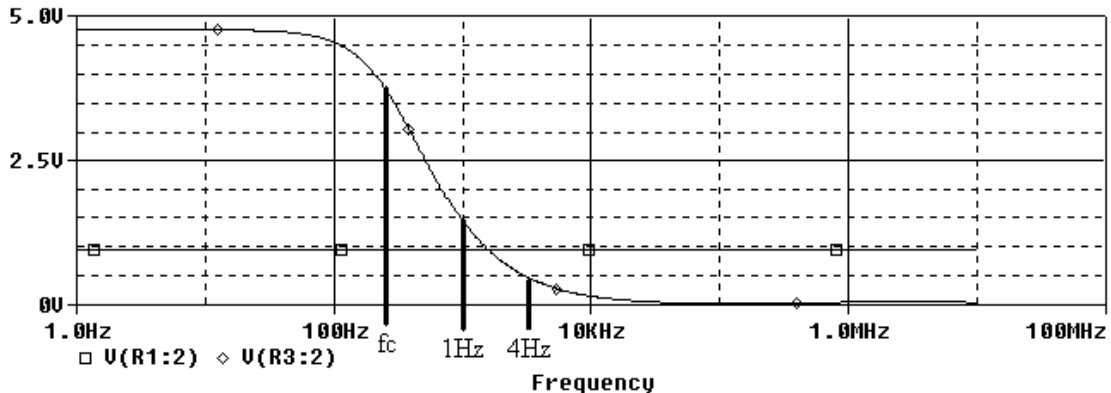
(Note how the small gain compensates for  $\omega$  in the differentiation.  $\omega$  is large, so the gain must be small or the op-amp will saturate.)

iii. The circuit when it is acting as an inverting amplifier (3 points)

$$v_{out} = \frac{-R_f}{R_{in}} v_{in} = \frac{-300K}{100K} v_{in} = -3v_{in}$$

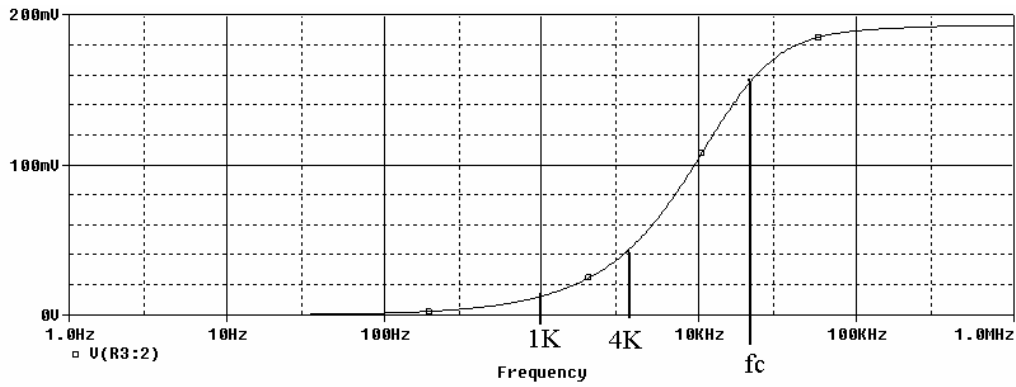
(Note that the input amplitude is 1V and at C, where the circuit is an inverting amplifier, the output amplitude is 3)

cA. Sketch the AC sweep of an integrator that integrates between 1K and 4K hertz. Give a ballpark estimate of the corner frequency. Mark 1K, 4K and the corner frequency on the sketch. Justify your decisions. (6 points)



The corner frequency in this case is about 200 Hertz. Note that answers may vary. Corner frequencies should be at or below 500Hz.

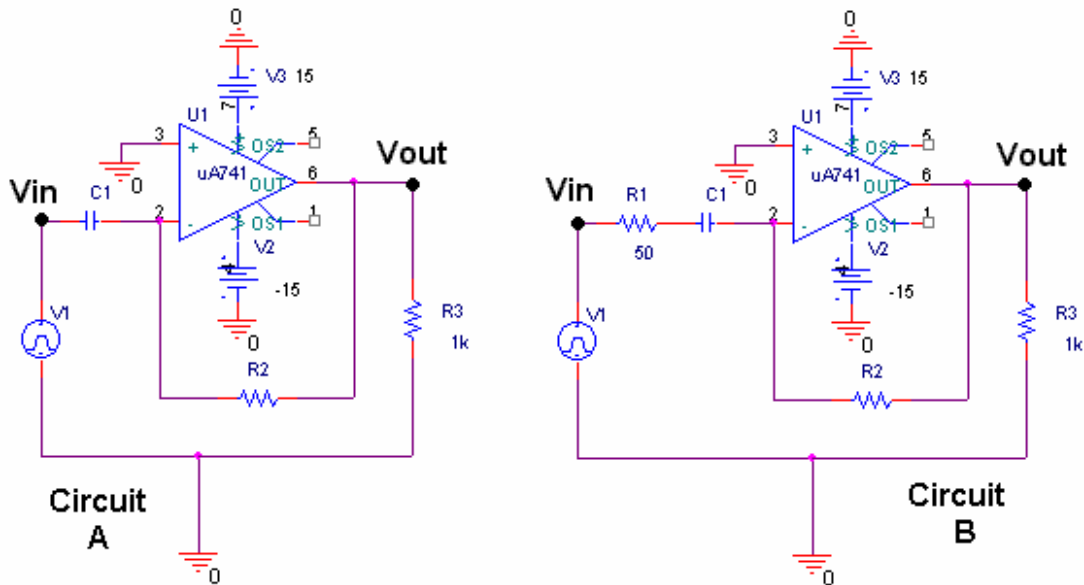
cB. Sketch the AC sweep of an differentiator that differentiates between 1K and 4K hertz. Give a ballpark estimate of the corner frequency. Mark 1K, 4K and the corner frequency on the sketch. Justify your decisions. (6 points)



The corner frequency in this case is about 20K Hertz. Note that answers may vary. Corner frequencies should be at or above 8K Hz

Fall 2003 solution

Question 3 -- Op Amp Differentiator (20 points)



Test A: Given:  $C1=0.001 \mu\text{F}$   $R2= 500 \Omega$

Test B: Given:  $C1=0.0022 \mu\text{F}$   $R2= 1000 \Omega$

In this question, we will look at the effect of the 50 ohm input resistance on the op-amp differentiator.

- a) What is the expression for  $V_{out}/V_{in}$  for circuit A above. Give this in terms of  $C1$ ,  $R2$ ,  $j$  and  $\omega$ . (3 points)

$$\frac{V_{out}}{V_{in}} = -j\omega R2C1$$

- b) Determine the expression for  $V_{out}/V_{in}$  for circuit B above. (Be sure to include the impedance of the function generator,  $R1$ .) Give this in terms of  $C1$ ,  $R1$ ,  $R2$ ,  $j$  and  $\omega$ . (3 points)

$$\frac{V_{out}}{V_{in}} = \frac{-j\omega R2C1}{1 + j\omega R1C1}$$

- c) Find the corner frequency  $f_c$  for circuit B. This should be a numerical value in Hertz. (4 points)

$$\text{Test A: } f_c = 1/[2\pi R1C1] = 1/[2\pi(50)(0.001EE-6)] = 3.183EE+6$$

$$f_c = 3,183K \text{ Hz} = 3.2 \text{ Meg Hz}$$

$$\text{Test B: } f_c = 1/[2\pi R1C1] = 1/[2\pi(50)(0.0022\text{EE-6})] = 1.447\text{EE+6}$$

$$f_c = 1,447\text{K Hz} = 1.4\text{Meg Hz}$$

d) Does circuit B behave like circuit A above or below the corner frequency? Use limits to show how you know this. (6 points)

$$\frac{V_{out}}{V_{in}} = \frac{-j\omega R2C1}{1 + j\omega R1C1}$$

$$\text{at low frequencies: } \frac{V_{out}}{V_{in}} = \frac{-j\omega R2C1}{1} = -j\omega R2C1 \quad \text{differentiator (like A)}$$

$$\text{at high frequencies: } \frac{V_{out}}{V_{in}} = \frac{-j\omega R2C1}{j\omega R1C1} = \frac{-R2}{R1} \quad \text{(inverting amplifier)}$$

**Circuit B will behave like circuit A below the corner frequency.**

e) At about what frequencies does the 50 ohm impedance of the function generator prevent this circuit from differentiating? (4 points)

*Circuit B should act like a differentiator at frequencies much less than  $f_c$ . Therefore, it will not differentiate above  $f_c$ . Minimally we could say that the resistance prevents this circuit from working at frequencies above about 3 Meg Hz for Test A and 1.5 Meg Hz for Test B. Even better, we could estimate that it would not be differentiating even above frequencies a bit lower, perhaps 1 Meg Hz for Test A and 1.5 Meg Hz for Test B. For a conservative estimate, we have plenty of frequency range available to go down an entire decade. Therefore, I would say conservatively,*

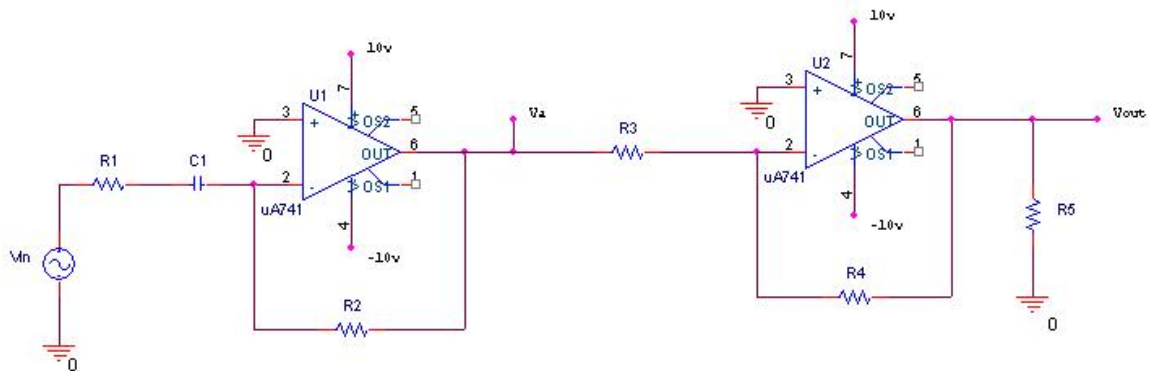
**Test A: 300K Hz and above**

**Test B: 150K Hz and above**

**(answers may vary)**

Spring 2003

3. Integrator/Differentiator (30 pts)

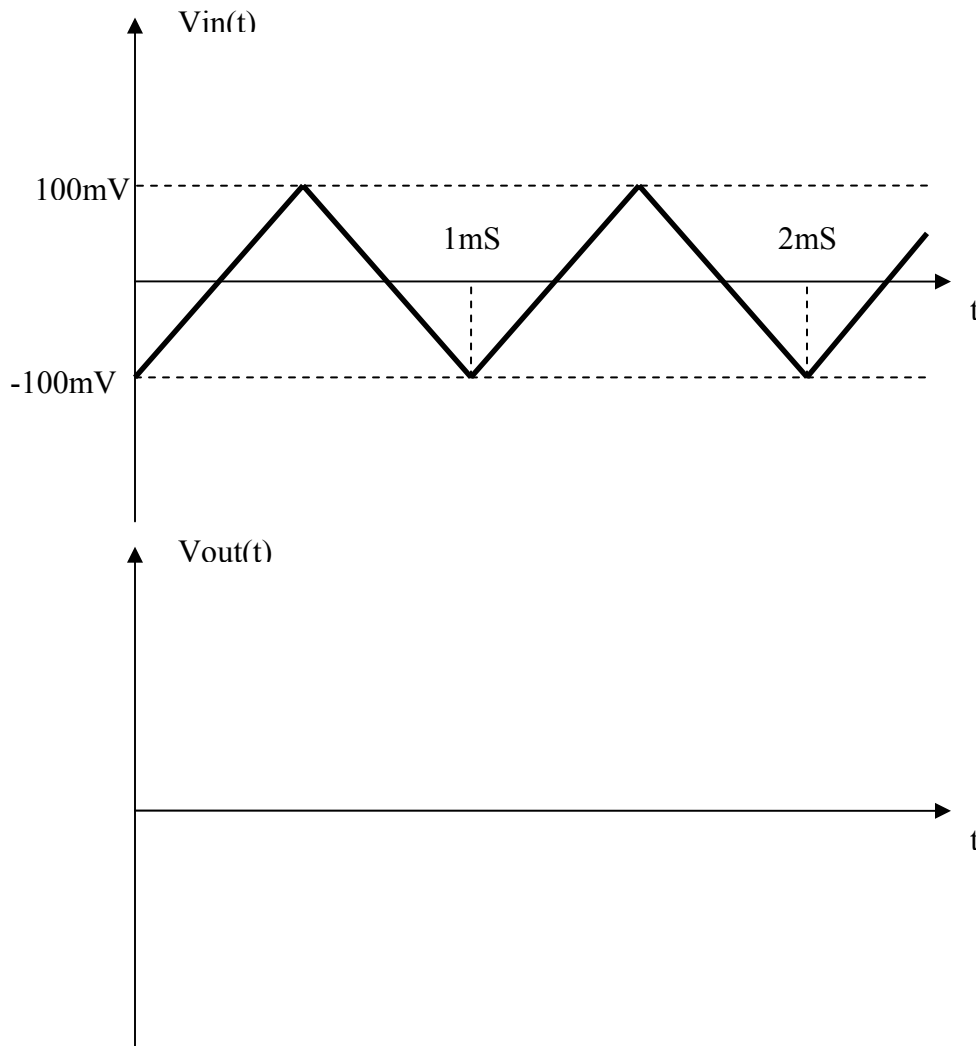


- a) Write down the transfer function for the first circuit  $H_1(j\omega)=V_a/V_{in}$ ? (4 pts)
  
- b) Write down the transfer function for the second circuit  $H_2(j\omega)=V_{out}/V_a$ ? (4 pts)
  
- c) Find the total transfer function  $H(j\omega)=V_{out}/V_{in}$ . (4 pts)
  
  
- d) By calculating the approximate transfer function, show that at frequencies much lower than  $\omega_c=1/(R_1C_1)$ , the circuit acts as a differentiator and at frequencies much higher than  $\omega_c$ , it acts as an amplifier. (6 pts)

- e) In the range where the circuit works as a differentiator, what is the time domain equation for the whole circuit? i.e. write an expression for  $V_{out}(t)$  as a function of  $V_{in}(t)$  (and the necessary component values). (5 pts)

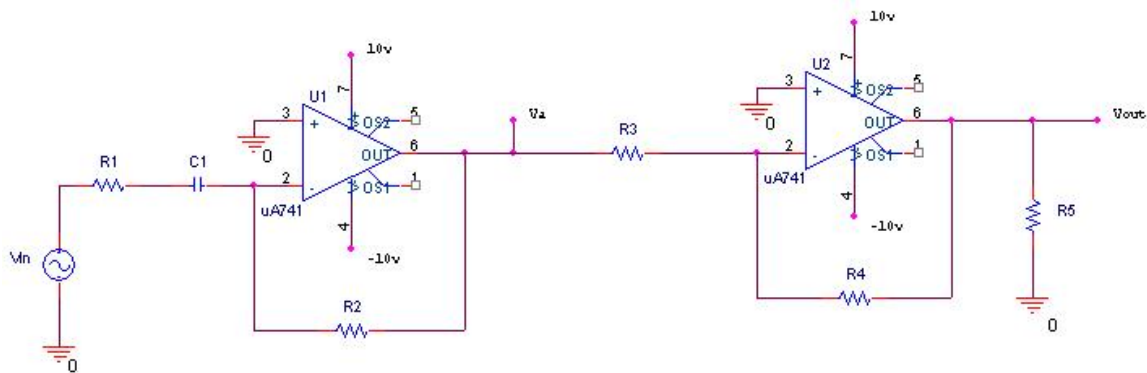
Assume  $R_1 = 100\Omega$ ,  $R_2 = 1K\Omega$ ,  $R_3 = 1K\Omega$ ,  $R_4 = 10K\Omega$ ,  $R_5 = 1M\Omega$  and  $C_1 = 1\mu F$ .

- f) Assume that the input voltage is the triangular wave shown below, draw the waveform of the output signal  $V_{out}(t)$ . Make sure that you clearly label all important times and voltages on the plot. (7 pts) (*Hint: Compare the frequency of the signal with  $\omega_c$  and decide how does the circuit act at this frequency.*)



Spring 2003 solution

3. Integrator/Differentiator (30 pts)



d) Write down the transfer function for the first circuit  $H_1(j\omega) = V_a/V_{in}$ ? (4 pts)

$$H_1(j\omega) = -j\omega R_2 C_1 / (1 + j\omega R_1 C_1)$$

e) Write down the transfer function for the second circuit  $H_2(j\omega) = V_{out}/V_a$ ? (4 pts)

$$H_2(j\omega) = -R_4/R_3$$

f) Find the total transfer function  $H(j\omega) = V_{out}/V_{in}$ . (4 pts)

$$H(j\omega) = H_1(j\omega) \times H_2(j\omega) = j\omega R_2 R_4 C_1 / (R_3 + j\omega R_1 R_3 C_1)$$

d) By calculating the approximate transfer function, show that at frequencies much lower than  $\omega_c = 1/(R_1 C_1)$ , the circuit acts as a differentiator and at frequencies much higher than  $\omega_c$ , it acts as an amplifier. (6 pts)

$$H_{Lo}(j\omega) = j\omega R_2 R_4 C_1 / R_3 \quad \text{This is a differentiator.}$$

$$H_{Hi}(j\omega) = j\omega R_2 R_4 C_1 / j\omega R_1 R_3 C_1 = R_2 R_4 / R_1 R_3 \quad \text{This is an amplifier.}$$

f) In the range where the circuit works as a differentiator, what is the time domain equation for the whole circuit? i.e. write an expression for  $V_{out}(t)$  as a function of  $V_{in}(t)$  (and the necessary component values). (5 pts)

The time domain equation for a differentiator is  $V_{out}(t) = -RC(dV_{in}(t)/dt)$  and  $H(j\omega) = -j\omega RC$ .

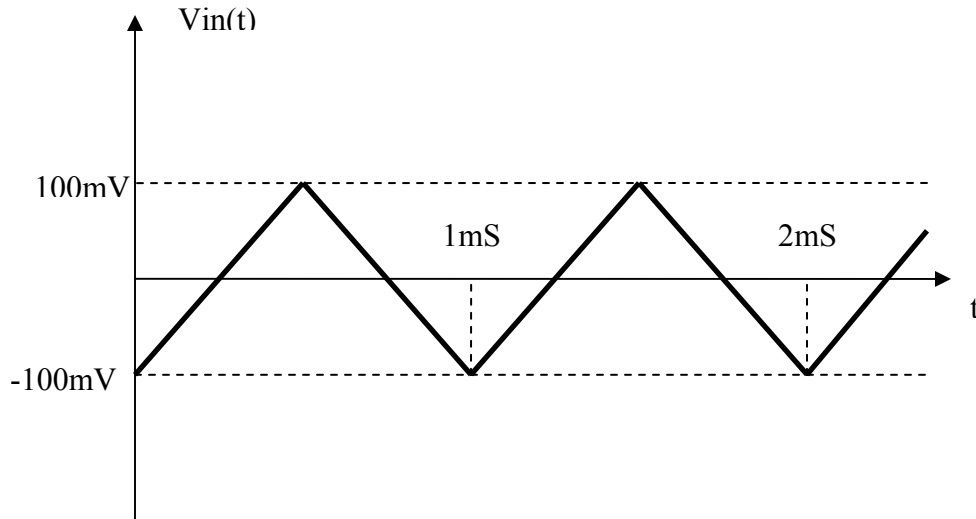
Our integrator has a transfer function of  $H(j\omega) = +j\omega R_2 R_4 C_1 / R_3$ , therefore the time domain equation must be

$$V_{out}(t) = (R_2 R_4 C_1 / R_3)(dV_{in}(t)/dt) = 0.01(dV_{in}(t)/dt)$$

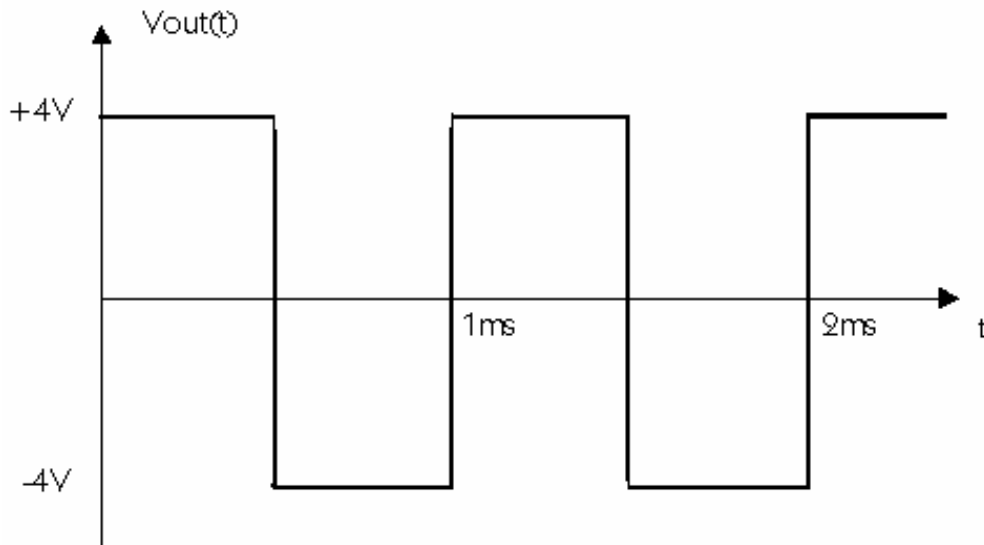
(If we substitute the values, we get  $(R_2 R_4 C_1 / R_3) = 1K \times 10K \times 1\mu / 1K = .01$ )



Assume  $R_1 = 100\Omega$ ,  $R_2 = 1K\Omega$ ,  $R_3 = 1K\Omega$ ,  $R_4 = 10K\Omega$ ,  $R_5 = 1M\Omega$  and  $C_1 = 1\mu F$ .  
 f) Assume that the input voltage is the triangular wave shown below, draw the waveform of the output signal  $V_{out}(t)$ . Make sure that you clearly label all important times and voltages on the plot. (7 pts) (Hint: Compare the frequency of the signal with  $\omega_c$  and decide how does the circuit act at this frequency.)



$\omega_c = 1/RC_1 = 1/(100 \times 1\mu) = 0.01 \times 10^6 = 10,000 \text{ rad/sec}$   
 $f_c = \omega_c / (2\pi) = 10000 / (2\pi) = 1600 \text{ Hz}$   
 $T = 1 \text{ ms}$   $f = 1000 \text{ Hz}$   $1000 \ll 1600$  Therefore, we have a differentiator.  
 $\text{slope} = \Delta y / \Delta x = 200 \text{ mV} / 0.5 \text{ ms} = 400 \text{ mV/ms} = 400 \text{ V/s}$   
 $V_{out}(t) = 0.01 (dV_{in}(t)/dt) = 0.01 \times 400 = 4 \text{ V}$ .

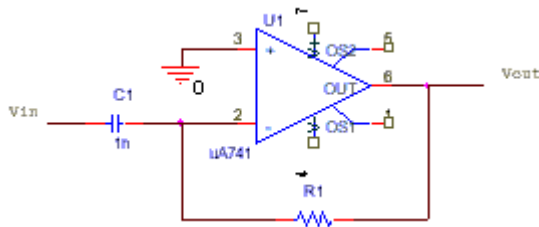


Spring 2002

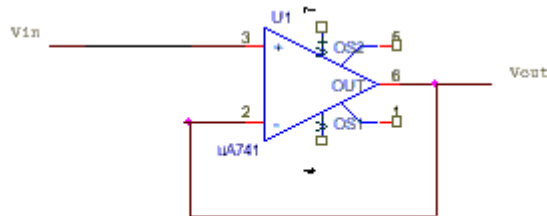
4) Op-Amp Configurations (20 points)

a) Seven circuits are shown below. (Connections to power supply are not displayed but assumed). For each circuit, identify the function (1 pnt each) and give the mathematical relationship that relates the input and output voltages (1 pnt each).

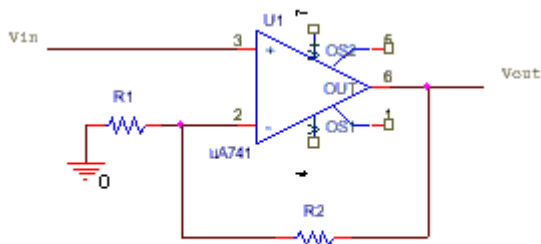
1.



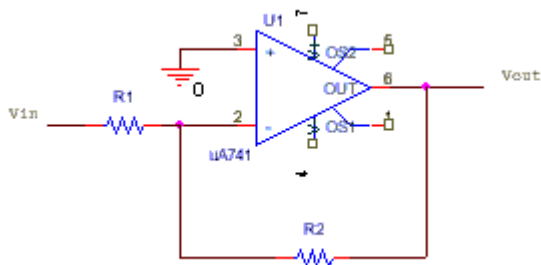
2.



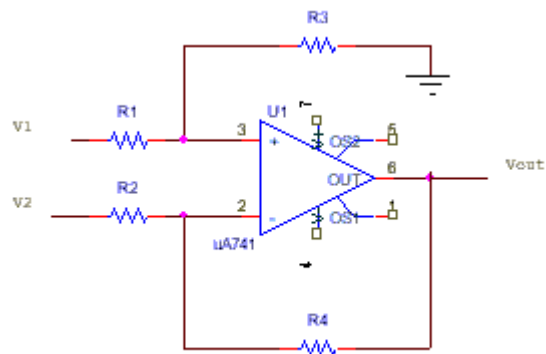
3.



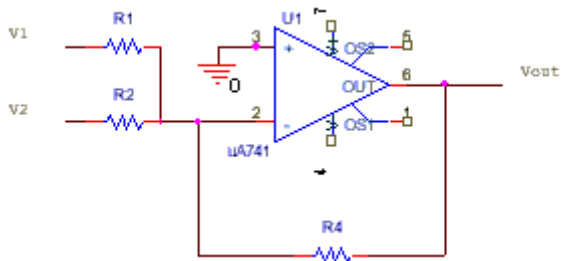
4.



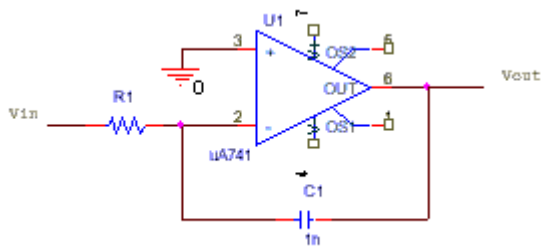
5.



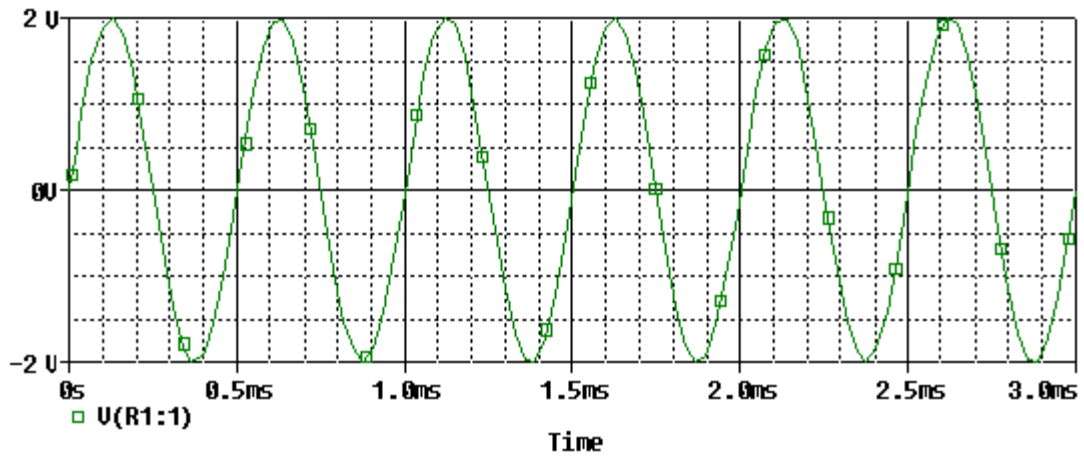
6.



7.



b) Consider the following input plot:



If this was used as the input to the circuit in 1 of this question, sketch what the graph of the output would be. Make sure the frequency and the amplitude and the value at 0 seconds are clearly indicated. Assume  $C1=1$  nF and  $R1=1$  K ohms. Show your work.

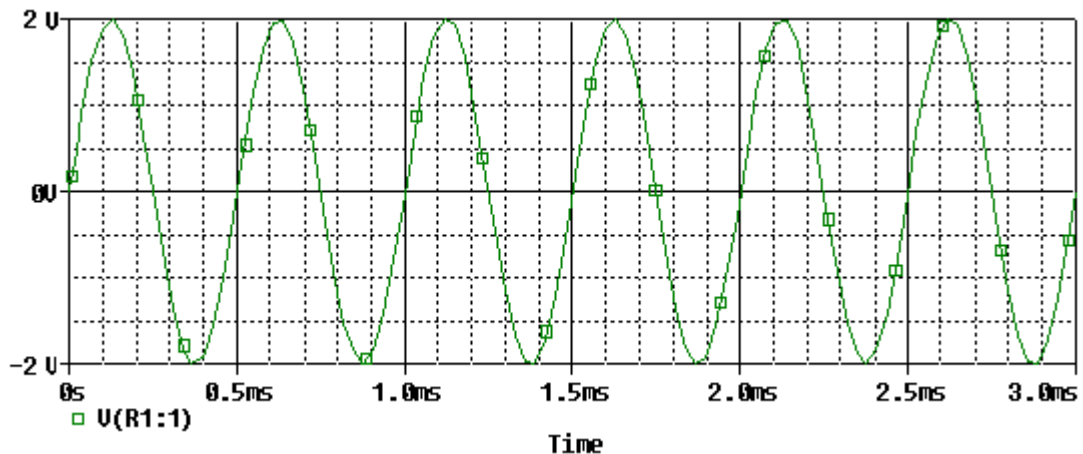
*Spring 2002 solution*

**4) Op-Amp Configurations (20 points)**

a) Seven circuits are shown below. (Connections to the power supply are not displayed but assumed). For each circuit, identify the function (1 pnt each) and give the mathematical relationship that relates the input and output voltages (1 pnt each).

1. ideal differentiator  $V_{out}/V_{in} = -j\omega R1C1$
2. voltage follower  $V_{out}/V_{in} = 1$
3. non-inverting op-amp  $V_{out}/V_{in} = (1+R2/R1)$
4. inverting op-amp  $V_{out}/V_{in} = -R2/R1$
5. differential amplifier  $V_{out}/(V1-V2) = -R3/R1$  (iff  $R1=R2, R3=R4$ )
6. weighted adder  $V_{out} = -R4(V1/R1 + V2/R2)$
- or-
- adder  $V_{out}/(V1+V2) = -R4/R1$  (iff  $R1=R2$ )
7. ideal integrator  $V_{out}/V_{in} = -1/(j\omega R1C1)$

b) Consider the following input plot:



If this was used as the input to the circuit in 1 of this question, sketch what the graph of the output would be. Make sure the frequency and the amplitude and the value at 0 seconds are clearly indicated. Assume  $C1=1$  nF and  $R1=1$  K ohms. Show your work.

*The circuit in 1 is an ideal differentiator.  $H(j\omega) = -j\omega RC$*

*$\omega = 2\pi f = 2(3.14)(1/0.5m) = 12.566$  K rad/sec*

*The plot would have an amplitude of  $(2V)(\omega)(R)(C) = (2)(12.566K)(1K)(1 \times 10^{-9})$   
amplitude =  $0.025$  V = 25 mV*

*The plot would have a phase shift of -90 degrees (negative j).*

*The frequency would not change.*

*Therefore, it would look like a negative cosine with an amplitude of 25 mV and the same frequency. At 0, 0.5ms, 1.0ms ..., the amplitude would be -25 mV. At .25ms, .75ms, 1.25ms ....., the amplitude would be +25 mV. You can sketch it.*

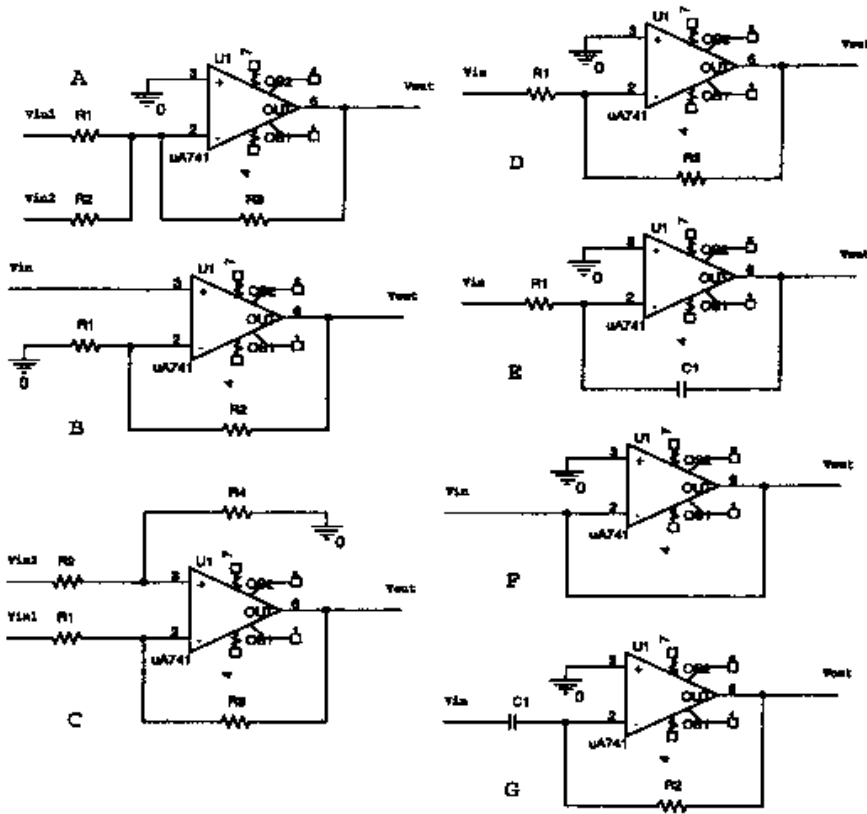
EI TEST 3A Fall 2001

Name \_\_\_\_\_ Sect \_\_\_\_\_

Please show all work on all questions for full credit, some explanation of your answer is required.

4. Op-Amp Configurations (20 points)

a) Seven circuits (A to G) are shown below. (Connections to power supply are not displayed but assumed).



10.6

For each circuit, determine the type (function) of the circuit (1 point each) and the mathematical relationship that relates Output and Input voltages (1 point each).

A. Adder

$$V_{out} = -R_3 \left( \frac{V_{in1}}{R_1} + \frac{V_{in2}}{R_2} \right)$$

B. Non-Inv. Amp.

$$V_{out} = \left( 1 + \frac{R_2}{R_1} \right) V_{in}$$

C. Differential Amp

$$V_{out} = \frac{R_3}{R_1} (V_{in2} - V_{in1})$$

Please show all work on all questions for full credit, some explanation of your answer is required.

D. Inverting Amp.

$$V_{out} = -\frac{R_f}{R_i} V_{in}$$

E. Ideal integrator

$$V_{out} = -\frac{1}{RC} \int V_{in} dt$$

F. Voltage follower (buffer)

$$V_{out} = V_{in}$$

G. Ideal differentiator

$$V_{out} = -RC \frac{dV_{in}}{dt}$$

b) Assume that for an ideal differentiator circuit,  $R = 200\Omega$ , and  $C = 5\mu F$ . If the input voltage is  $V_{in} = 5v + 1v \sin(\omega t + \phi)$ , what is the output voltage. (6 points)

$$V_{out} = -RC \frac{dV_{in}}{dt}$$

$$\frac{dV_{in}}{dt} = 0 + (-\omega \times 1v) \cos(\omega t + \phi)$$

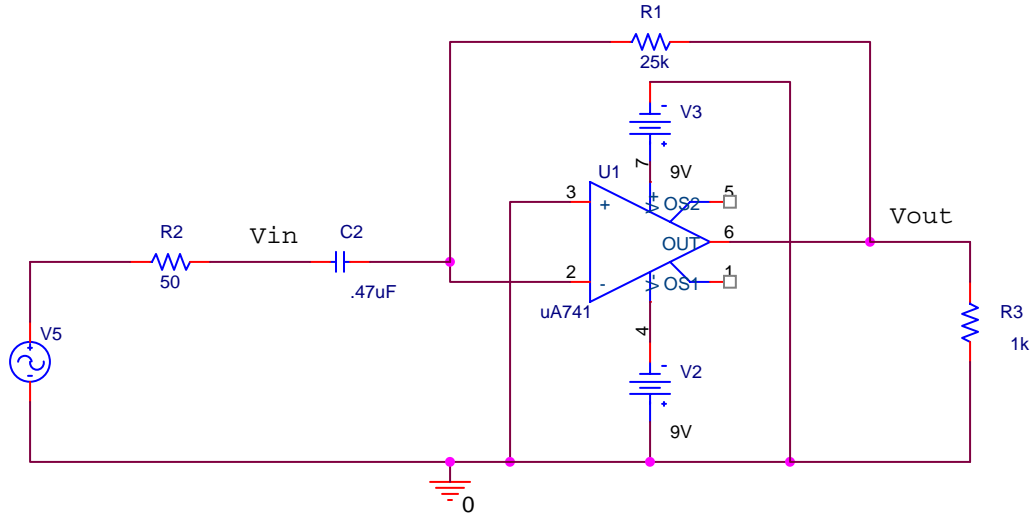
$$\Rightarrow V_{out} = -200 \times 5 \times 10^{-6} \omega \cos(\omega t + \phi)$$

$$\Rightarrow \boxed{V_{out} = -0.001 \omega \cos(\omega t + \phi)}$$

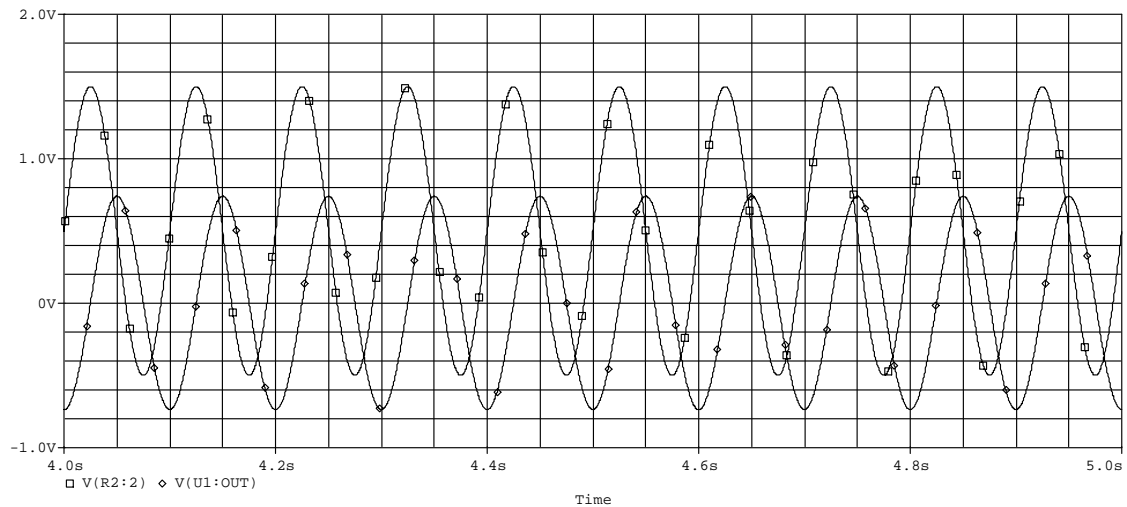
Fall 2000

#### 4. Op-Amp Configuration

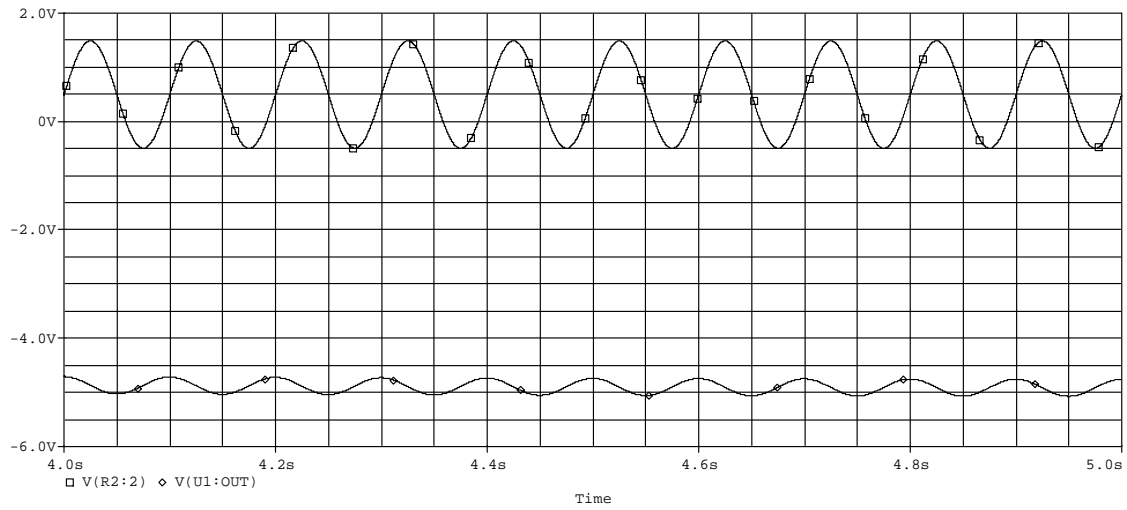
The following circuit is simulated using PSpice. One of the two probe plots shown below shows the correct voltages at  $V_{in}$  and  $V_{out}$  in this circuit. Which one is the correct plot? Both the input and output voltages are plotted in each case.



What kind of op-amp configuration is this circuit?







What relationship is supposed to exist between the two voltages that are plotted, if the circuit is working more-or-less in an ideal sense?

Show, using the information in the plot, that the circuit is working reasonably correctly.