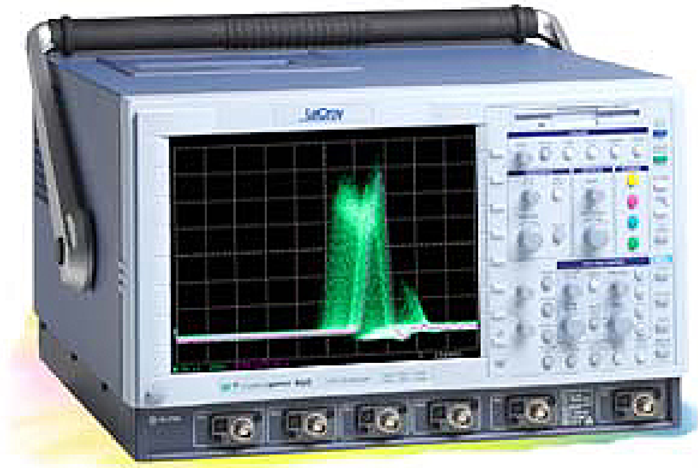


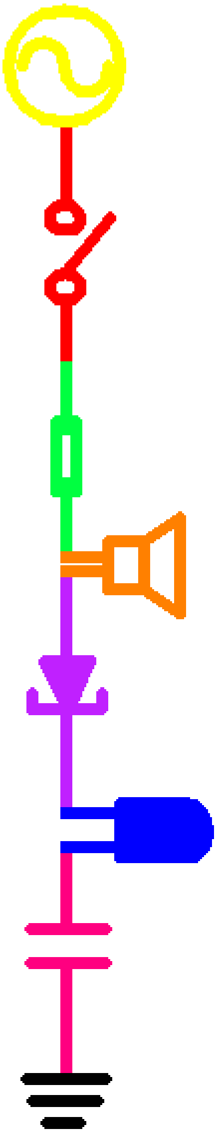
Electronic Instrumentation

Experiment 5

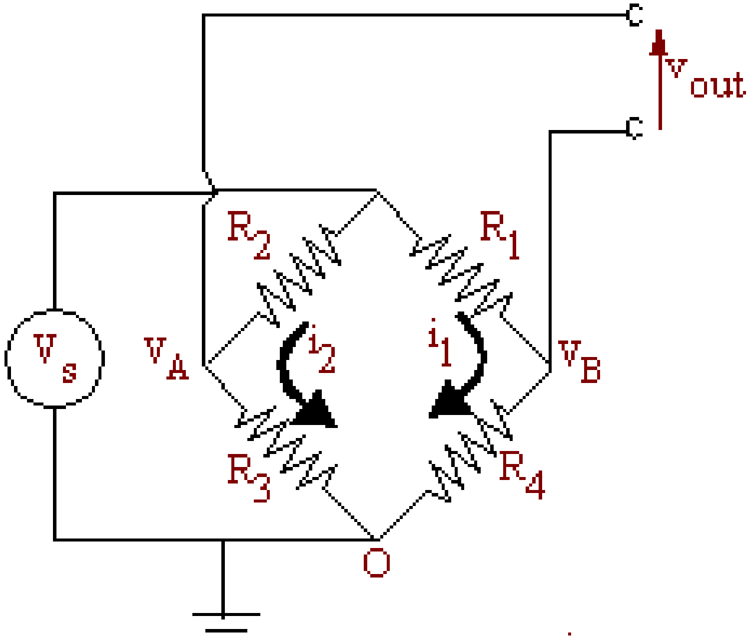
* Part A: Bridge Circuits
* Part B: Potentiometers and Strain Gauges
* Part C: Oscillation of an Instrumented Beam
* Part D: Oscillating Circuits

Part A

- Bridges
- Thevenin Equivalent Circuits



Wheatstone Bridge

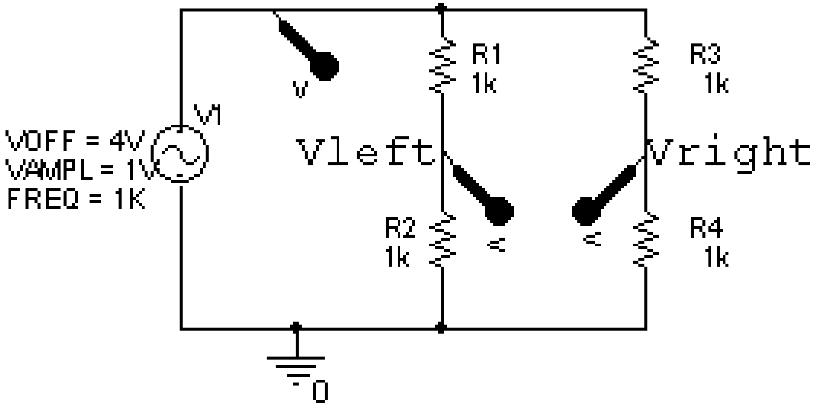


The Wheatstone Bridge

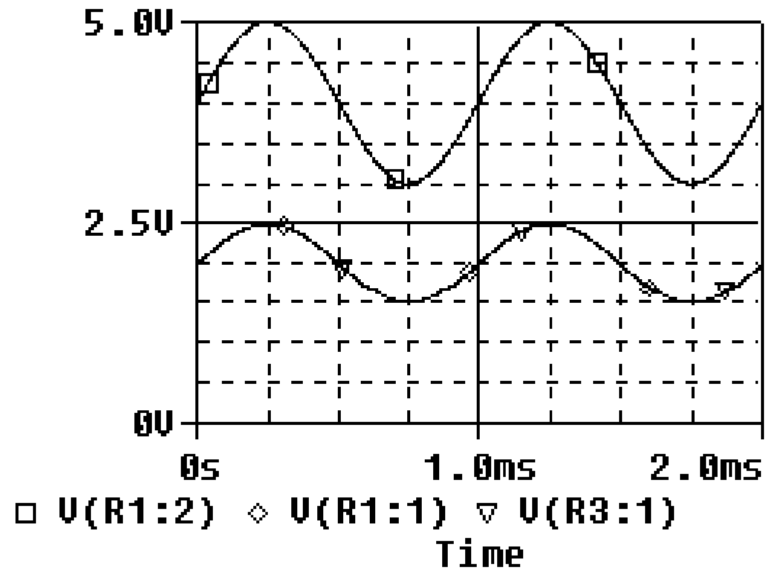
A bridge is just two voltage dividers in parallel. The output is the difference between the two dividers.

$$V_A = \frac{R3}{R2 + R3} V_S \qquad V_B = \frac{R4}{R1 + R4} V_S \qquad V_{out} = dV = V_A - V_B$$

A Balanced Bridge Circuit



Vleft = Vright

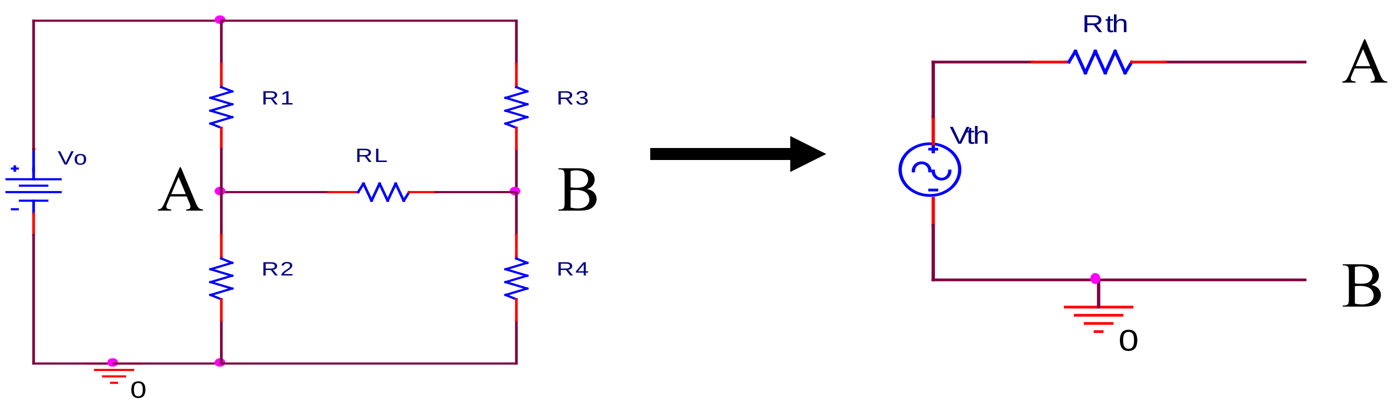


$$V_{left} = \frac{1K}{1K + 1K} V1 \qquad V_{right} = \frac{1K}{1K + 1K} V1$$

$$dV = V_{left} - V_{right} = \frac{V1}{2} - \frac{V1}{2} = 0$$

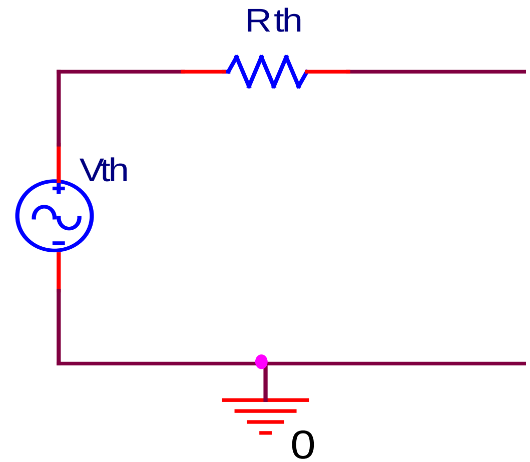
Thevenin Voltage Equivalents

- In order to better understand how bridges work, it is useful to understand how to create Thevenin Equivalents of circuits.
- Thevenin invented a model called a Thevenin Source for representing a complex circuit using
  - A single “pseudo” source, Vth
  - A single “pseudo” resistance, Rth



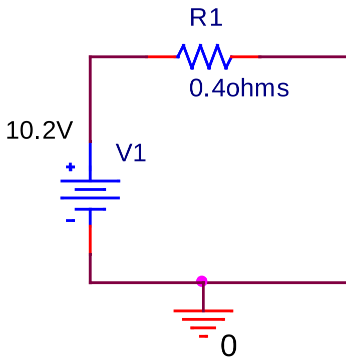
Thevenin Voltage Equivalents

The Thevenin source, "looks" to the load on the circuit like the actual complex combination of resistances and sources.

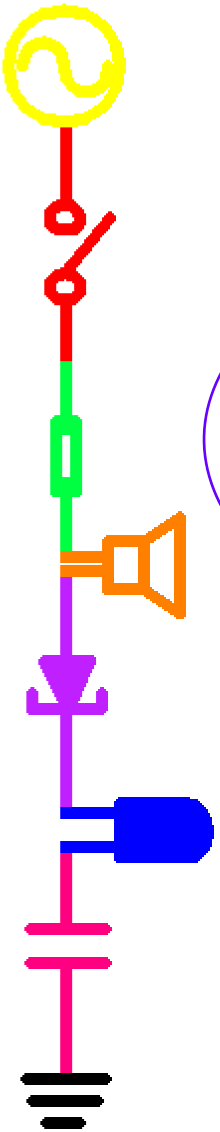


This model can be used interchangeably with the original (more complex) circuit when doing analysis.

The Battery Model

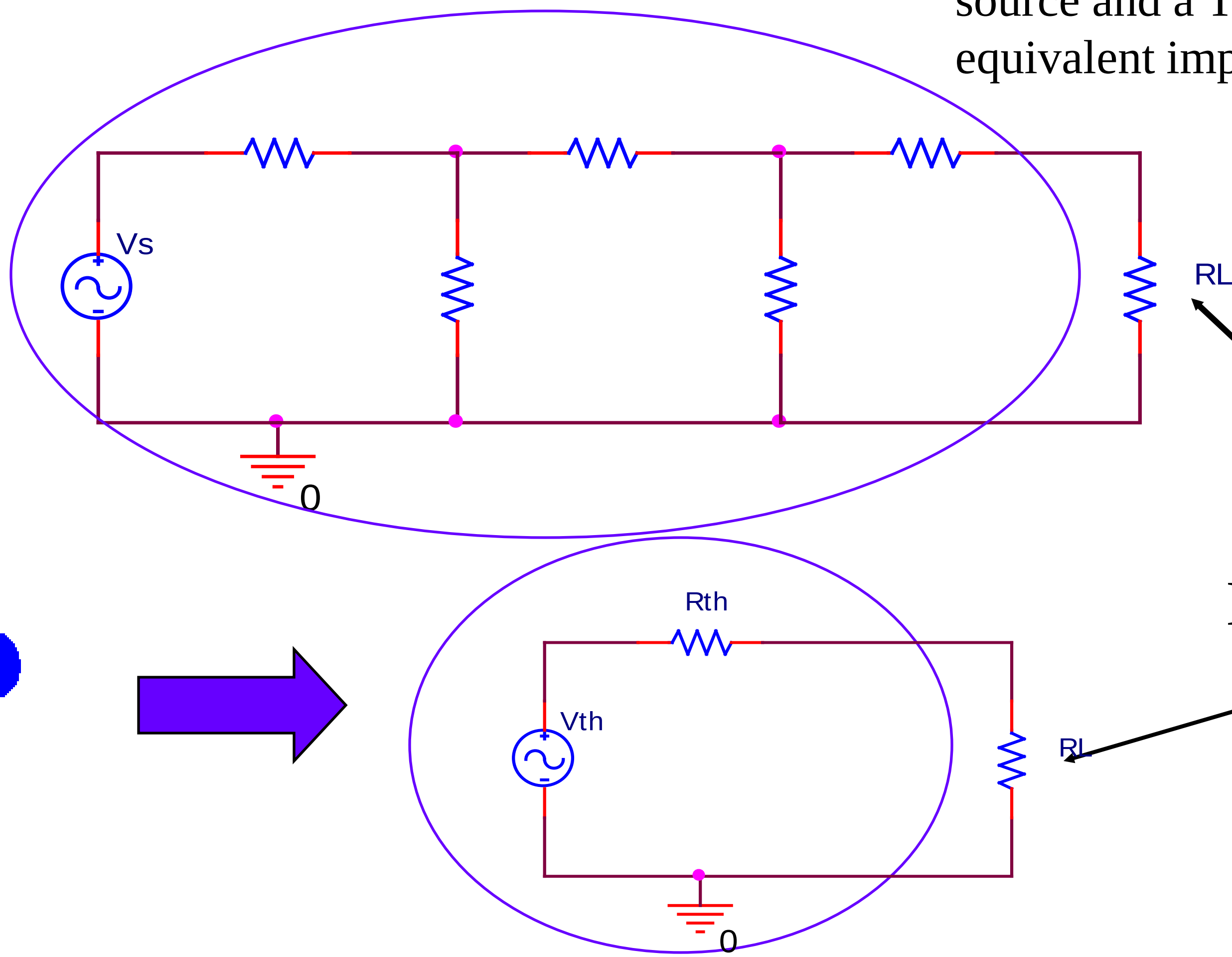


- Recall that we measured the internal resistance of a battery.
- This is actually the Thevenin equivalent model for the battery.
- The actual battery is more complicated – including chemistry, aging, …



Thevenin Model

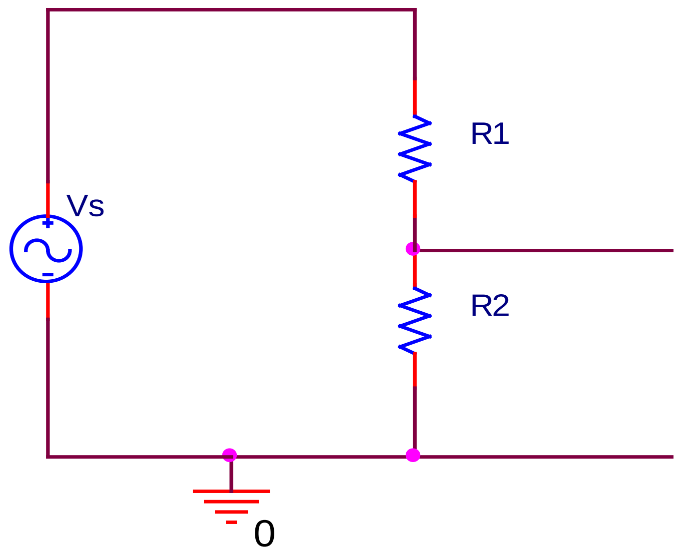
Any linear circuit connected to a load can be modeled as a Thevenin equivalent voltage source and a Thevenin equivalent impedance.

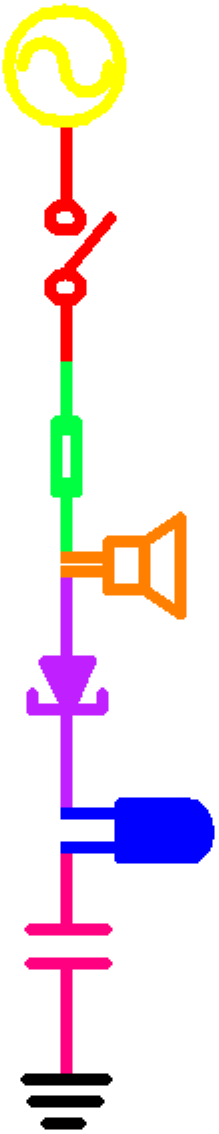


Load Resistor

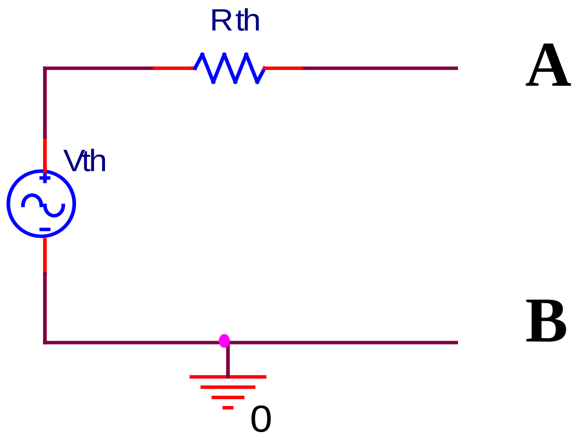
Note:

- We might also see a circuit with no load resistor, like this voltage divider.





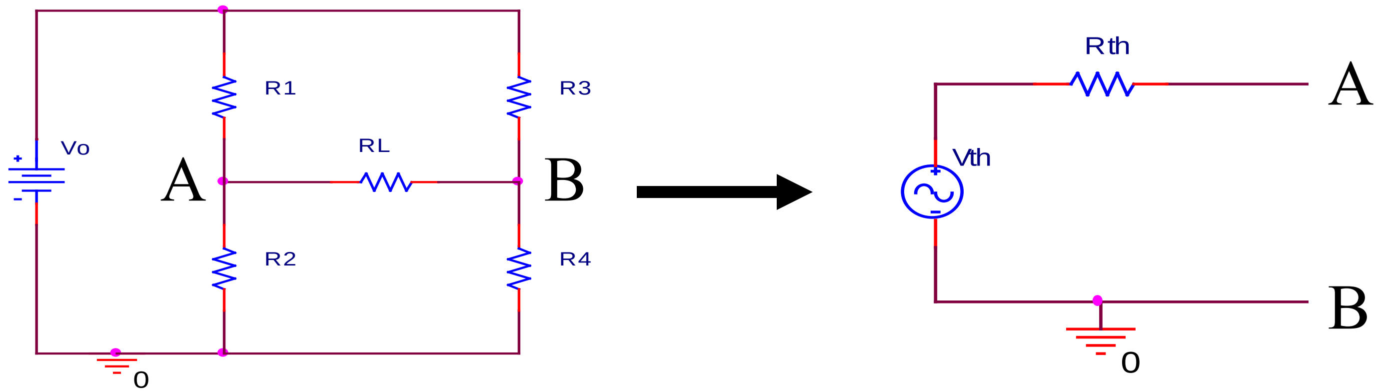
Thevenin Method



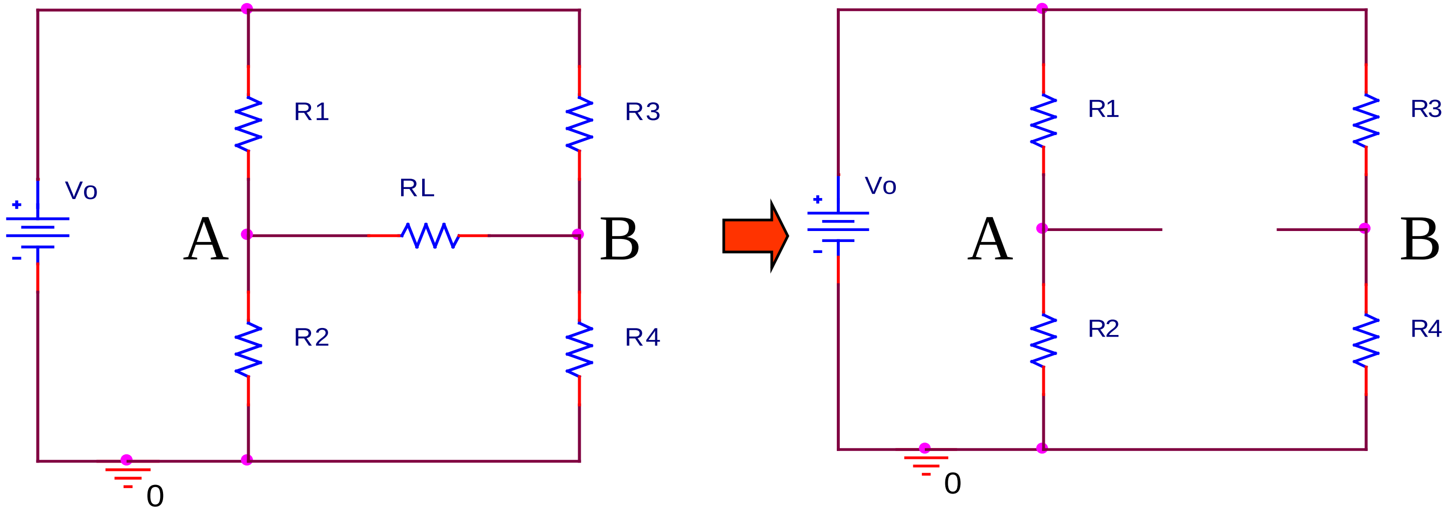
- Find Vth (open circuit voltage)
  - Remove load if there is one so that load is open
  - Find voltage across the open load
- Find Rth (Thevenin resistance)
  - Set voltage sources to zero (current sources to open) – in effect, shut off the sources
  - Find equivalent resistance from A to B

Example: The Bridge Circuit

- We can remodel a bridge as a Thevenin Voltage source



Find Vth by removing the Load



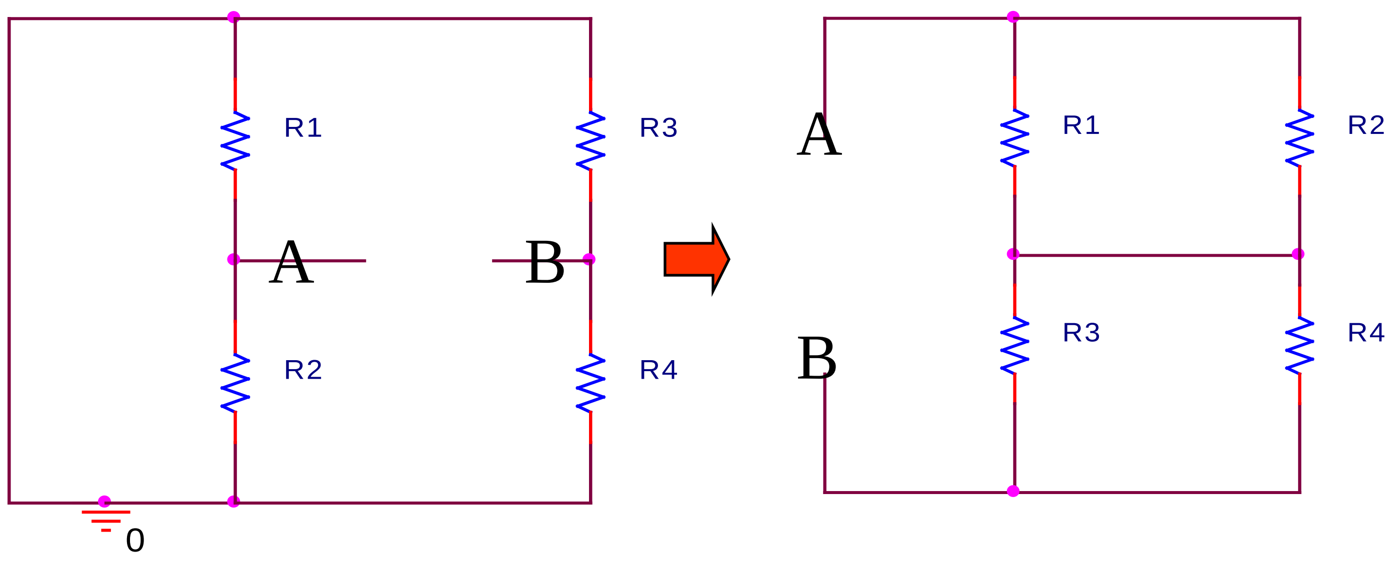
Let Vo=12, R1=2k, R2=4k, R3=3k, R4=1k

$$VB = \left(\frac{1k}{1k + 3k}\right)12 = 3V \quad VA = \left(\frac{4k}{4k + 2k}\right)12 = 8V$$

$$Vth = VA - VB = 8 - 3 = 5V$$

To find Rth

- First, short out the voltage source (turn it off) & redraw the circuit for clarity.



Find Rth

- Find the parallel combinations of R1 & R2 and R3 & R4.

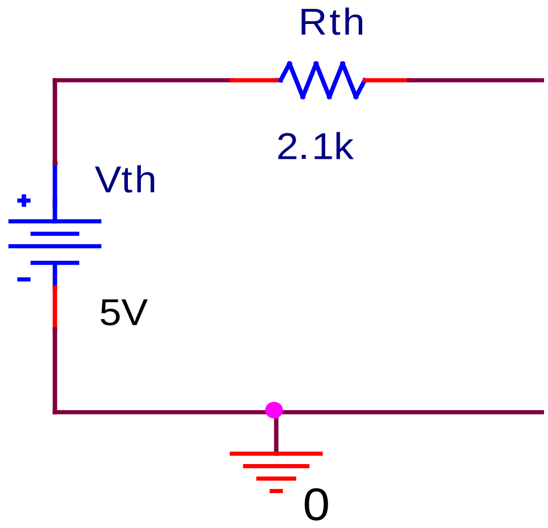
$$R12 = \frac{R1R2}{R1 + R2} = \frac{4k2k}{4k + 2k} = \frac{8k}{6} = 1.33k$$

$$R34 = \frac{R3R4}{R3 + R4} = \frac{1k3k}{1k + 3k} = \frac{3k}{4} = 0.75k$$

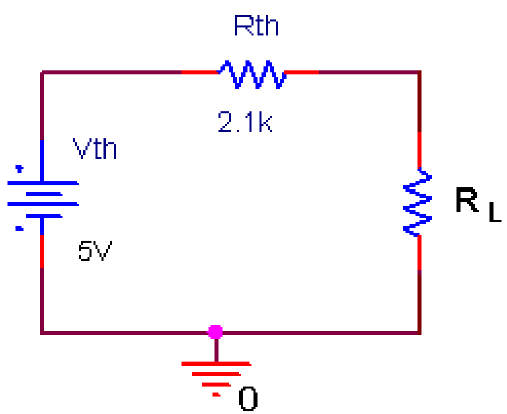
- Then find the series combination of the results.

$$Rth = R12 + R34 = \left(\frac{4}{3} + \frac{3}{4}\right)k = 2.1k$$

Redraw Circuit as a Thevenin Source

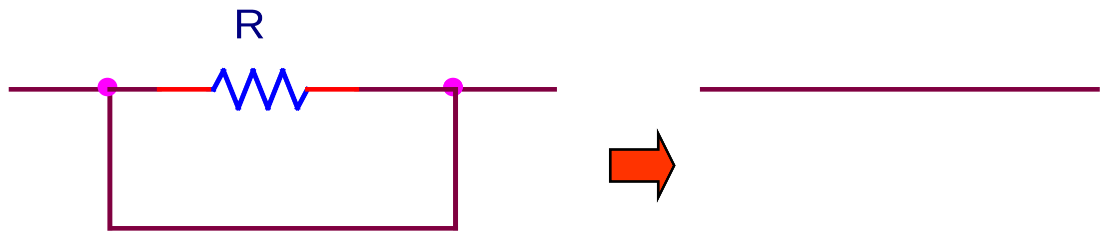


- Then add any load and treat it as a voltage divider.



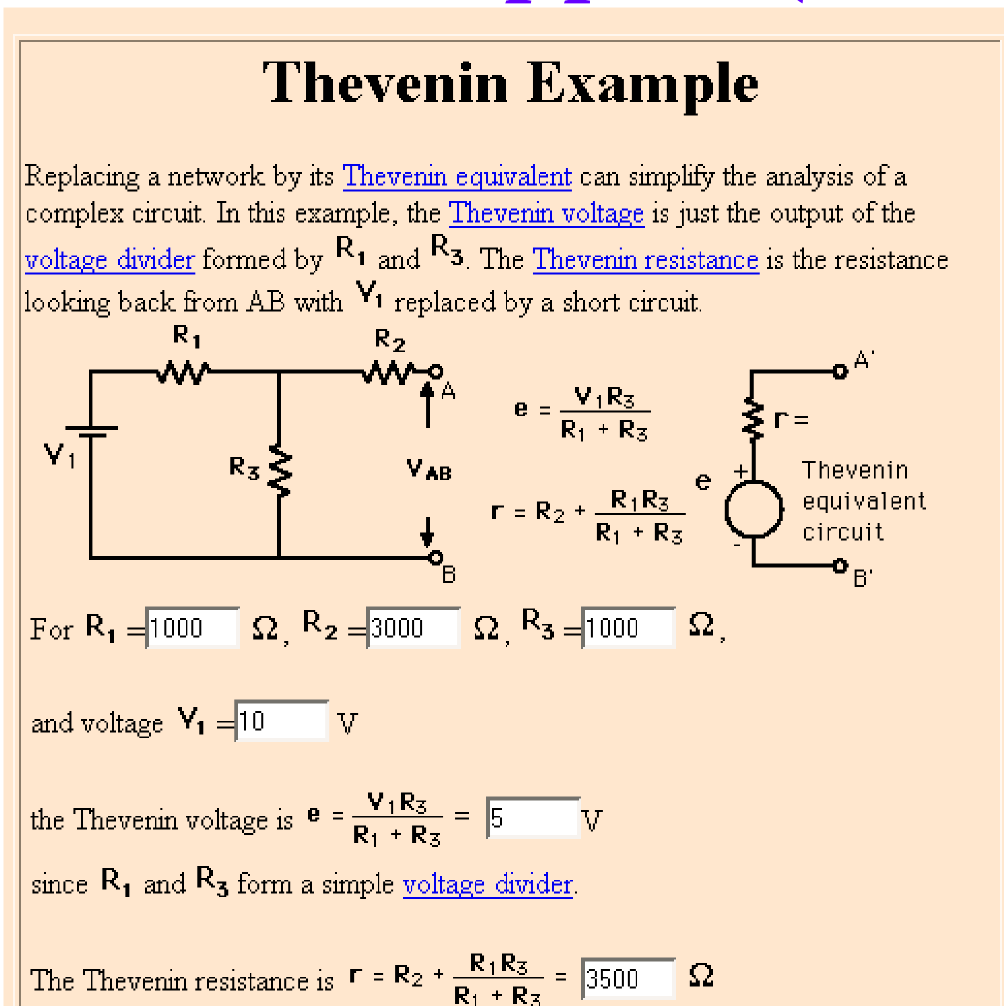
$$V_L = \frac{R_L}{R_{th} + R_L} V_{th}$$

Thevenin Method Tricks



- Note
  - When a short goes across a resistor, that resistor is replaced by a short.
  - When a resistor connects to nothing, there will be no current through it and, thus, no voltage across it.

Thevenin Applet (see webpage)



Thevenin Example

Replacing a network by its Thevenin equivalent can simplify the analysis of a complex circuit. In this example, the Thevenin voltage is just the output of the voltage divider formed by R_1 and R_3. The Thevenin resistance is the resistance looking back from AB with V_1 replaced by a short circuit.

For R_1 = 1000 Ω, R_2 = 3000 Ω, R_3 = 1000 Ω,

and voltage V_1 = 10 V

the Thevenin voltage is $e = \frac{V_1 R_3}{R_1 + R_3}$ = 5 V

since R_1 and R_3 form a simple voltage divider.

The Thevenin resistance is $r = R_2 + \frac{R_1 R_3}{R_1 + R_3}$ = 3500 Ω

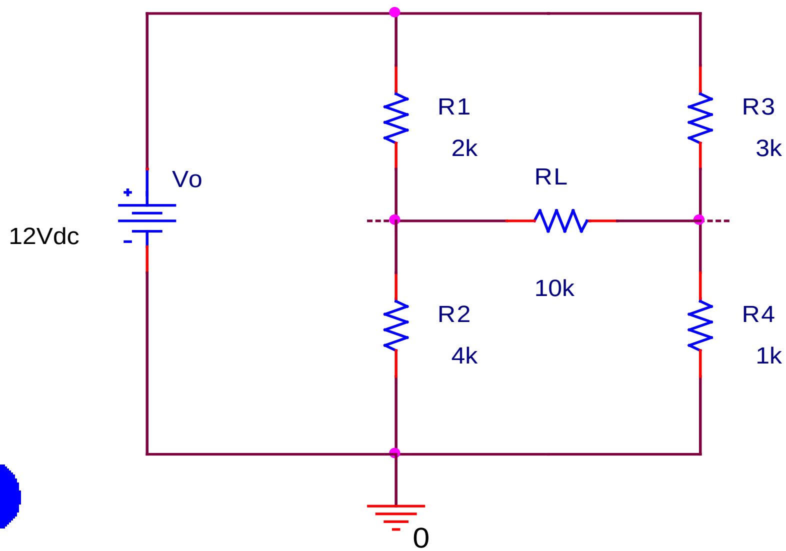
- Test your Thevenin skills using this applet from the links for Exp 3

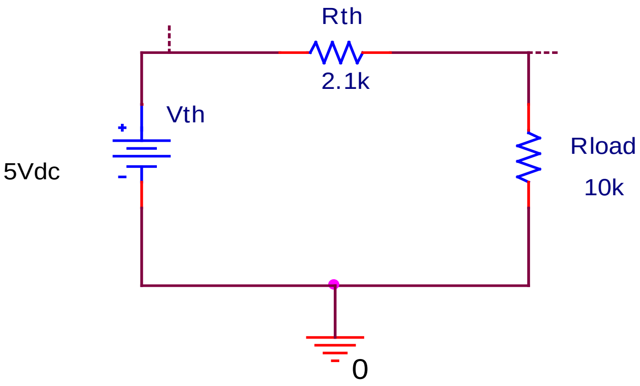
Does this really work?

- To confirm that the Thevenin method works, add a load and check the voltage across and current through the load to see that the answers agree whether the original circuit is used or its Thevenin equivalent.
- If you know the Thevenin equivalent, the circuit analysis becomes much simpler.

Thevenin Method Example

- Checking the answer with PSpice



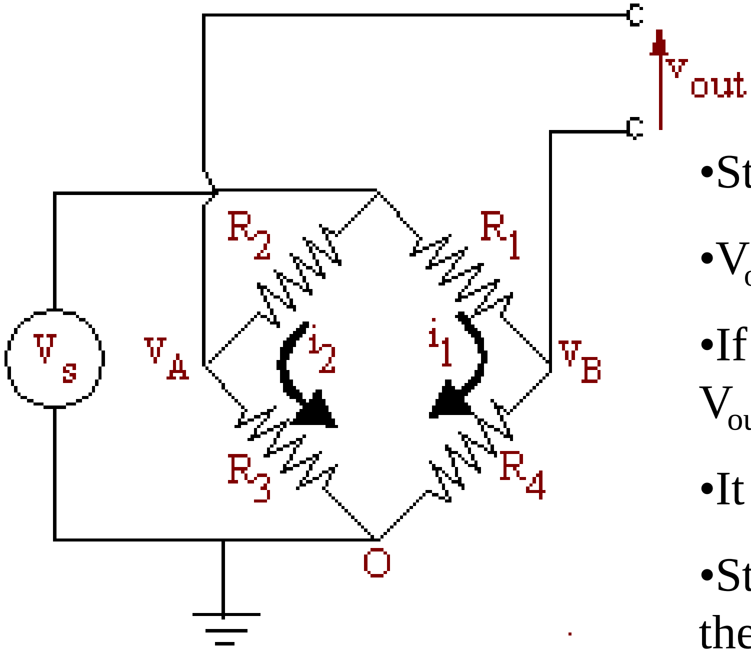


- Note the identical voltages across the load.
  - 7.4 - 3.3 = 4.1 (only two significant digits in Rth)

Thevenin's method is extremely useful and is an important topic.

- But back to bridge circuits – for a balanced bridge circuit, the Thevenin equivalent voltage is zero.
- An unbalanced bridge is of interest. You can also do this using Thevenin's method.
- Why are we interested in the bridge circuit?

Wheatstone Bridge

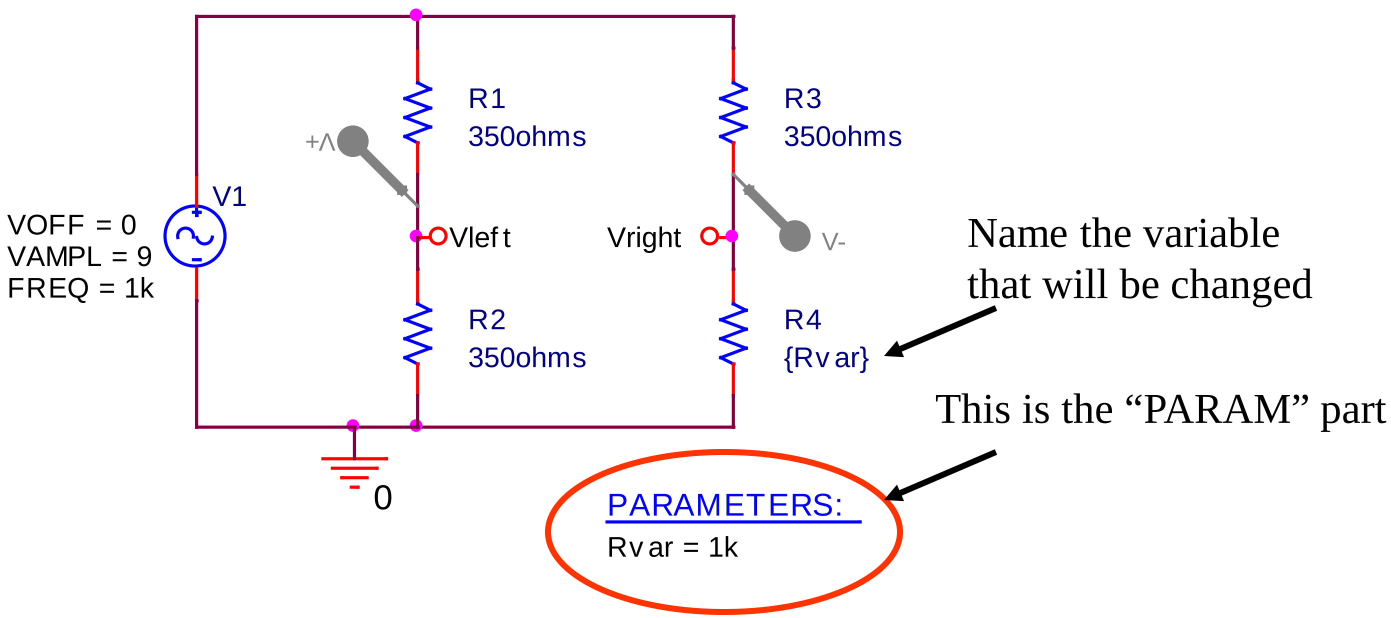


The Wheatstone Bridge

- Start with R1=R4=R2=R3
- $V_{out}=0$
- If one R changes, even a small amount, $V_{out} \neq 0$
- It is easy to measure this change.
- Strain gauges look like resistors and the resistance changes with the strain
- The change is very small.

$$V_A = \frac{R3}{R2 + R3} V_S \qquad V_B = \frac{R4}{R1 + R4} V_S \qquad V_{out} = dV = V_A - V_B$$

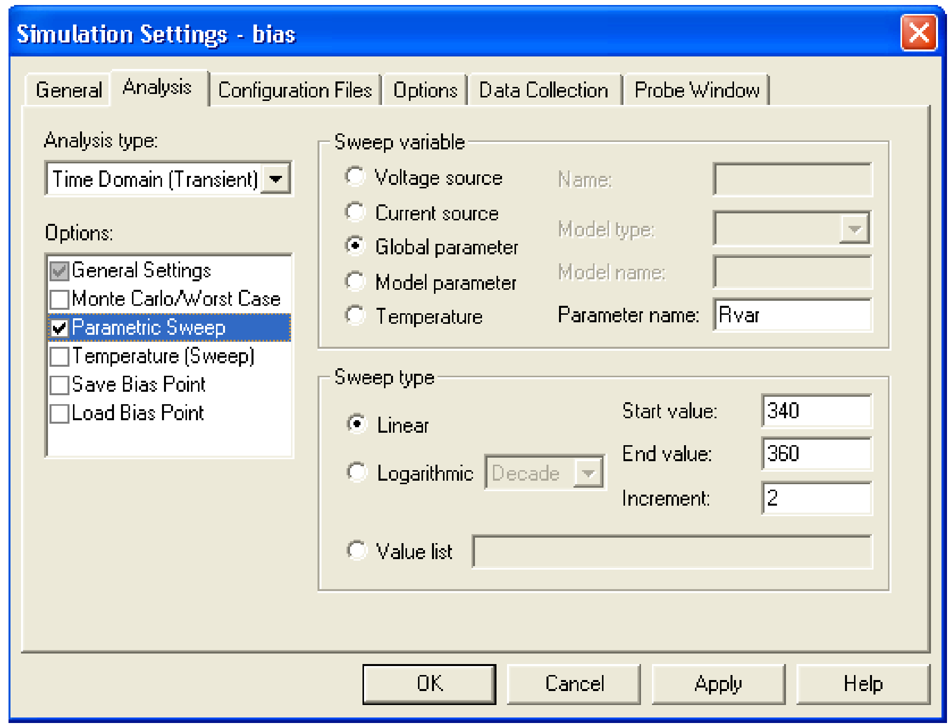
Using a parameter sweep to look at bridge circuits.



- PSpice allows you to run simulations with several values for a component.
- In this case we will "sweep" the value of R4 over a range of resistances.

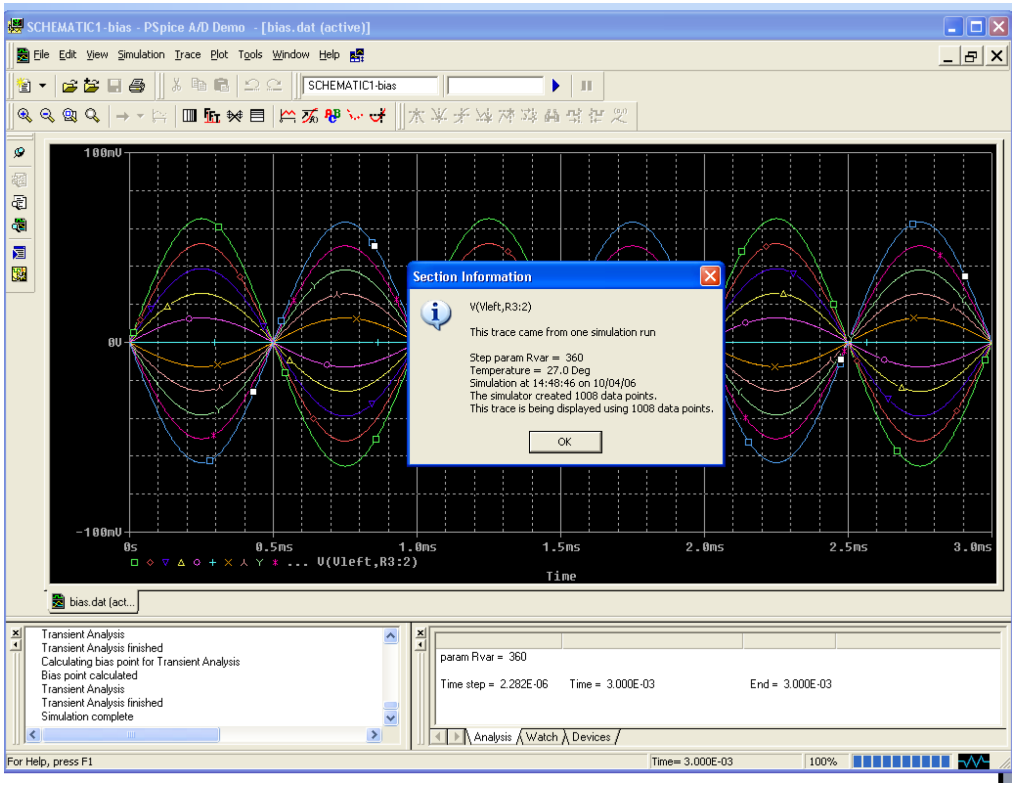
Parameter Sweep

- Set up the values to use.
- In this case, simulations will be done for 11 values for Rvar.



Parameter Sweep

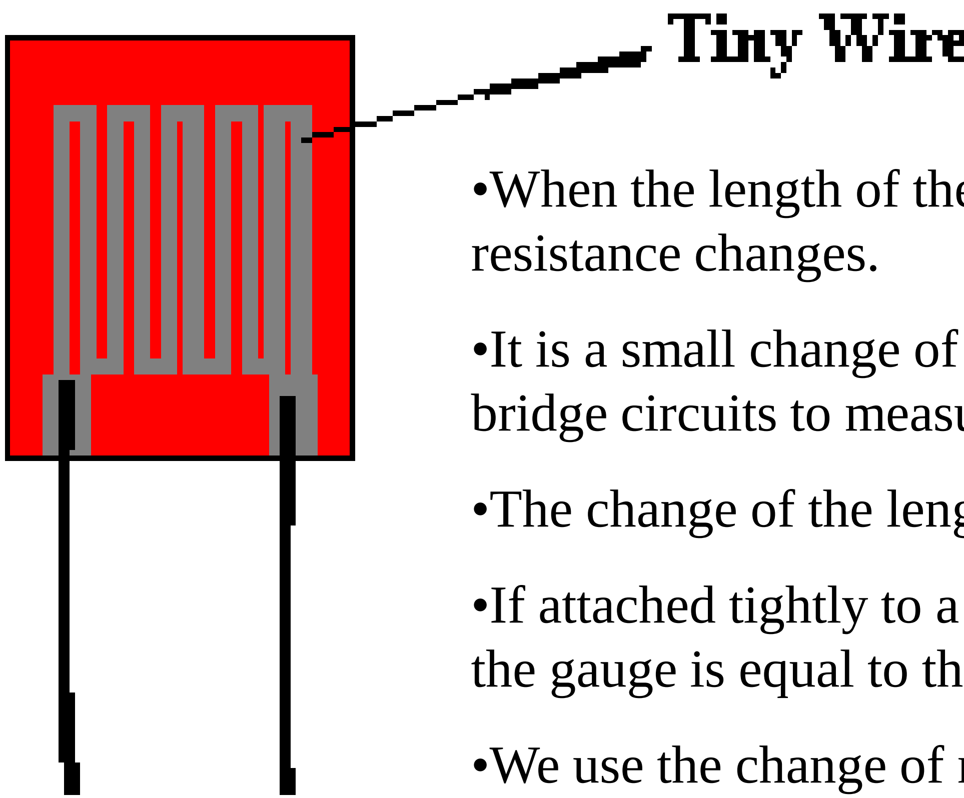
- All 11 simulations can be displayed
- Right click on one trace and select “information” to know which Rvar is shown.



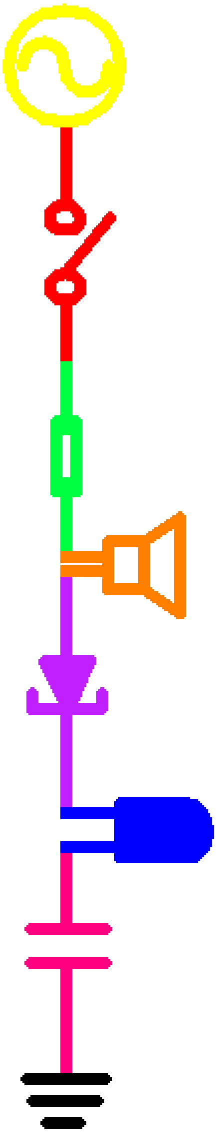
Part B

- Strain Gauges
- The Cantilever Beam
- Damped Sinusoids

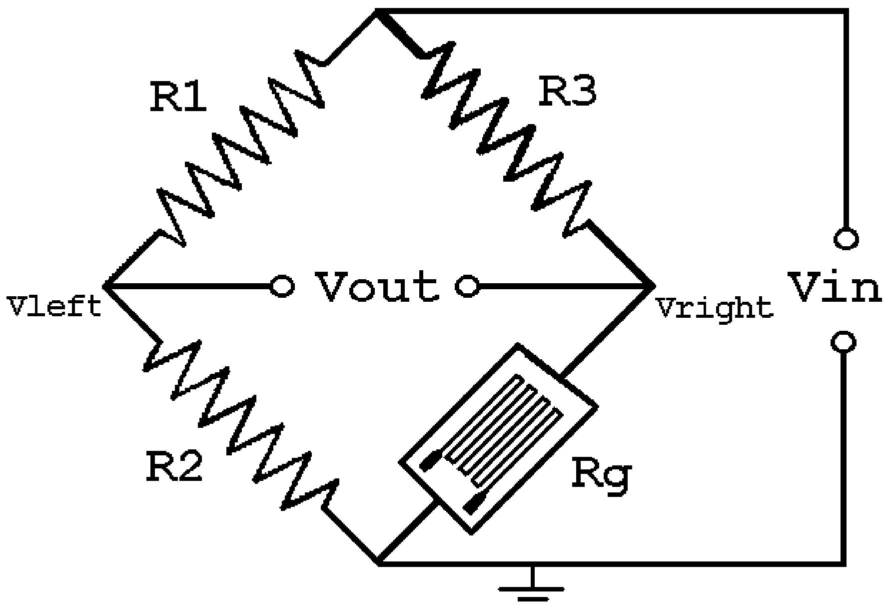
Strain Gauges



- When the length of the traces changes, the resistance changes.
- It is a small change of resistance so we use bridge circuits to measure the change.
- The change of the length is the strain.
- If attached tightly to a surface, the strain of the gauge is equal to the strain of the surface.
- We use the change of resistance to measure the strain of the beam.



Strain Gauge in a Bridge Circuit

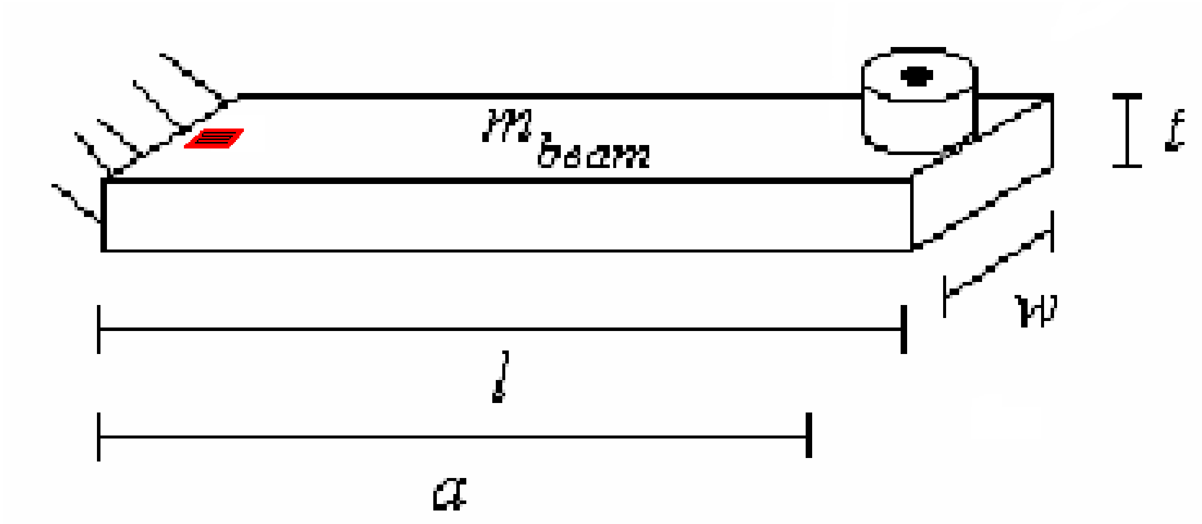


$$V_{out} = dV = V_{left} - V_{right}$$

$$V_{out} = V_{in}\left[\frac{R2}{R1+R2} - \frac{Rg}{R3+Rg}\right]$$

Two voltage dividers in parallel

Cantilever Beam

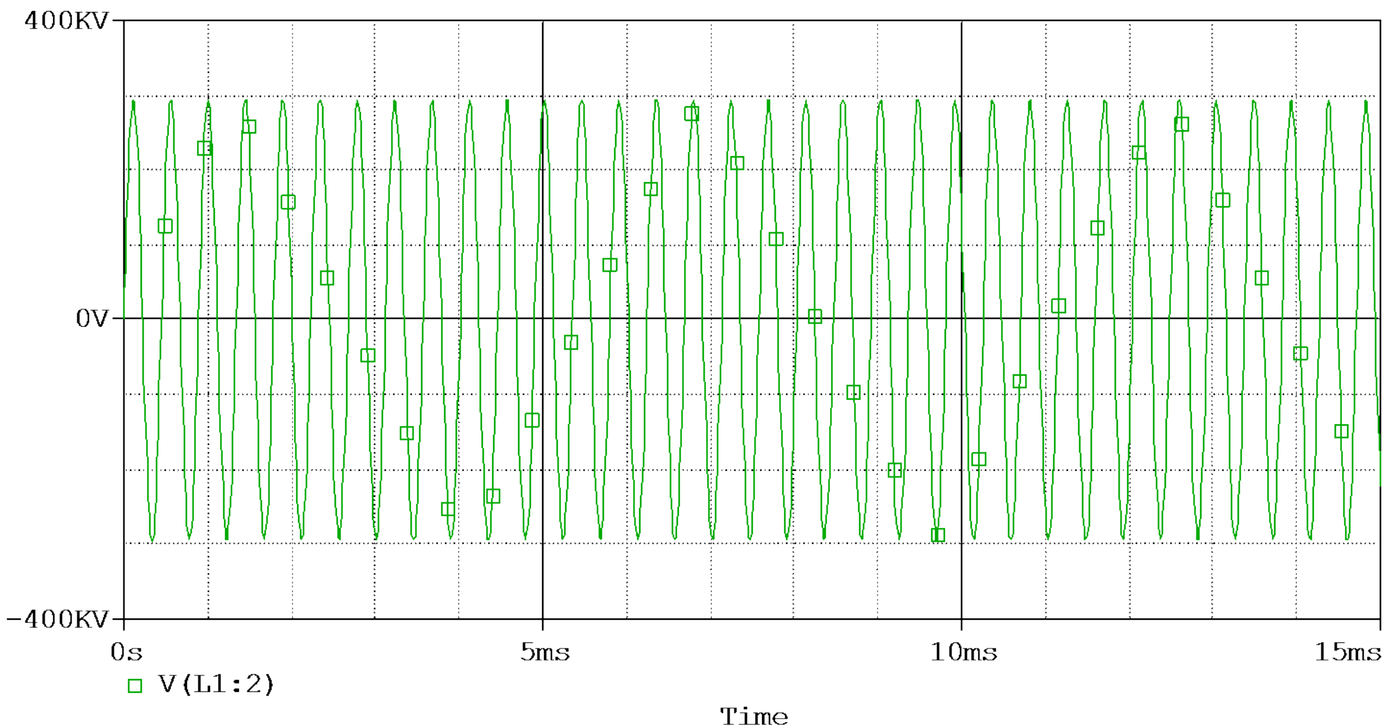


The beam has two strain gauges, one on the top of the beam and one on the bottom. The stain is approximately equal and opposite for the two gauges.

In this experiment, we will hook up the strain gauges in a bridge circuit to observe the oscillations of the beam.

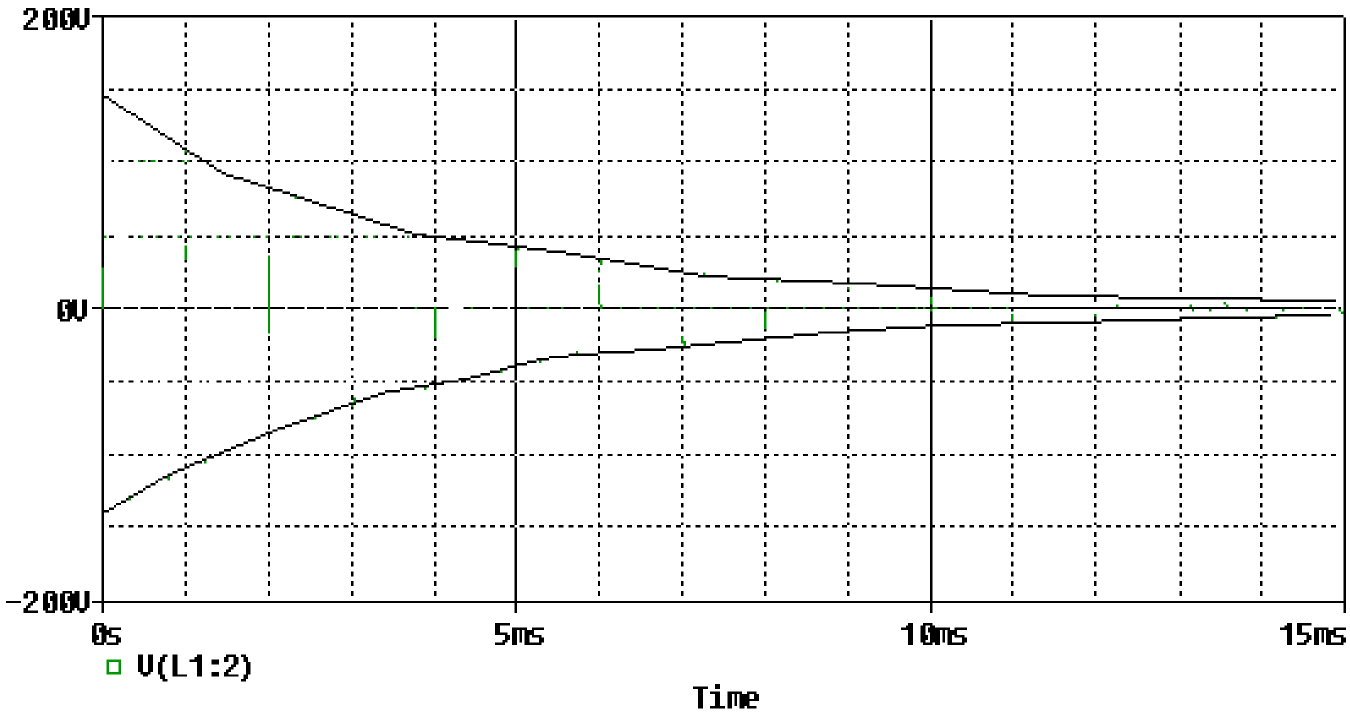
Modeling Damped Oscillations

- $v(t) = A \sin(\omega t)$



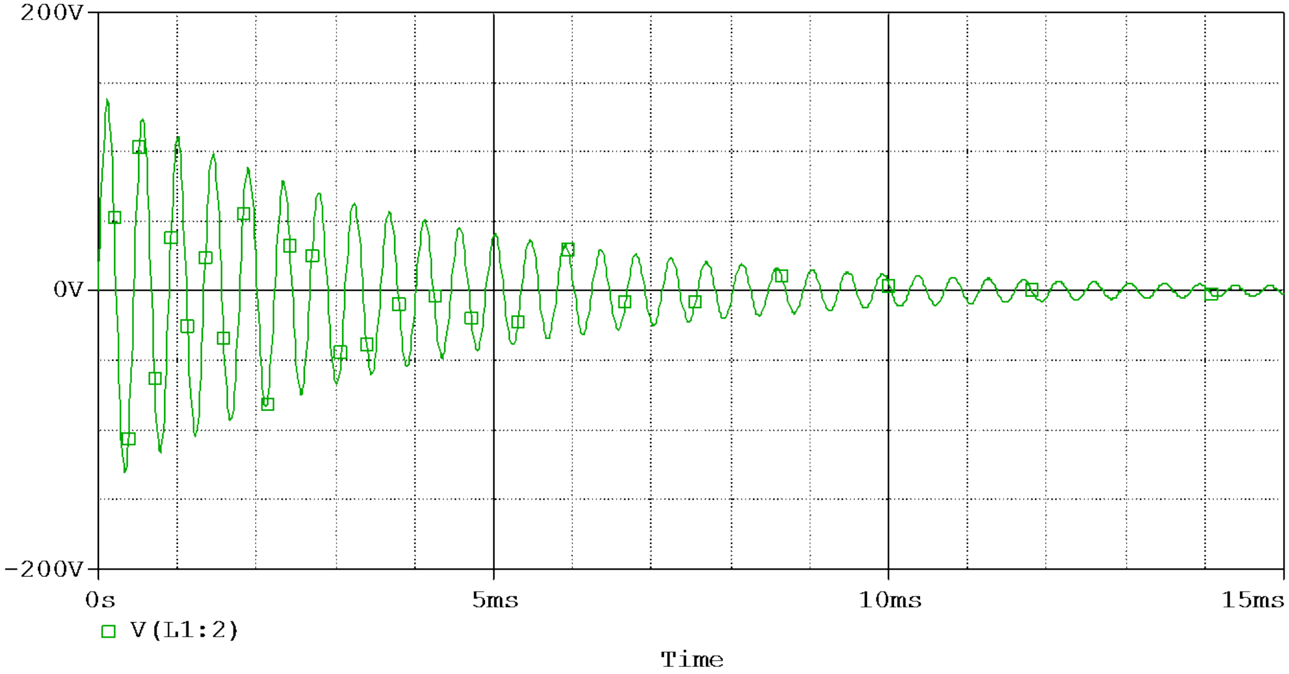
Modeling Damped Oscillations

- $v(t) = Be^{-\alpha t}$



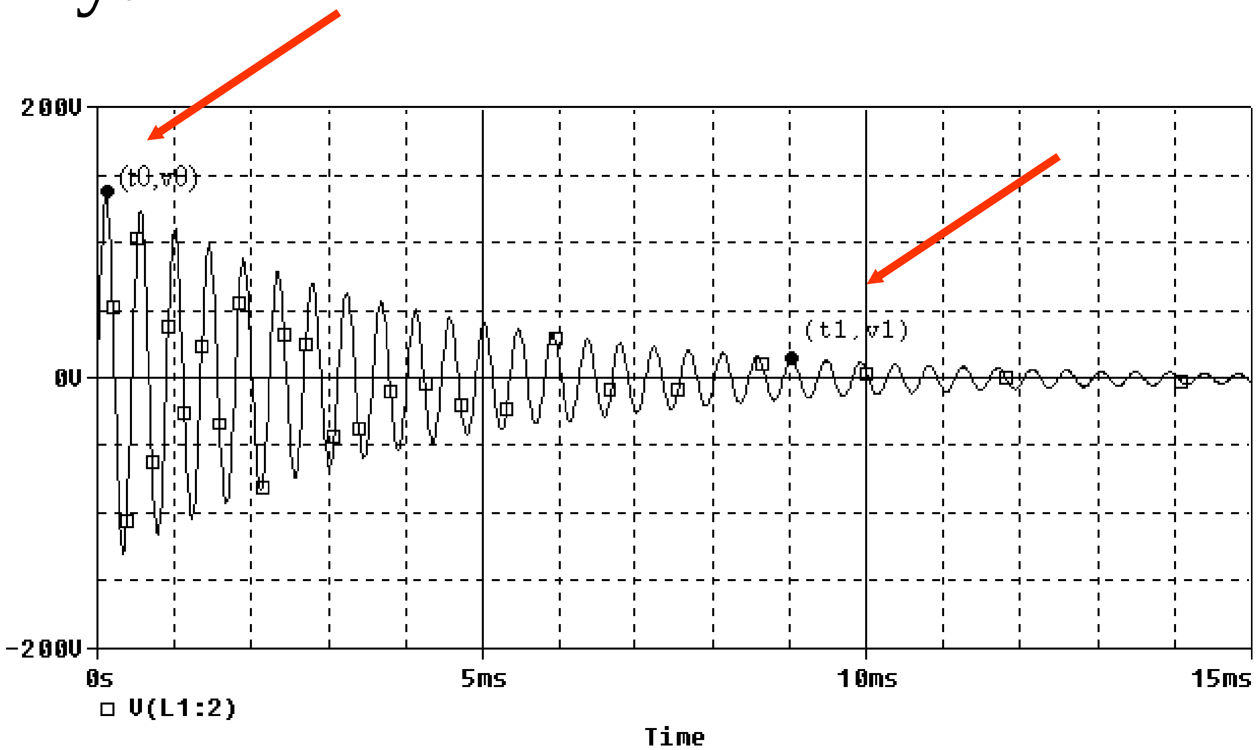
Modeling Damped Oscillations

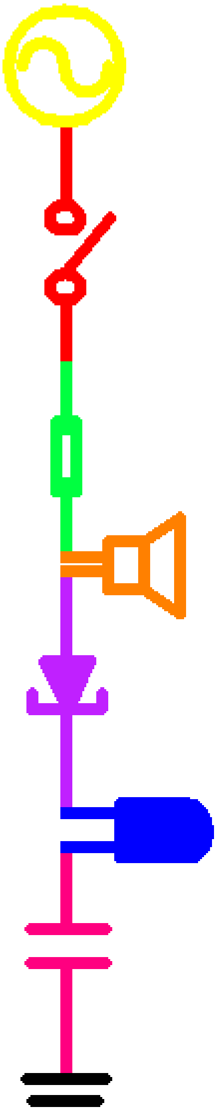
- $v(t) = A \sin(\omega t)\, Be^{-\alpha t} = Ce^{-\alpha t}\sin(\omega t)$



Finding the Damping Constant

- Choose two maxima at extreme ends of the decay.





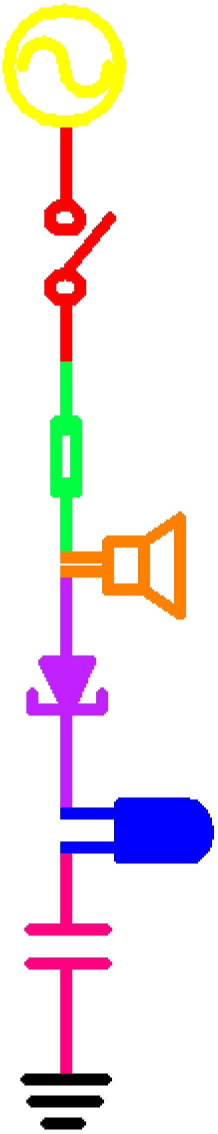
Finding the Damping Constant

- Assume (t0,v0) is the starting point for the decay.
- The amplitude at this point,v0, is C.
- $v(t) = Ce^{-\alpha t}\sin(\omega t)$ at (t1,v1):
  $v1 = v0e^{-\alpha(t1-t0)}\sin(\pi/2) = v0e^{-\alpha(t1-t0)}$
- Substitute and solve for α: $v1 = v0e^{-\alpha(t1-t0)}$

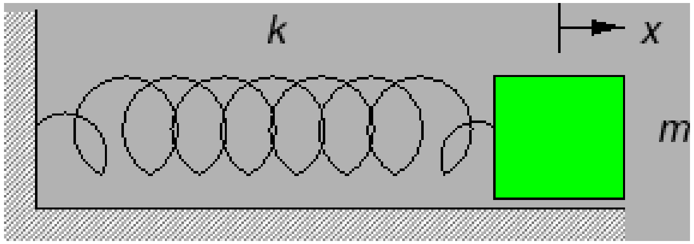
Part C

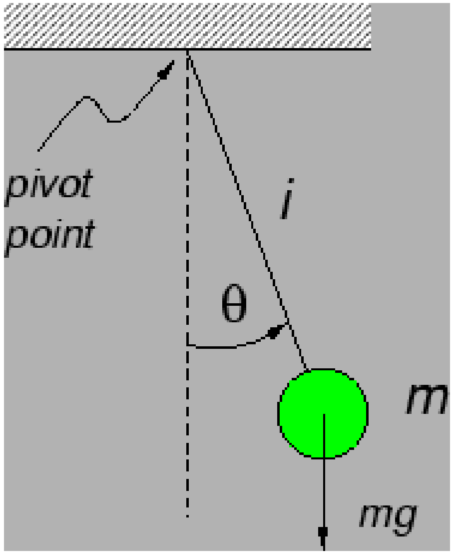
- Harmonic Oscillators
- Analysis of Cantilever Beam Frequency Measurements

Examples of Harmonic Oscillators



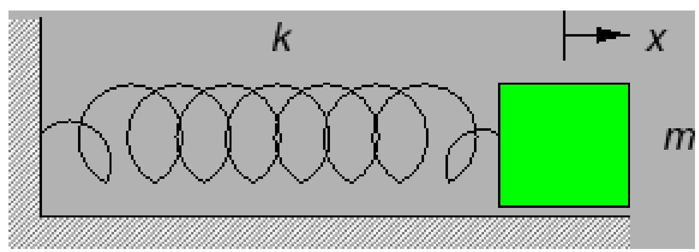
- Spring-mass combination
- Violin string
- Wind instrument
- Clock pendulum
- Playground swing
- LC or RLC circuits
- Others?





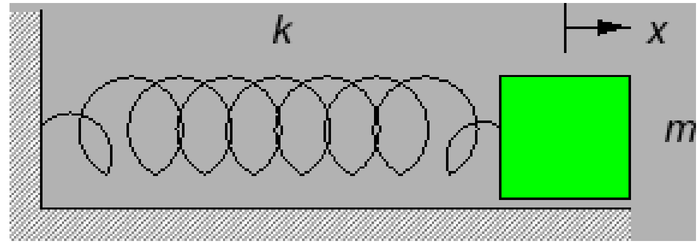
Harmonic Oscillator

- Equation $$\frac{d^2x}{dt^2} + \omega^2 x = 0$$
- Solution $x = A\sin(\omega t)$



- x is the displacement of the oscillator while A is the amplitude of the displacement

Spring



- *Spring Force*

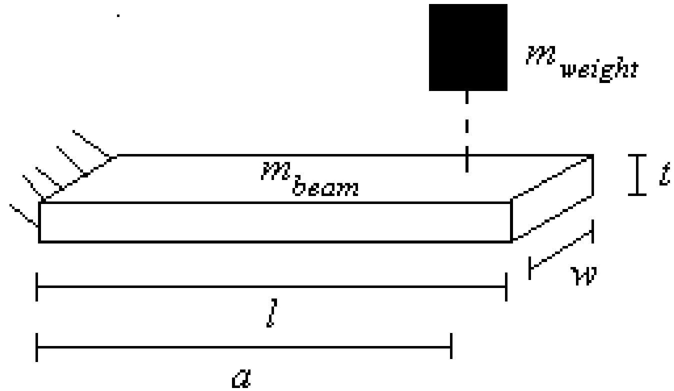
$$F = ma = -kx$$

- Oscillation Frequency

$$\omega = \sqrt{\frac{k}{m}}$$

- This expression for frequency holds for a massless spring with a mass at the end, as shown in the diagram.

Spring Model for the Cantilever Beam



- Where *l* is the length, *t* is the thickness, *w* is the width, and m_{beam} is the mass of the beam. Where m_{weight} is the applied mass and *a* is the length to the location of the applied mass.

Finding Young's Modulus

- For a beam loaded with a mass at the end, *a* is equal to *l*. For this case:

$$k = \frac{Ewt^3}{4l^3}$$

where *E* is Young's Modulus of the beam.

- See experiment handout for details on the derivation of the above equation.
- If we can determine the spring constant, k, and we know the dimensions of our beam, we can calculate E and find out what the beam is made of.

Finding k using the frequency

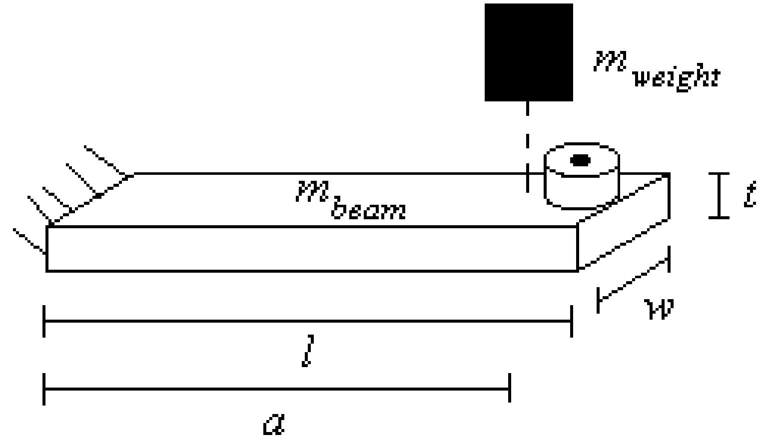
- Now we can apply the expression for the ideal spring mass frequency to the beam.

$$\frac{k}{m} = (2\pi f)^2$$

- The frequency, f_n, will change depending upon how much mass, m_n, you add to the end of the beam.

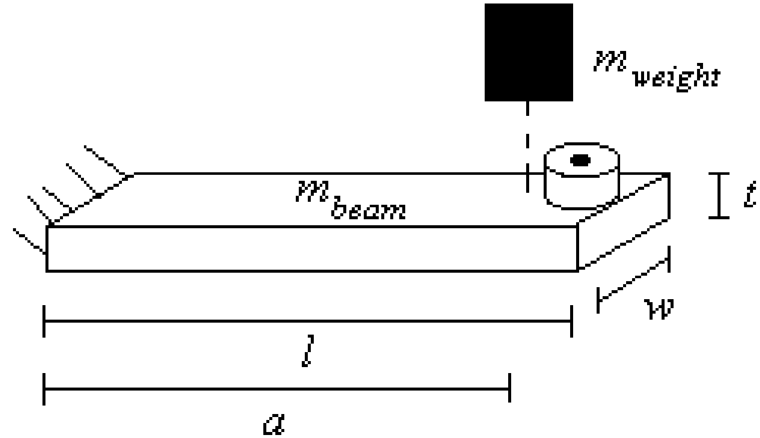
$$\frac{k}{m + m_n} = (2\pi f_n)^2$$

Our Experiment



- For our beam, we must deal with the beam mass and any extra load we add to the beam to observe how its performance depends on load conditions.
- Real beams have finite mass distributed along the length of the beam. We will model this as an equivalent mass at the end that would produce the same frequency response. This is given by $m = 0.23m_{beam}$.

Our Experiment



- To obtain a good measure of *k* and *m*, we will make 4 measurements of oscillation, one for just the beam and three others by placing an additional mass at the end of the beam.

$$k = (m)(2\pi f_0)^2 \qquad k = (m + m_1)(2\pi f_1)^2$$

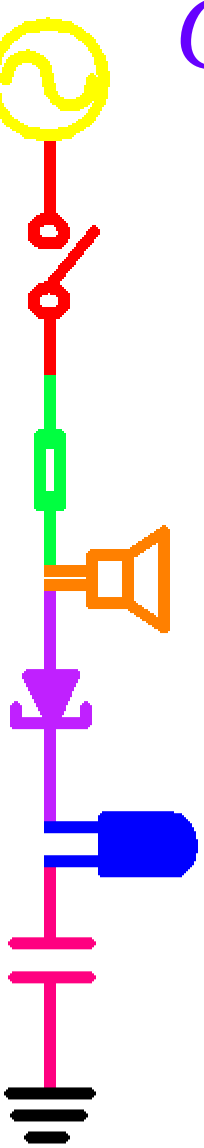
$$k = (m + m_2)(2\pi f_2)^2 \qquad k = (m + m_3)(2\pi f_3)^2$$

Our Experiment

- Once we obtain values for *k* and *m* we can plot the following function to see how we did.

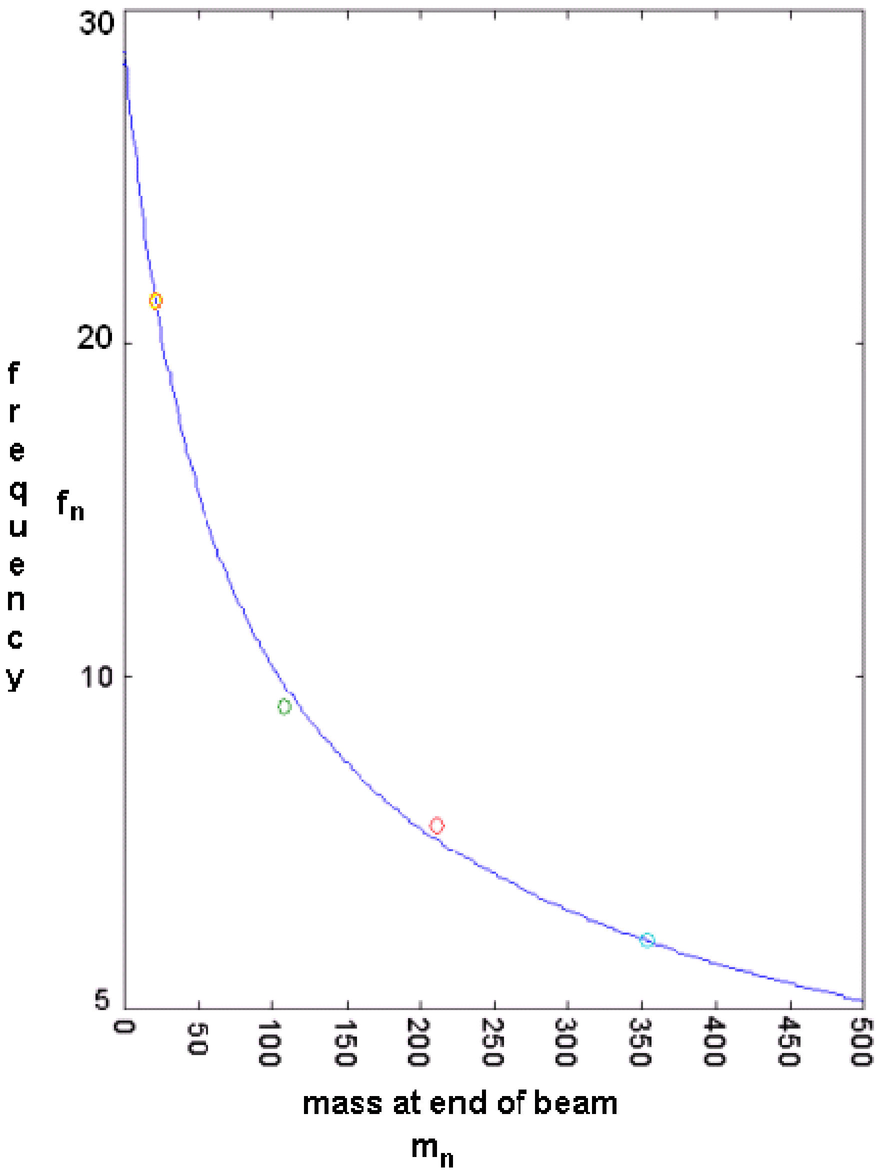
$$f_n = \frac{1}{2\pi}\sqrt{\frac{k_{guess}}{m_{guess} + m_n}}$$

- In order to plot m_n vs. f_n, we need to obtain a guess for *m*, m_{guess}, and *k*, k_{guess}. Then we can use the guesses as constants, choose values for m_n (our domain) and plot f_n (our range).



Our Experiment

The output plot should look something like this. The blue line is the plot of the function and the points are the results of your four trials.

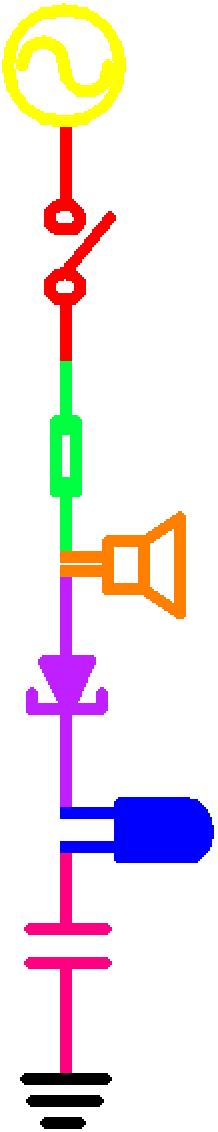


Our Experiment

- How to find final values for *k* and *m*.
  - Solve for k_{guess} and m_{guess} using only two of your data points and two equations. (The larger loads work best.)
  - Plot *f* as a function of load mass to get a plot similar to the one on the previous slide.
  - Change values of *k* and *m* until your function and data match.

Our Experiment

- Can you think of other ways to more systematically determine k_{guess} and m_{guess} ?
- Experimental hint: make sure you keep the center of any mass you add as near to the end of the beam as possible. It can be to the side, but not in front or behind the end.



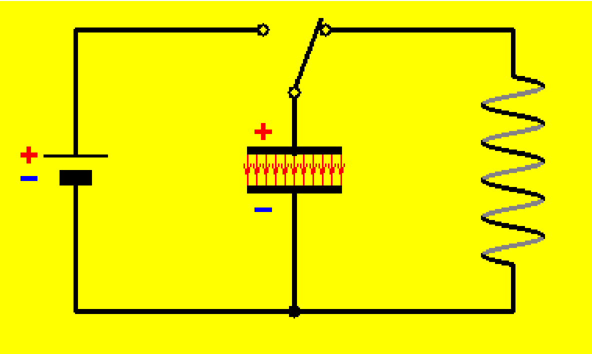
Part D

- Oscillating Circuits
- Comparative Oscillation Analysis
- Interesting Oscillator Applications

Oscillating Circuits

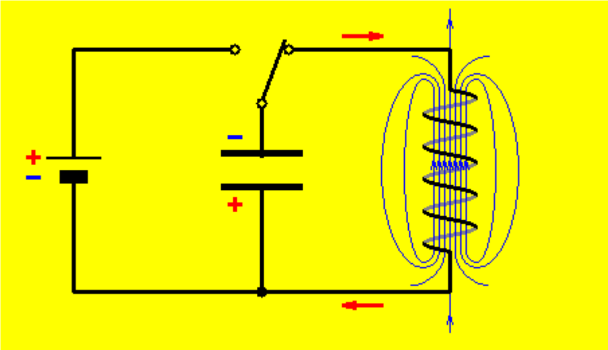
- Energy Stored in a Capacitor

$$CE = \tfrac{1}{2}CV^2$$



- Energy stored in an Inductor

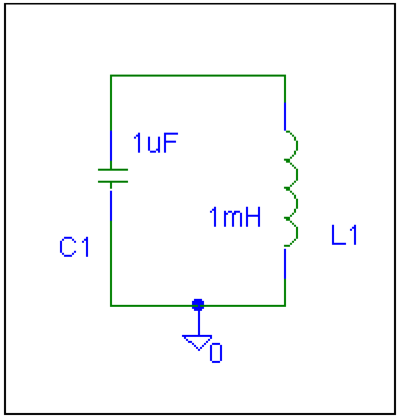
$$LE = \tfrac{1}{2}LI^2$$



- An Oscillating Circuit transfers energy between the capacitor and the inductor.

http://www.walter-fendt.de/ph11e/osccirc.htm

Voltage and Current

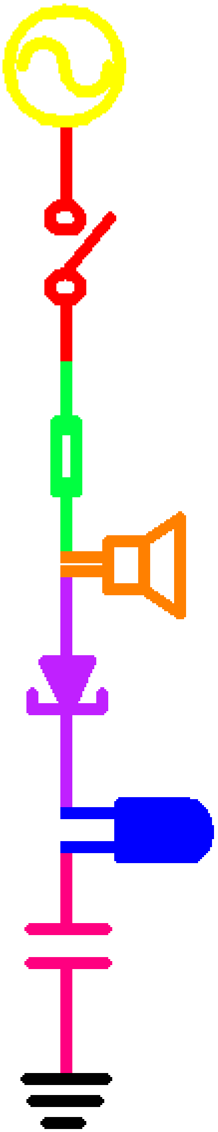


- Note that the circuit is in series, so the current through the capacitor and the inductor are the same.

$$I = I_L = I_C$$

- Also, there are only two elements in the circuit, so, by Kirchoff’s Voltage Law, the voltage across the capacitor and the inductor must be the same.

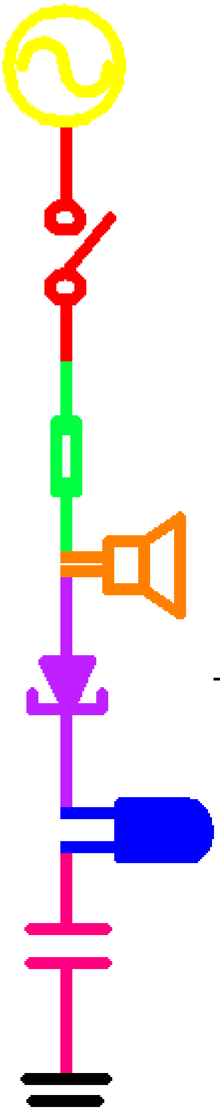
$$V = V_L = V_C$$



Oscillator Analysis

- Spring-Mass
- W = KE + PE
- KE = kinetic energy=$\frac{1}{2}mv^2$
- PE = potential energy(spring)=$\frac{1}{2}kx^2$
- W = $\frac{1}{2}mv^2 + \frac{1}{2}kx^2$

- Electronics
- W = LE + CE
- LE = inductor energy=$\frac{1}{2}LI^2$
- CE = capacitor energy=$\frac{1}{2}CV^2$
- W = $\frac{1}{2}LI^2 + \frac{1}{2}CV^2$



Oscillator Analysis

- Take the time derivative

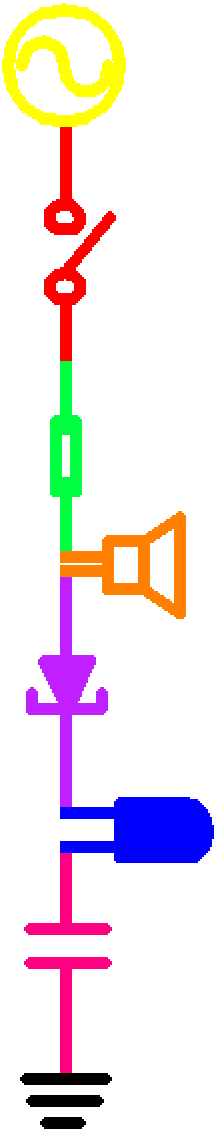
$$\frac{dW}{dt} = \tfrac{1}{2} m 2v \frac{dv}{dt} + \tfrac{1}{2} k 2x \frac{dx}{dt}$$

$$\frac{dW}{dt} = mv \frac{dv}{dt} + kx \frac{dx}{dt}$$

- Take the time derivative

$$\frac{dW}{dt} = \tfrac{1}{2} L 2I \frac{dI}{dt} + \tfrac{1}{2} C 2V \frac{dV}{dt}$$

$$\frac{dW}{dt} = LI \frac{dI}{dt} + CV \frac{dV}{dt}$$



Oscillator Analysis

- W is a constant. Therefore, $\frac{dW}{dt} = 0$
- Also

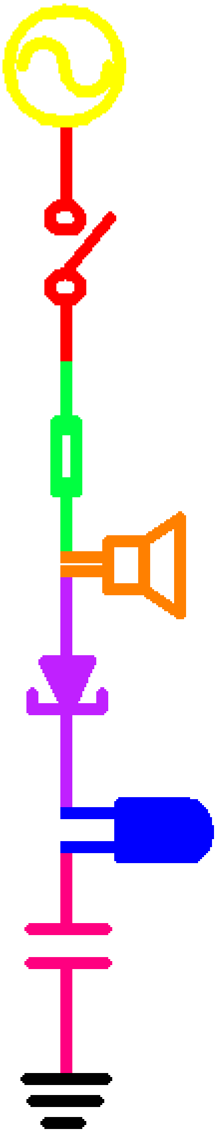
$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- W is a constant. Therefore, $\frac{dW}{dt} = 0$
- Also

$$I = C\frac{dV}{dt} \qquad \frac{dV}{dt} = \frac{I}{C}$$

$$\frac{dI}{dt} = C\frac{d^2V}{dt^2}$$



Oscillator Analysis

- Simplify

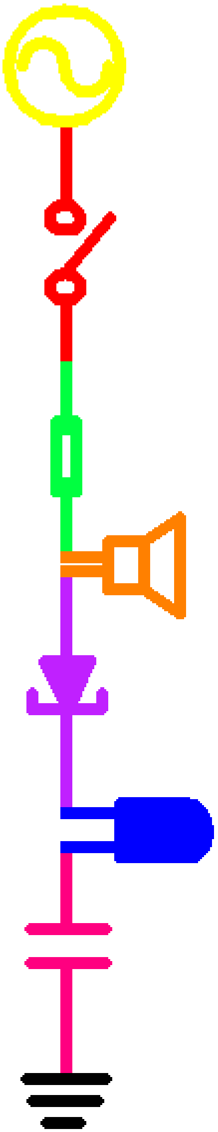
$$0 = mv\frac{d^2x}{dt^2} + kxv$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

- Simplify

$$0 = LIC\frac{d^2V}{dt^2} + CV\frac{I}{C}$$

$$\frac{d^2V}{dt^2} + \frac{1}{LC}V = 0$$



Oscillator Analysis

- Solution

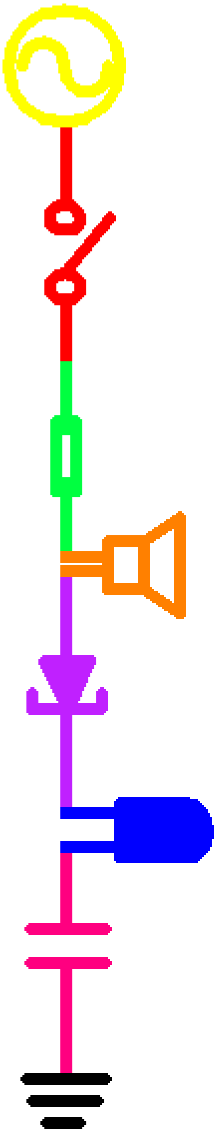
$$x = A\sin(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

- Solution

$$V = A\sin(\omega t)$$

$$\omega = \frac{1}{\sqrt{LC}}$$



Using Conservation Laws

- Please also see the write up for experiment 3 for how to use energy conservation to derive the equations of motion for the beam and voltage and current relationships for inductors and capacitors.
- Almost everything useful we know can be derived from some kind of conservation law.

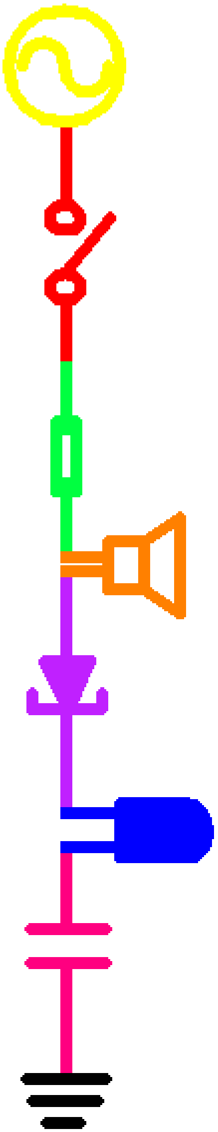
Large Scale Oscillators

Petronas Tower (452m)

CN Tower (553m)

Tall buildings are like cantilever beams, they all have a natural resonating frequency.

Deadly Oscillations



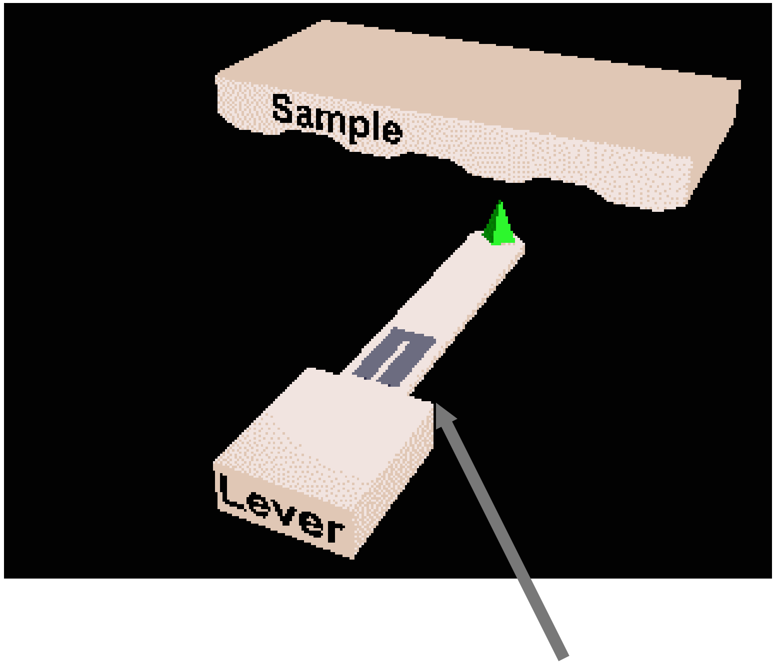
The Tacoma Narrows Bridge went into oscillation when exposed to high winds. The movie shows what happened.

http://www.slcc.edu/schools/hum_sci/physics/tutor/2210/mechanical_oscillations/

In the 1985 Mexico City earthquake, buildings between 5 and 15 stories tall collapsed because they resonated at the same frequency as the quake. Taller and shorter buildings survived.

Atomic Force Microscopy -AFM

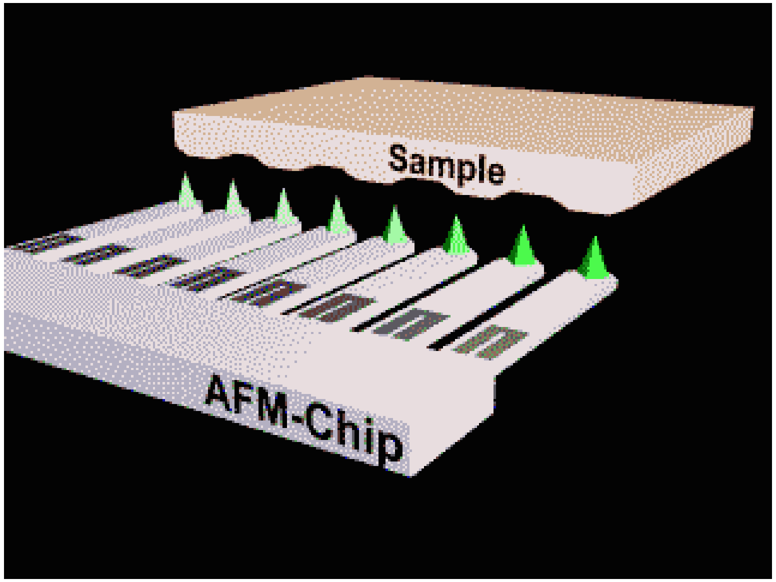
- This is one of the key instruments driving the nanotechnology revolution
- Dynamic mode uses frequency to extract force information



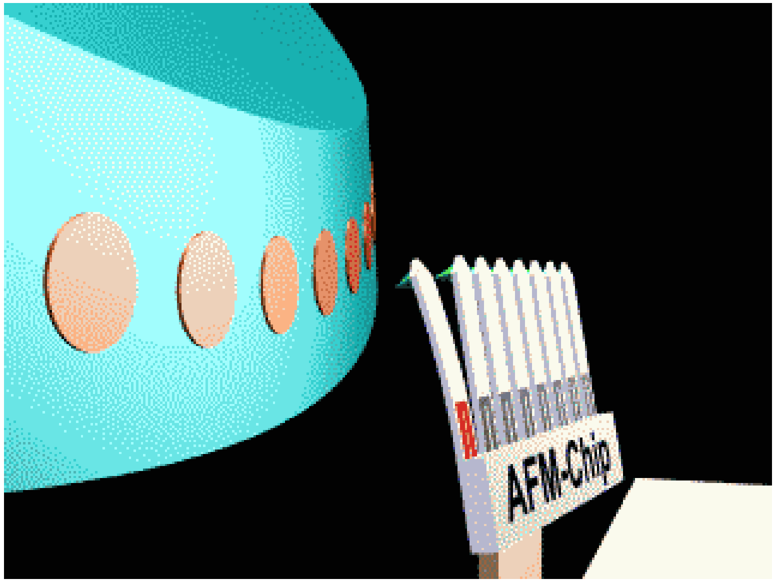
Note Strain Gage

AFM on Mars

- Redundancy is built into the AFM so that the tips can be replaced remotely.



AFM on Mars

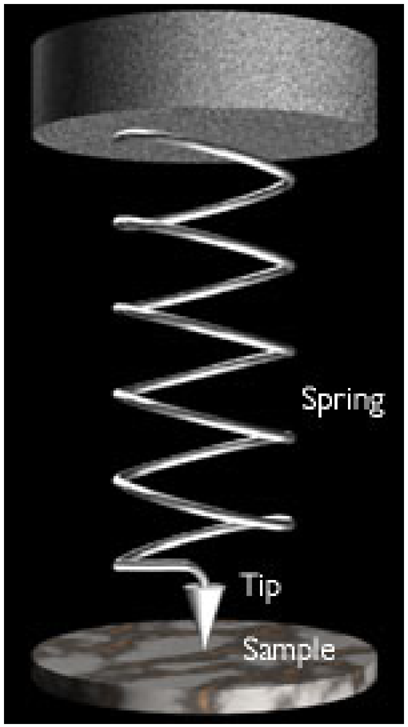


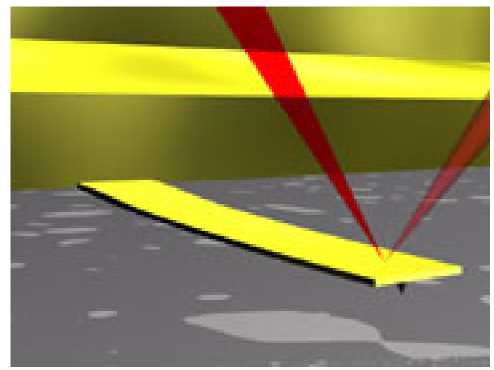
- Soil is scooped up by robot arm and placed on sample. Sample wheel rotates to scan head. Scan is made and image is stored.

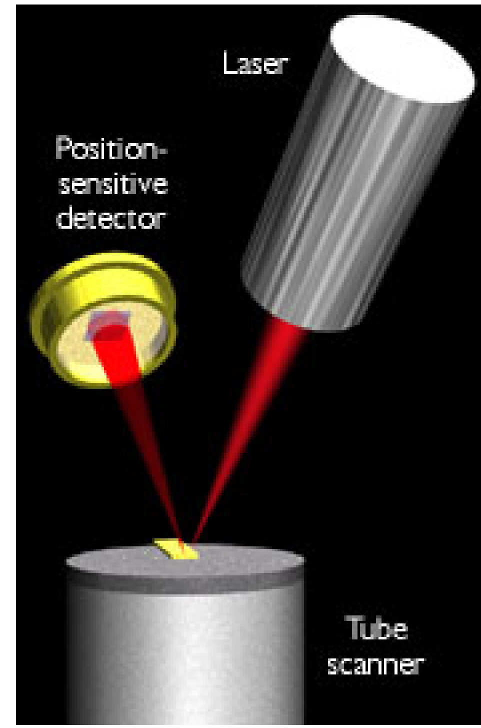
AFM Image of Human Chromosomes

- There are other ways to measure deflection.

AFM Optical Pickup

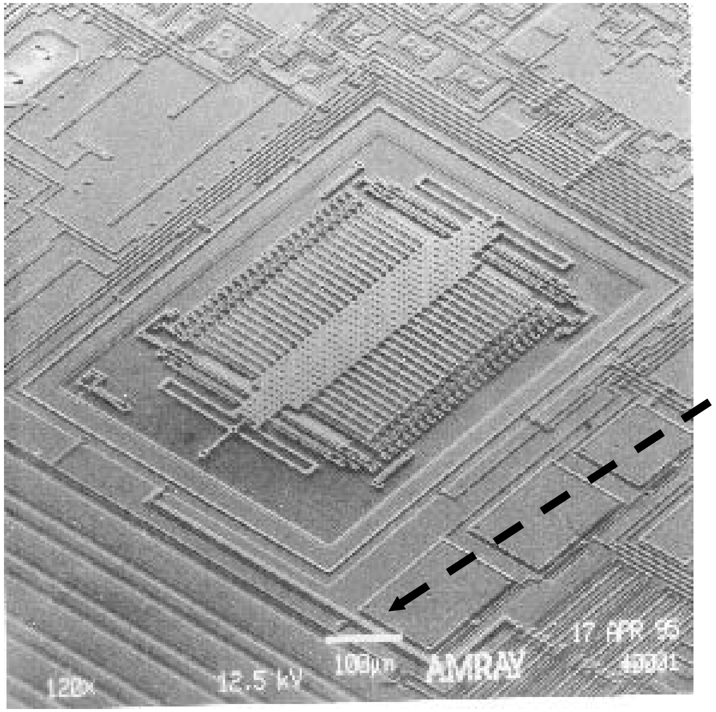






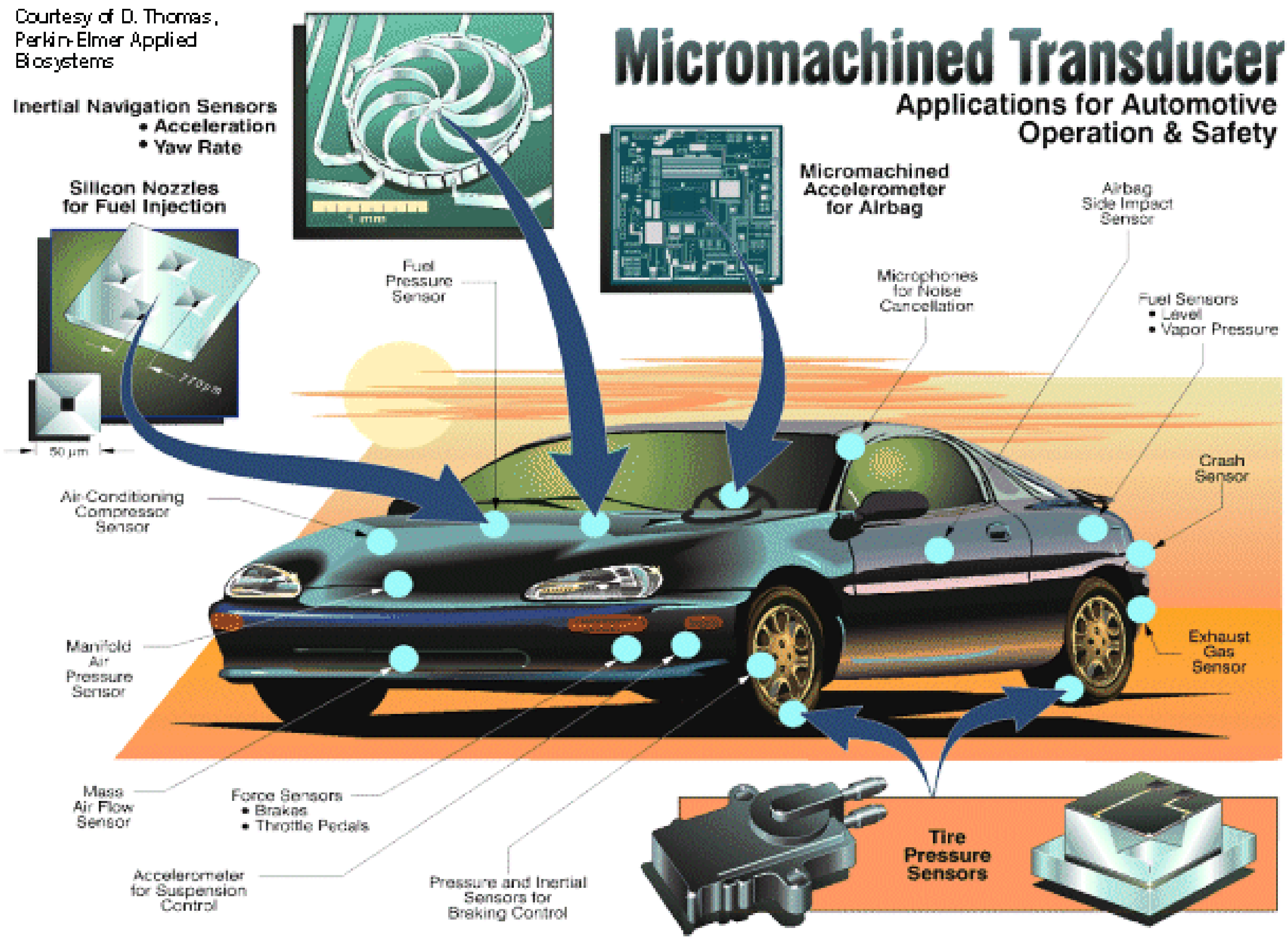
- On the left is the generic picture of the beam. On the right is the optical sensor.

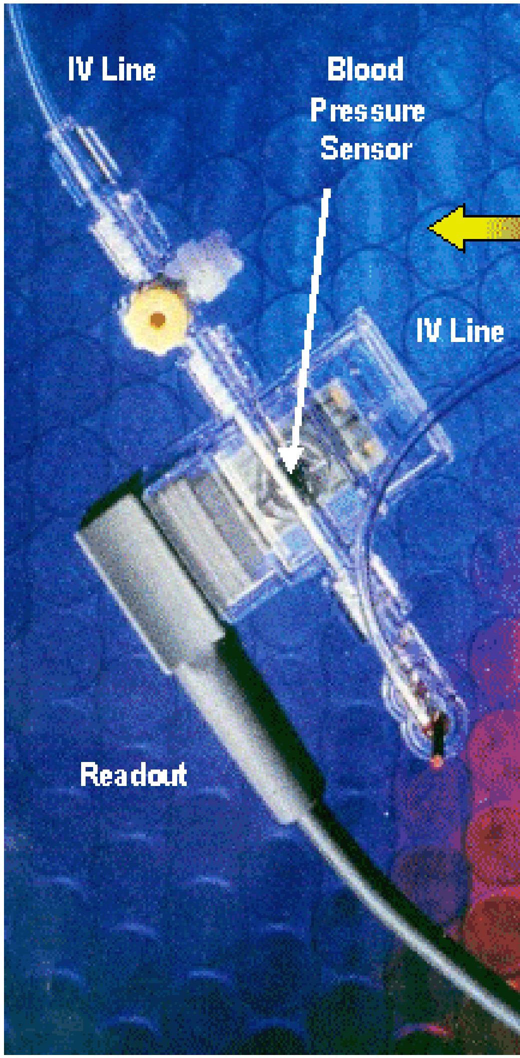
MEMS Accelerometer



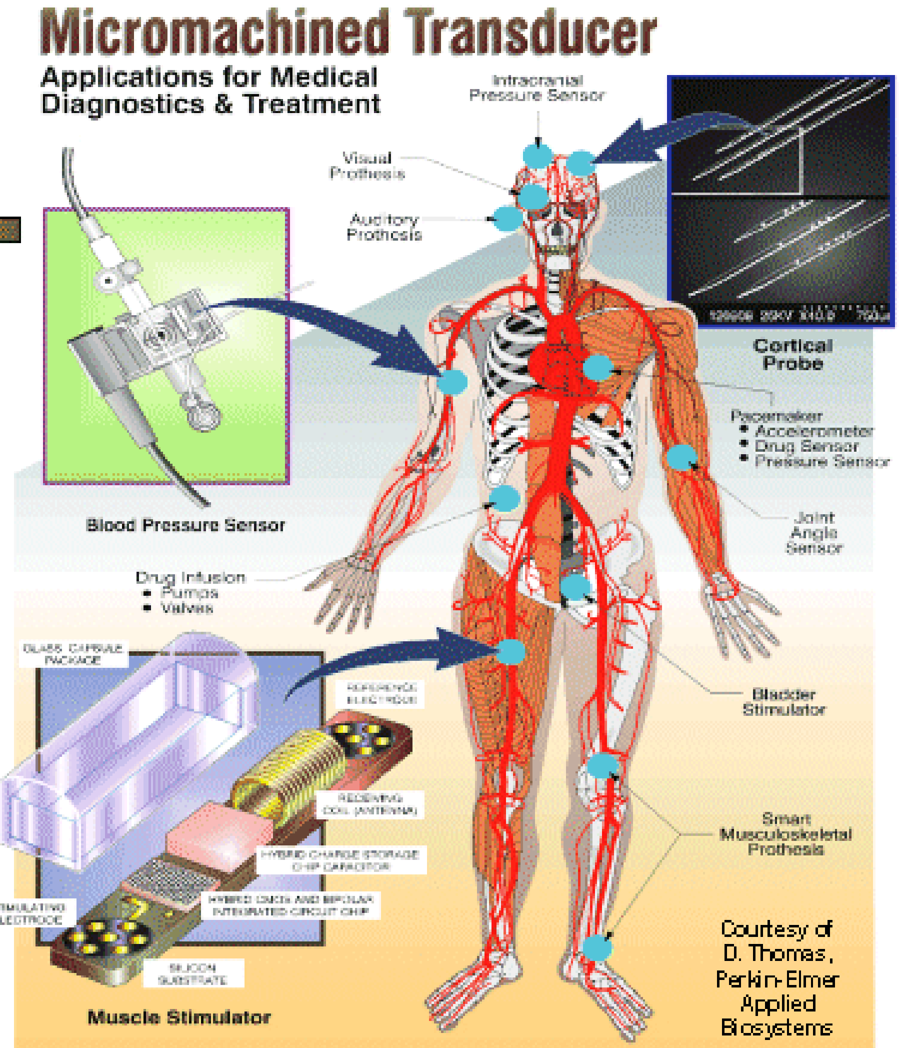
Note Scale

- An array of cantilever beams can be constructed at very small scale to act as accelerometers.


Micromachined Transducer
Applications for Automotive Operation & Safety
Courtesy of D. Thomas, Perkin-Elmer Applied Biosystems
Inertial Navigation Sensors
• Acceleration
• Yaw Rate
Silicon Nozzles for Fuel Injection
1 mm
770 µm
50 µm
Micromachined Accelerometer for Airbag
Fuel Pressure Sensor
Microphones for Noise Cancellation
Airbag Side Impact Sensor
Fuel Sensors
• Level
• Vapor Pressure
Crash Sensor
Air-Conditioning Compressor Sensor
Manifold Air Pressure Sensor
Exhaust Gas Sensor
Mass Air Flow Sensor
Force Sensors
• Brakes
• Throttle Pedals
Accelerometer for Suspension Control
Pressure and Inertial Sensors for Braking Control
Tire Pressure Sensors



IV Line
Blood Pressure Sensor
IV Line
Readout



Micromachined Transducer
Applications for Medical Diagnostics & Treatment
Intracranial Pressure Sensor
Visual Prothesis
Auditory Prothesis
Cortical Probe
Pacemaker
• Accelerometer
• Drug Sensor
• Pressure Sensor
Joint Angle Sensor
Blood Pressure Sensor
Drug Infusion
• Pumps
• Valves
Bladder Stimulator
Smart Musculoskeletal Prothesis
GLASS CAPSULE PACKAGE
REFERENCE ELECTRODE
RECEIVING COIL (ANTENNA)
HYBRID CHARGE STORAGE CHIP CAPACITOR
HYBRID CMOS AND BIPOLAR INTEGRATED CIRCUIT CHIP
STIMULATING ELECTRODE
SILICON SUBSTRATE
Muscle Stimulator
Courtesy of D. Thomas, Perkin-Elmer Applied Biosystems

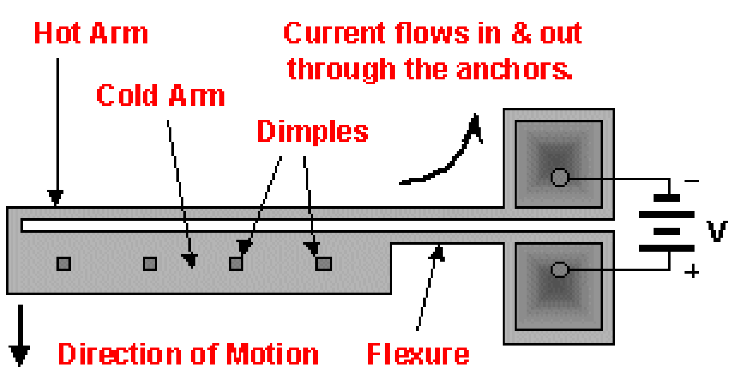
Typical Actuators

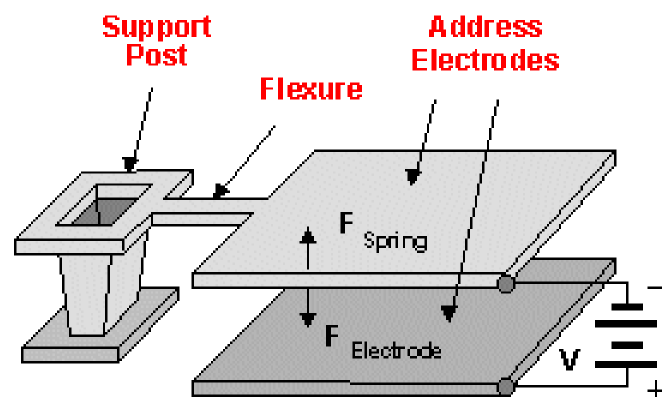
Thermal Actuators:

- Single layer lateral-motion actuator
- Motion caused by uneven ohmic heating
- Returns to resting position after cooling
- Basic structure used for vertical actuation
- Can be arrayed for combining force
- Larger deflections per length of device!
- Advantages:
  - Low Potentials, Large Forces, Large Deflections
- Disadvantages
  - Low Frequencies, Moderate Power, Large Arrays

Electrostatic Actuators:

- Opposing flexure-supported electrodes
- Continuous force balance operation
  - Spring force counteracts electrode force
- Basic structure used in many forms:
  - Comb drives, micromirrors, membranes
- Used for vertical or lateral actuation
- Advantages:
  - Low Power, High Frequency, Simple
- Disadvantages:
  - High Potentials, Low Force, Nonlinearity

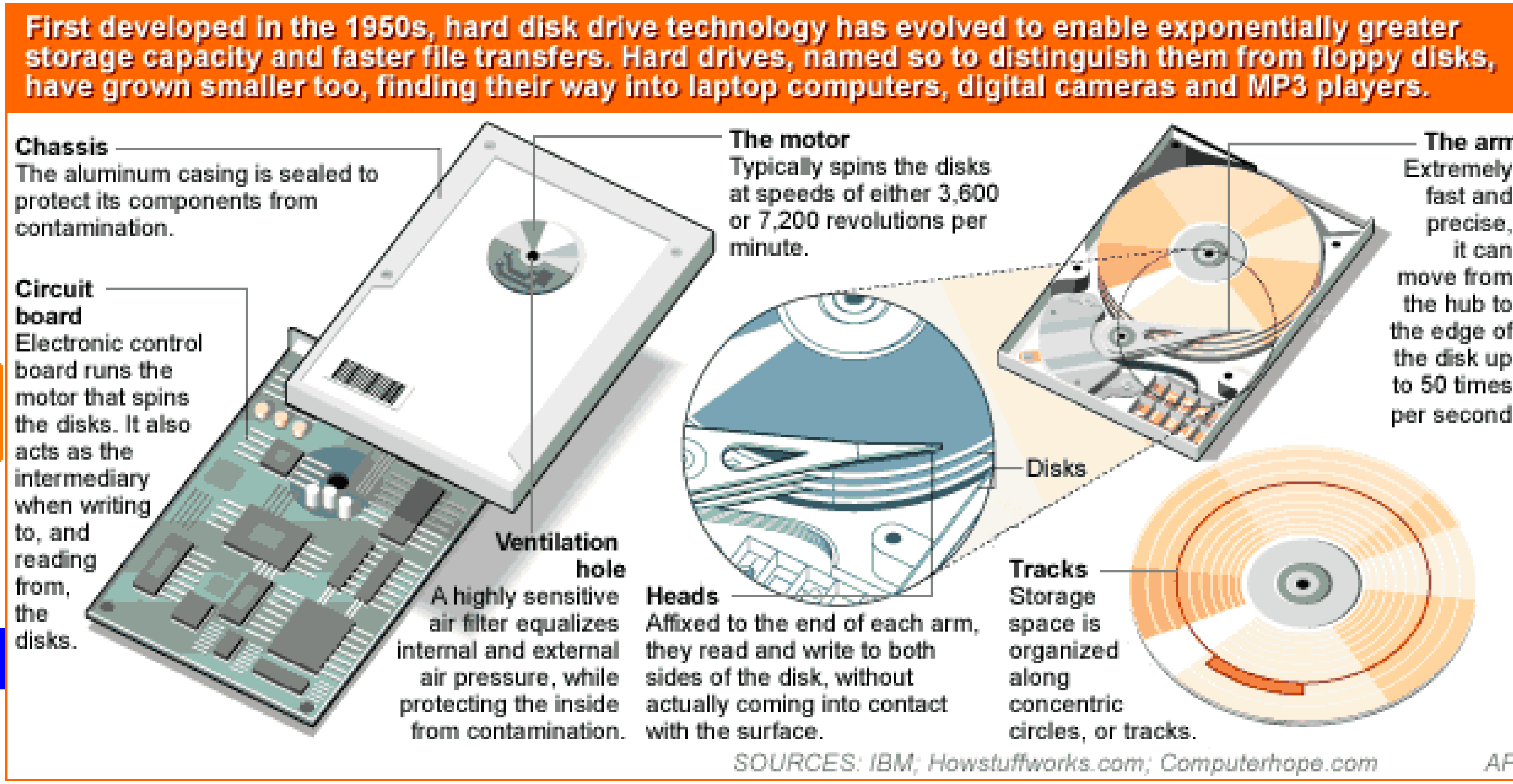




Hard Drive Cantilever

- The read-write head is at the end of a cantilever. This control problem is a remarkable feat of engineering.

More on Hard Drives



- A great example of Mechatronics.