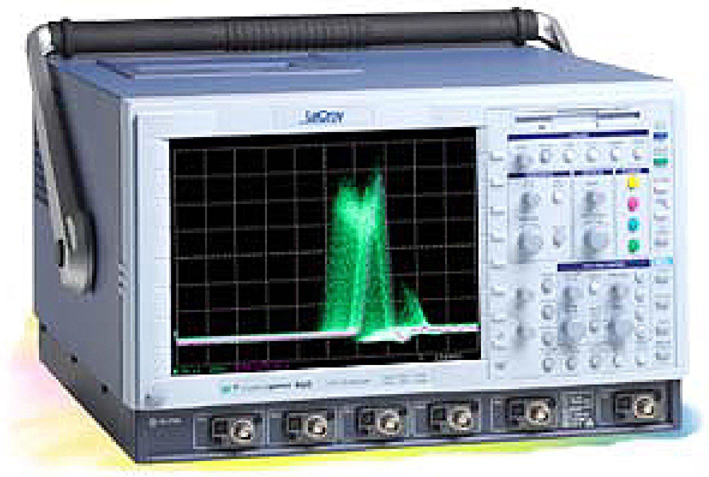
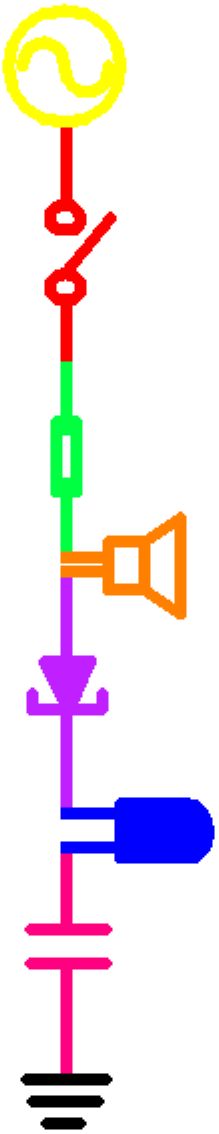


Electronic Instrumentation

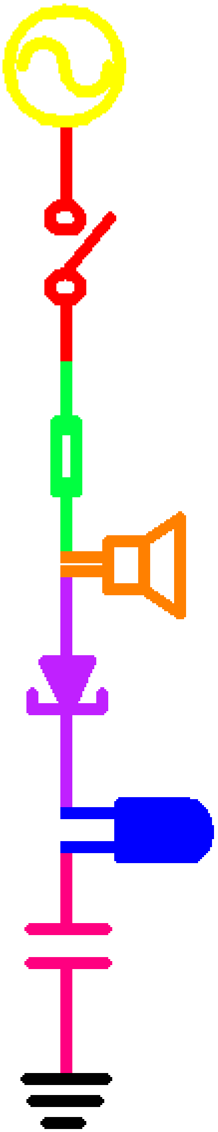
Experiment 2

* Part A: Intro to Transfer Functions and AC Sweeps
* Part B: Phasors, Transfer Functions and Filters
* Part C: Using Transfer Functions and RLC Circuits
* Part D: Equivalent Impedance and DC Sweeps



Part A

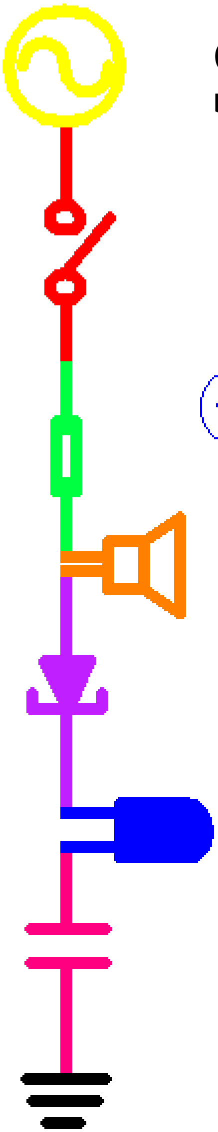
- Introduction to Transfer Functions and Phasors
- Complex Polar Coordinates
- Complex Impedance (Z)
- AC Sweeps



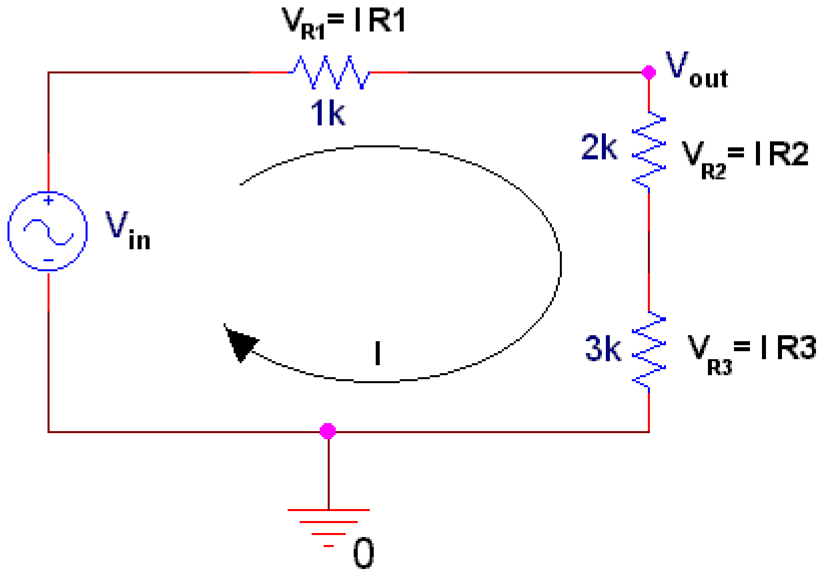
Transfer Functions

$$H \equiv \frac{V_{out}}{V_{in}}$$

- The transfer function describes the behavior of a circuit at V_{out} for all possible V_{in}.



Simple Example



$$V_{out} = V_{in} * \frac{R2 + R3}{R1 + R2 + R3}$$

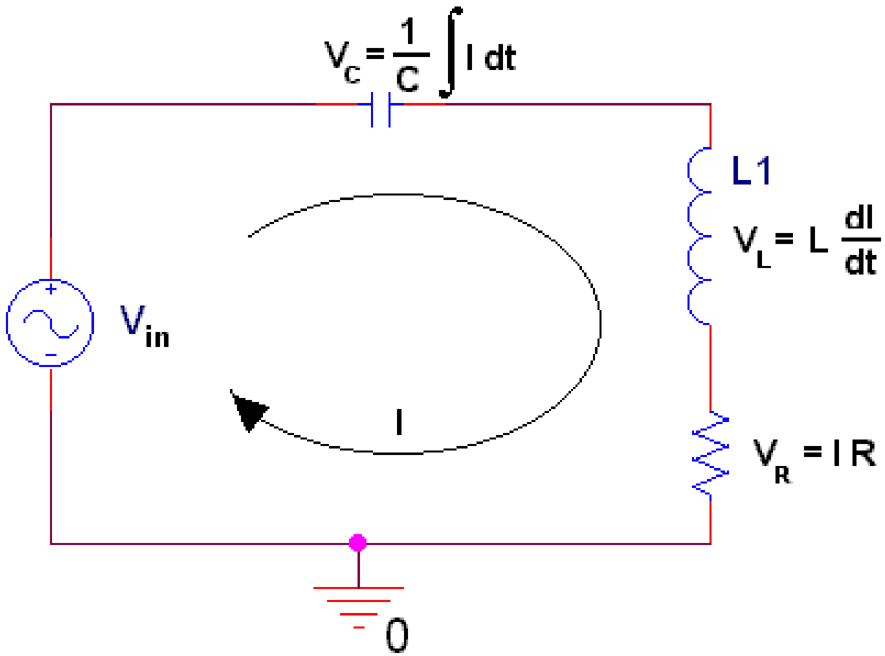
$$V_{out} = V_{in} * \frac{2k + 3k}{1k + 2k + 3k}$$

$$H \equiv \frac{V_{out}}{V_{in}} = \frac{5}{6}$$

$$if\ V_{in}(t) = 6V \sin(2kt + \frac{\pi}{2}) + 12V$$

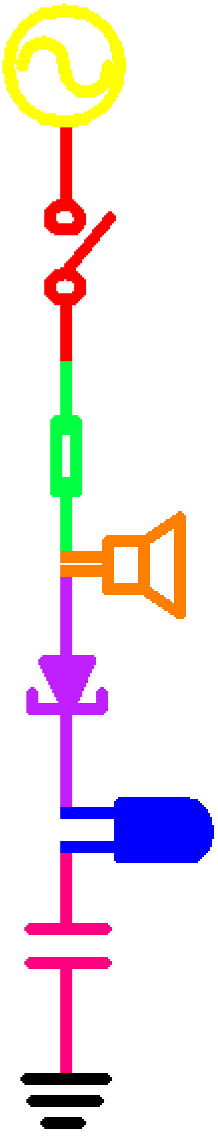
$$then\ V_{out}(t) = 5V \sin(2kt + \frac{\pi}{2}) + 10V$$

More Complicated Example



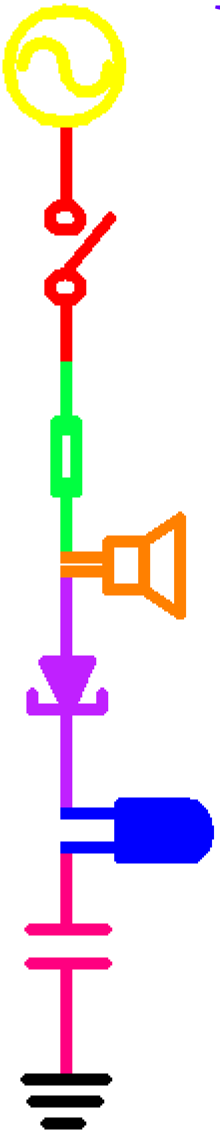
What is H now?

- H now depends upon the input frequency ($\omega = 2\pi f$) because the capacitor and inductor make the voltages change with the change in current.



How do we model H?

- We want a way to combine the effect of the components in terms of their influence on the amplitude and the phase.
- We can only do this because the signals are sinusoids
  - cycle in time
  - derivatives and integrals are just phase shifts and amplitude changes

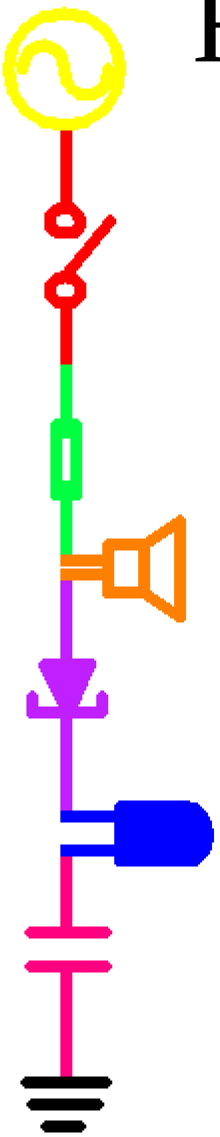


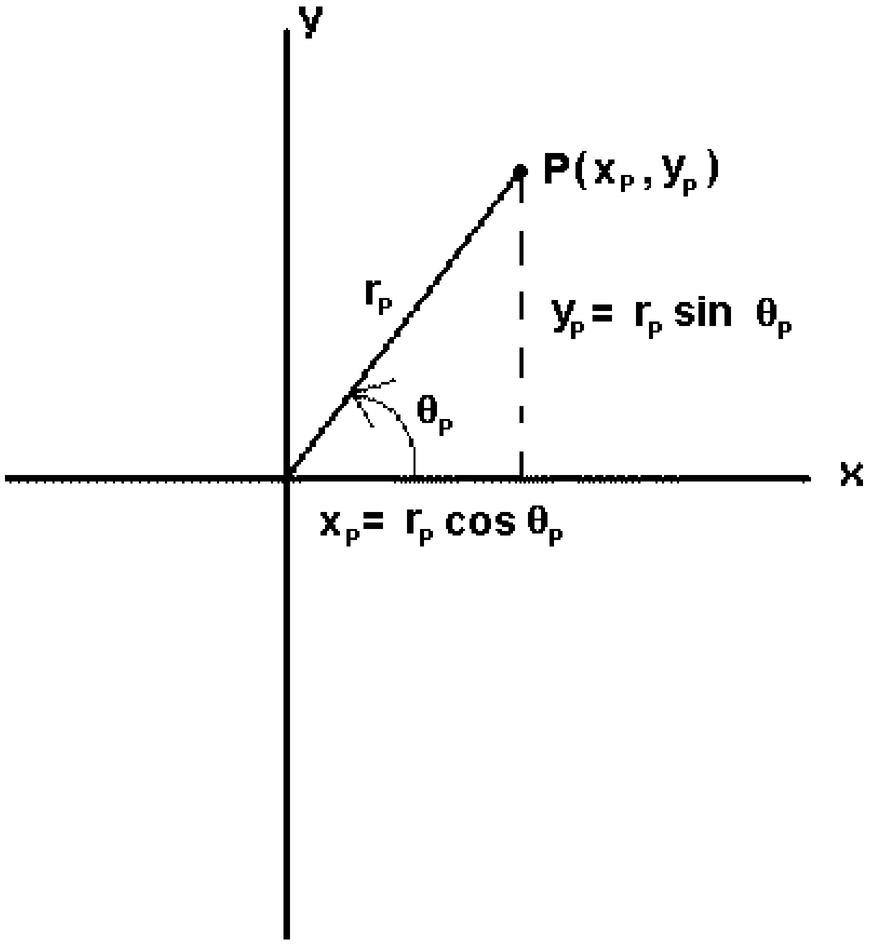
We will define Phasors

$$\vec{V} = f(A, \phi)$$

- A phasor is a function of the amplitude and phase of a sinusoidal signal
- Phasors allow us to manipulate sinusoids in terms of amplitude and phase changes.
- Phasors are based on complex polar coordinates.
- Using phasors and complex numbers we will be able to find transfer functions for circuits.

Review of Polar Coordinates



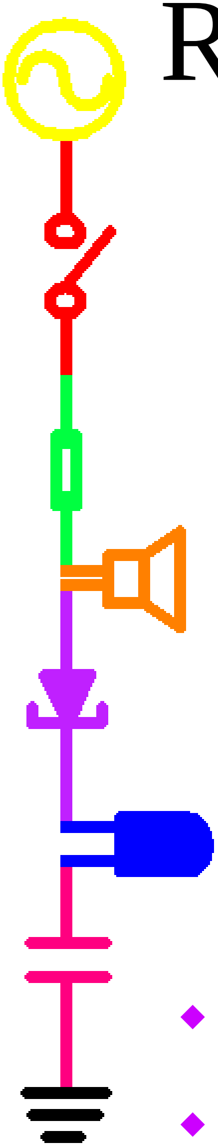


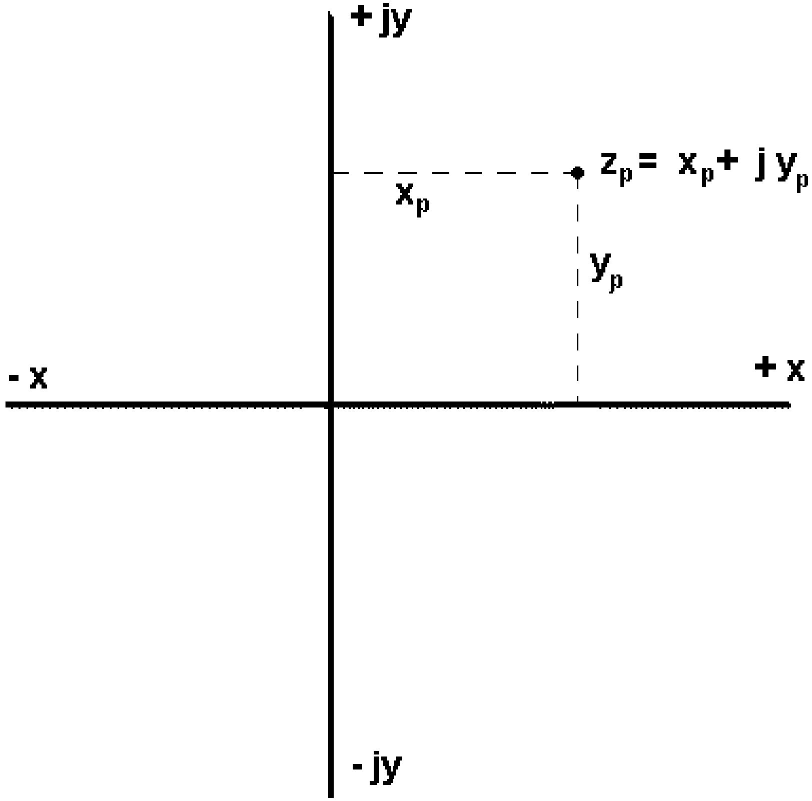
point P is at
($r_p \cos\theta_p$, $r_p \sin\theta_p$)

$$\theta_P = \tan^{-1}\left(\frac{y_P}{x_P}\right)$$

$$r_P = \sqrt{x_P^2 + y_P^2}$$

Review of Complex Numbers





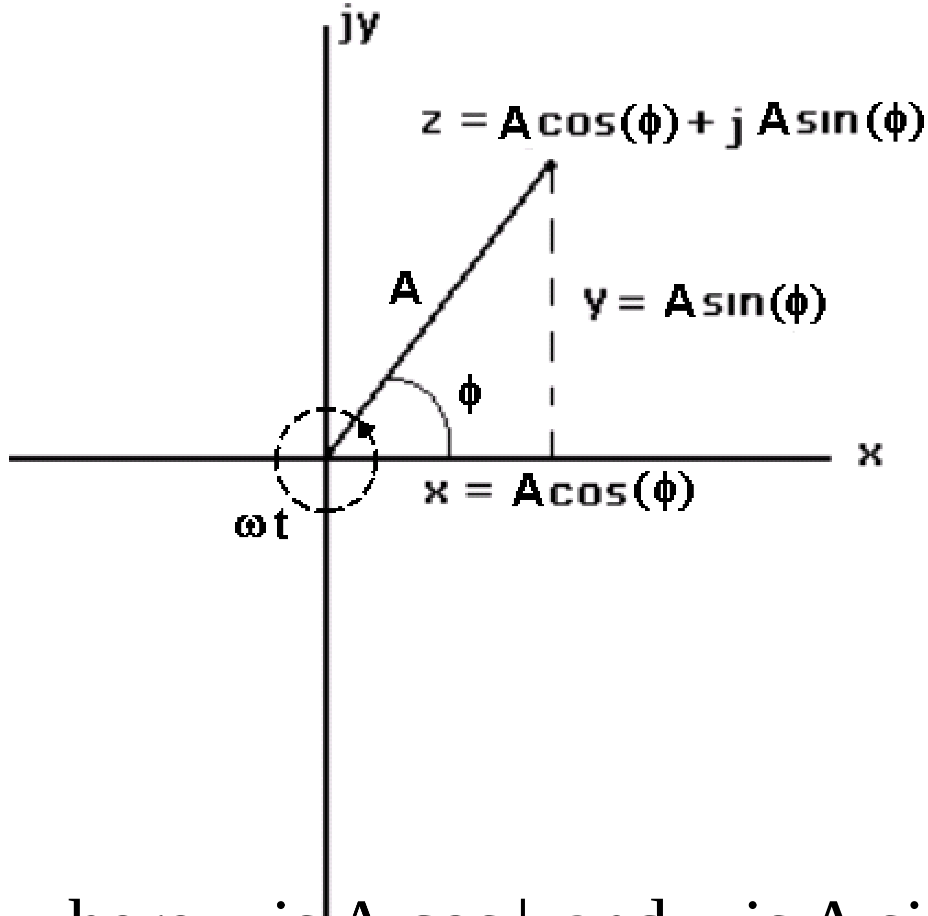
$$j \equiv \sqrt{-1}$$

$$j \cdot j = -1$$

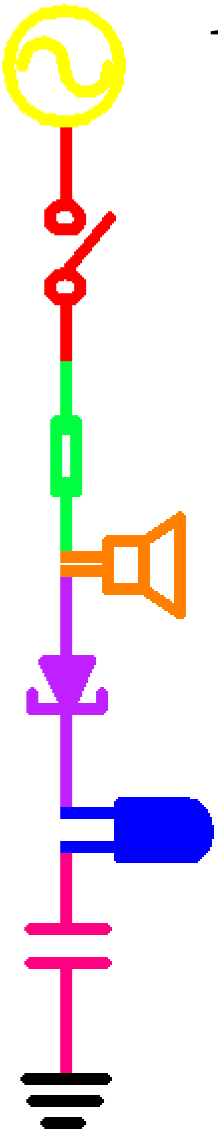
$$\frac{1}{j} = -j$$

- z_p is a single number represented by two numbers
- z_p has a “real” part (x_p) and an “imaginary” part (y_p)

Complex Polar Coordinates



- $z = x+jy$ where x is $A \cos\phi$ and y is $A \sin\phi$
- ωt cycles once around the origin once for each cycle of the sinusoidal wave ($\omega=2\pi f$)



Now we can define Phasors

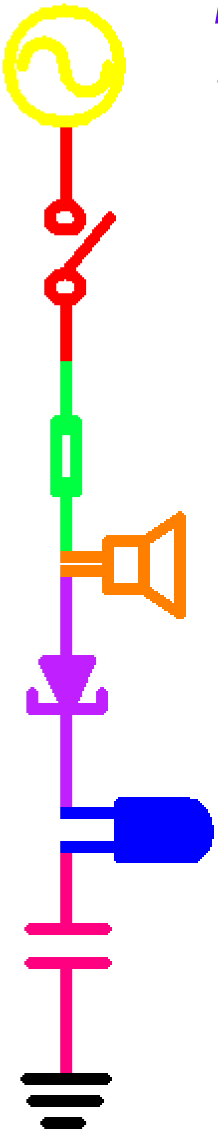
$$\textit{if } V(t) = A\cos(\omega t + \phi), \textit{ then let}$$

$$\vec{V} = A\cos(\omega t + \phi) + jA\sin(\omega t + \phi)$$

$$\textit{or simply}, \vec{V} = A\cos\phi + jA\sin\phi$$

$$(\omega t \textit{ is common to each term, so it is dropped.})$$

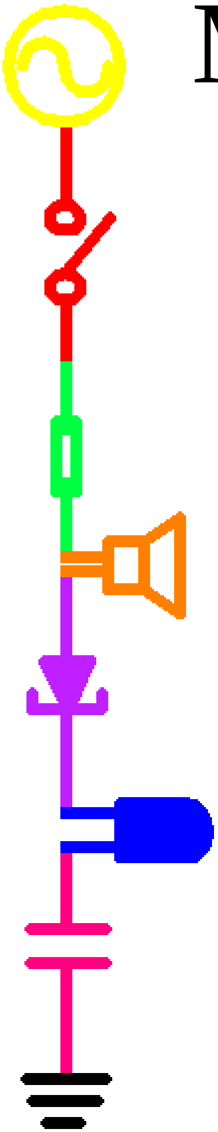
- The real part is our signal.
- The two parts allow us to determine the influence of the phase and amplitude changes mathematically.
- After we manipulate the numbers, we discard the imaginary part.



The "V=IR" of Phasors

$$\vec{V} = \vec{I}Z$$

- The influence of each component is given by Z, its complex impedance
- Once we have Z, we can use phasors to analyze circuits in much the same way that we analyze resistive circuits – except we will be using the complex polar representation.



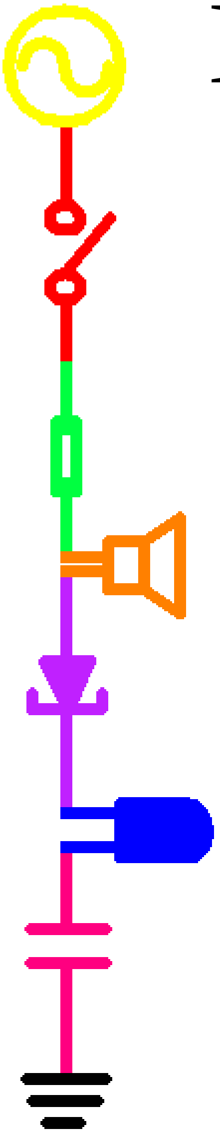
Magnitude and Phase

$$\vec{V} \equiv A\cos\phi + jA\sin\phi = x + jy$$

$$\left|\vec{V}\right| \equiv \sqrt{x^2 + y^2} = A \quad \textit{magnitude of } \vec{V}$$

$$\angle\vec{V} = \tan^{-1}\left(\frac{y}{x}\right) = \phi \quad \textit{phase of } \vec{V}$$

- Phasors have a magnitude and a phase derived from polar coordinates rules.



Influence of Resistor on Circuit

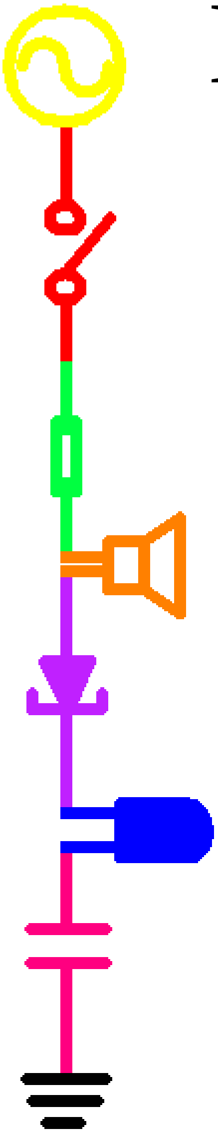
$$V_R = I_R R$$

$$if\ \ I_R(t) = A\sin(\omega t)$$

$$then\ \ V_R(t) = R * A\sin(\omega t)$$

- Resistor modifies the amplitude of the signal by R
- Resistor has no effect on the phase

Influence of Inductor on Circuit



$$V_L = L\frac{dI_L}{dt}$$

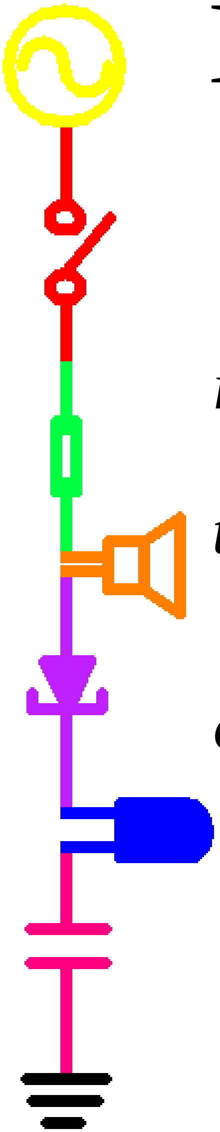
Note:
cosθ=sin(θ+π/2)

$$if\ \ I_L(t) = A\sin(\omega t)$$

$$then\ \ V_L(t) = \omega L * A\cos(\omega t)$$

$$or\ V_L(t) = \omega L * A\sin(\omega t + \frac{\pi}{2})$$

- Inductor modifies the amplitude of the signal by ωL
- Inductor shifts the phase by +π/2



Influence of Capacitor on Circuit

$$V_C = \frac{1}{C}\int I_C\,dt$$

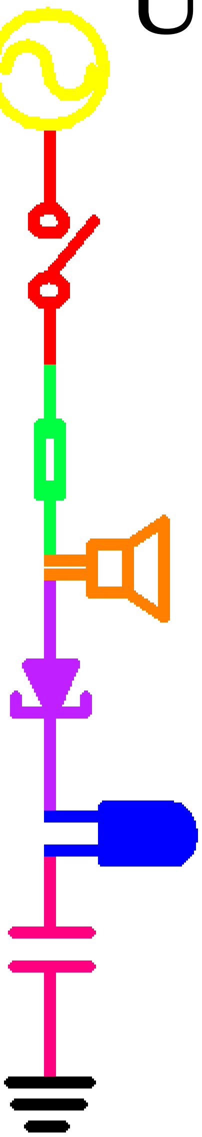
$$if\;\; I_C(t) = A\sin(\omega t)$$

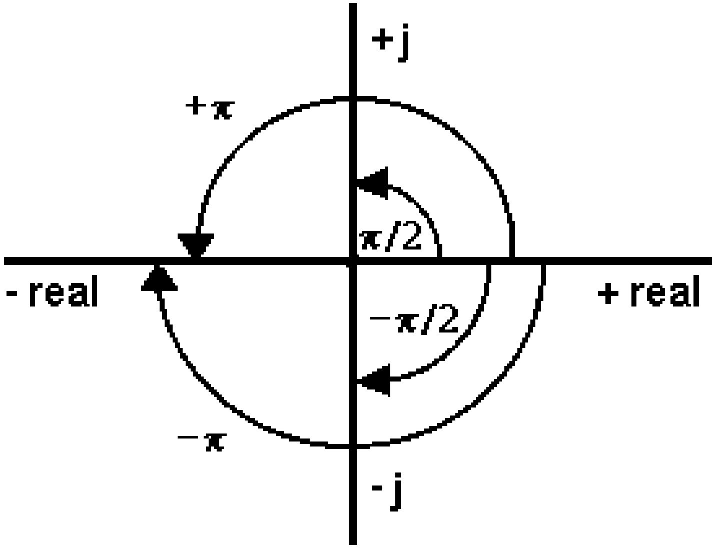
$$then\;\; V_C(t) = \frac{-1}{\omega C} * A\cos(\omega t) = \frac{1}{\omega C} * A\cos(\omega t - \pi)$$

$$or\; V_C(t) = \frac{1}{\omega C} * A\sin(\omega t + \frac{\pi}{2} - \pi) = \frac{1}{\omega C} * A\sin(\omega t - \frac{\pi}{2})$$

- Capacitor modifies the amplitude of the signal by $1/\omega C$
- Capacitor shifts the phase by $-\pi/2$

Understanding the influence of Phase





$$\angle\vec{V} = \tan^{-1}\left(\frac{y}{x}\right)$$

$+\,real:\quad if\ \ y = 0\ and\ x > 0$

$$then\ \angle\vec{V} = \tan^{-1}\left(\frac{0}{x^{+}}\right) = 0$$

$+\,j:\quad if\ \ x = 0\ and\ y > 0$

$$then\ \angle\vec{V} = \tan^{-1}\left(\frac{y^{+}}{0}\right) = \frac{\pi}{2} = 90^{\circ}$$

$-\,j:\quad if\ \ x = 0\ and\ y < 0$

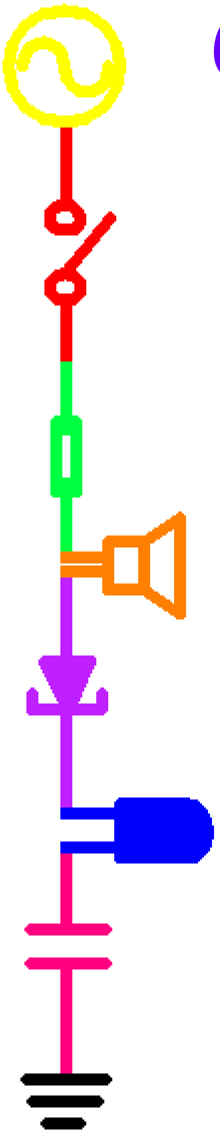
$$then\ \angle\vec{V} = \tan^{-1}\left(\frac{y^{-}}{0}\right) = -\frac{\pi}{2} = -90^{\circ}$$

$-\,real:\quad if\ \ y = 0\ and\ x < 0$

$$then\ \angle\vec{V} = \tan^{-1}\left(\frac{0}{x^{-}}\right) = \pi\ \ (or\ -\pi)$$

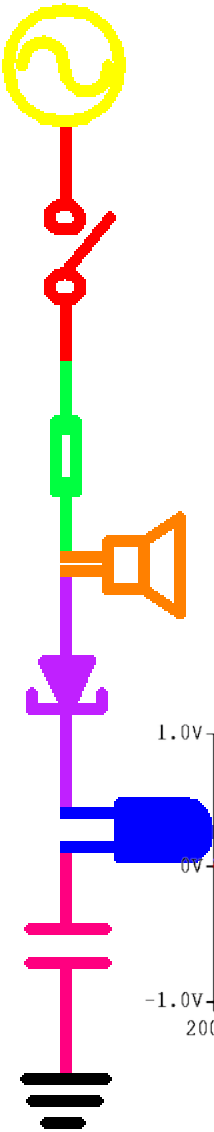
$$= \pm 180^{\circ}$$

Complex Impedance $\vec{V} = \vec{I}Z$

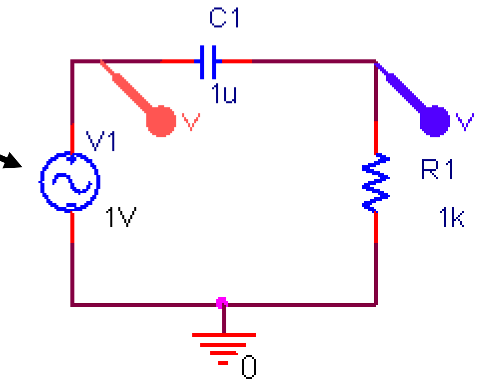


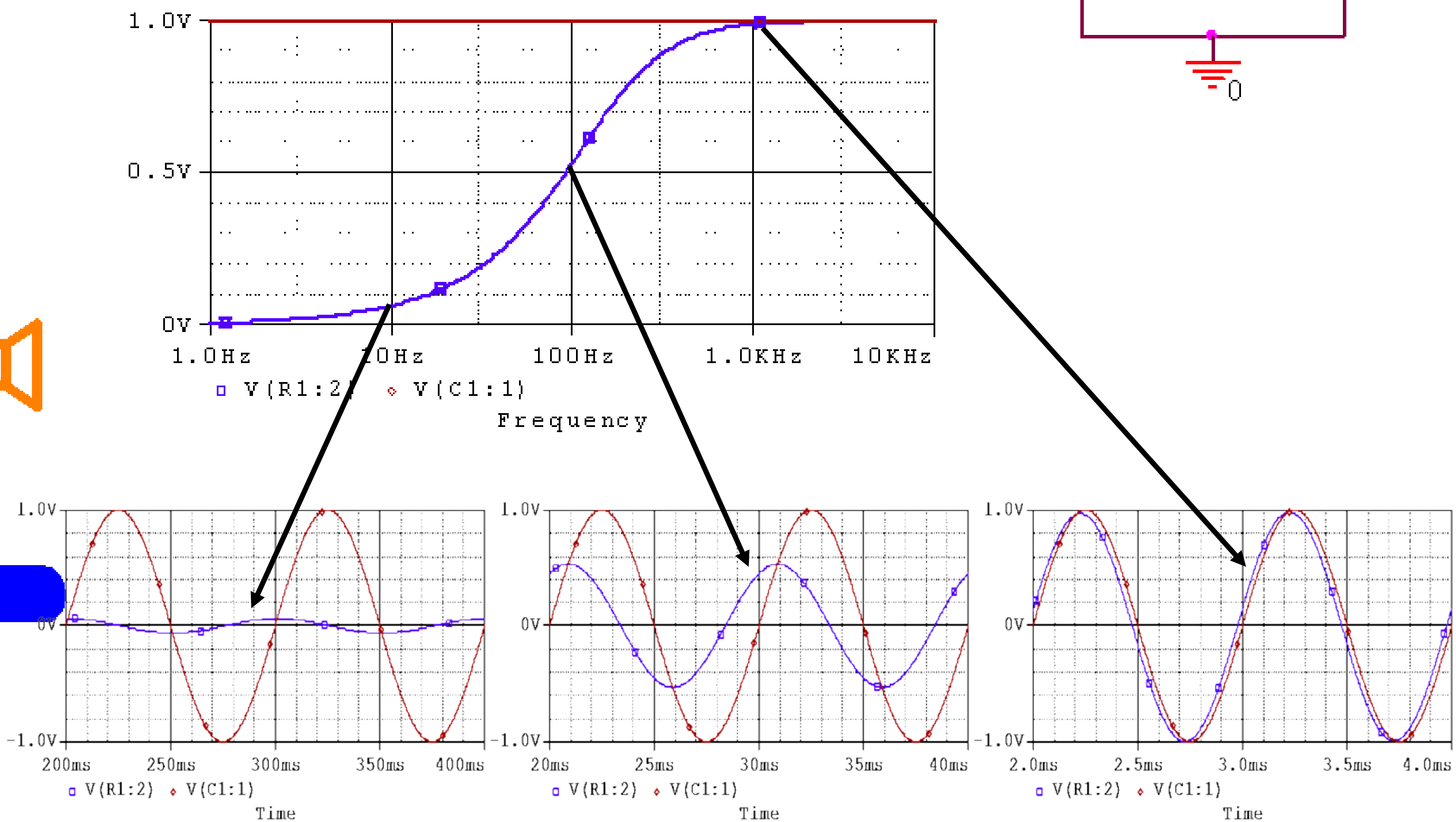
- Z defines the influence of a component on the amplitude and phase of a circuit
  - Resistors: $Z_R = R$
    - change the amplitude by R
  - Capacitors: $Z_C = 1/j\omega C$
    - change the amplitude by $1/\omega C$
    - shift the phase -90 ($1/j = -j$)
  - Inductors: $Z_L = j\omega L$
    - change the amplitude by ωL
    - shift the phase +90 (j)

AC Sweeps



AC Source sweeps from 1Hz to 10K Hz

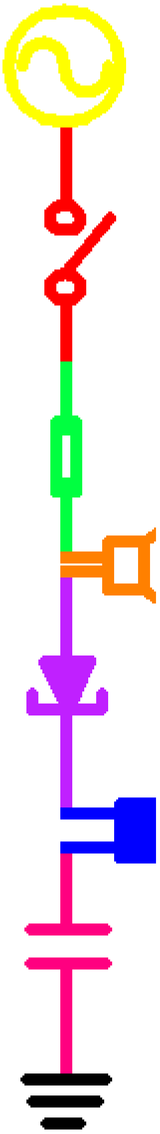




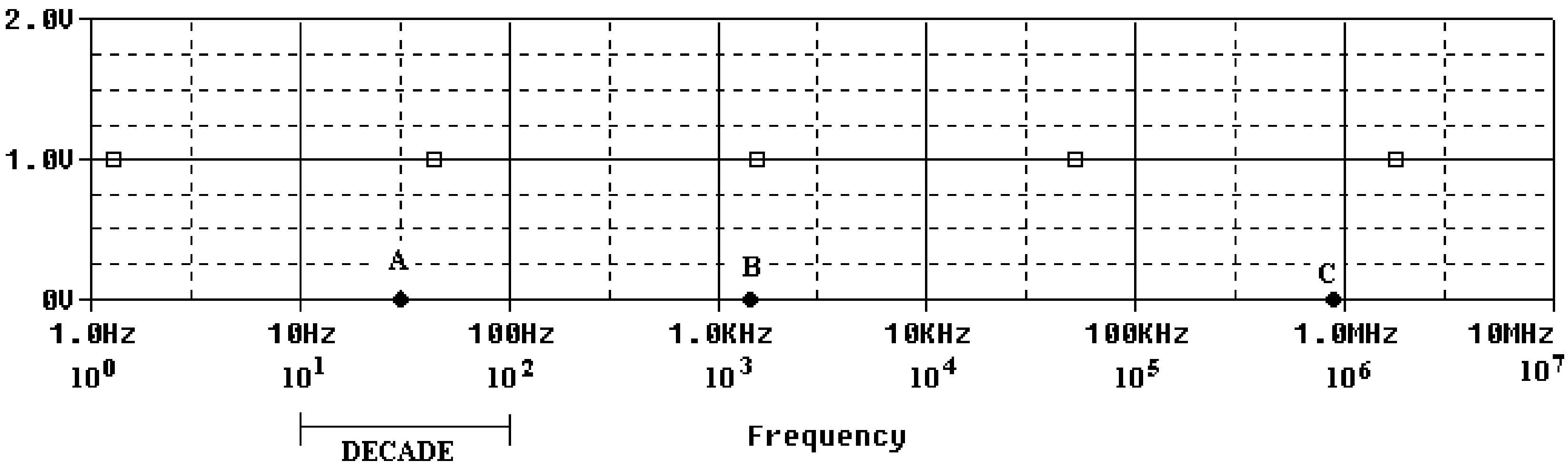
Transient at 10 Hz

Transient at 100 Hz

Transient at 1k Hz



Notes on Logarithmic Scales



Frequency = $10^{[\text{decade}].[\%\text{across decade}]}$

A: Frequency = $10^{1.5}$ = 32 Hz

B: Frequency = $10^{3.2}$ = 1600 Hz

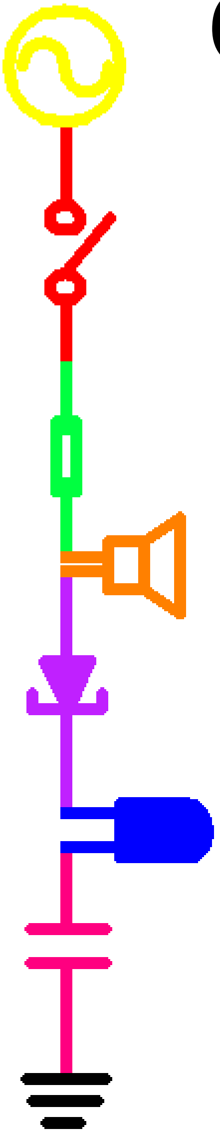
C: Frequency = $10^{5.9}$ = 790,000 Hz

[decade].[%across decade] = LOG [Frequency]

A: location = LOG (32) = 1.5   decade 1 - 50%

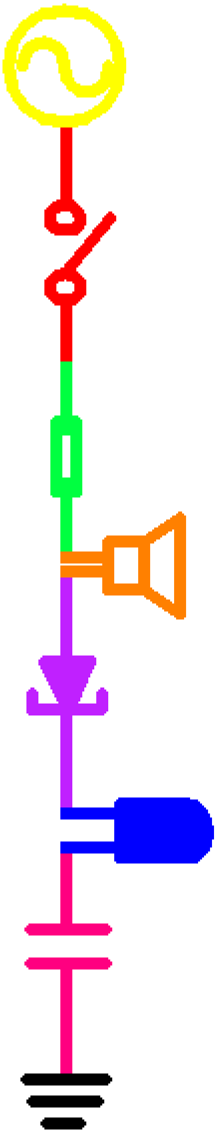
B: location = LOG(1600) = 3.2   decade 3 - 20%

C: location = LOG(790000)=5.9   decade 5 - 90%



Capture/PSpice Notes

- Showing the real and imaginary part of the signal
  - in Capture: PSpice->Markers->Advanced
    - ->Real Part of Voltage
    - ->Imaginary Part of Voltage
  - in PSpice: Add Trace
    - real part: R( )
    - imaginary part: IMG( )
- Showing the phase of the signal
  - in Capture:
    - PSpice->Markers->Advanced->Phase of Voltage
  - in PSPice: Add Trace
    - phase: P( )



Part B

- Phasors
- Complex Transfer Functions
- Filters

Definition of a Phasor

$$\text{if } V(t) = A\cos(\omega t + \phi), \text{ then let}$$

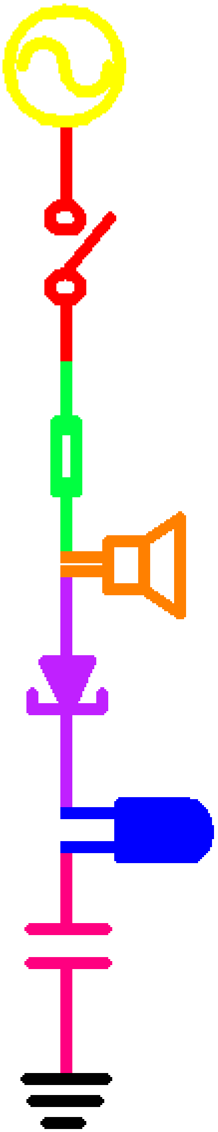
$$\vec{V} = A\cos\phi + jA\sin\phi$$

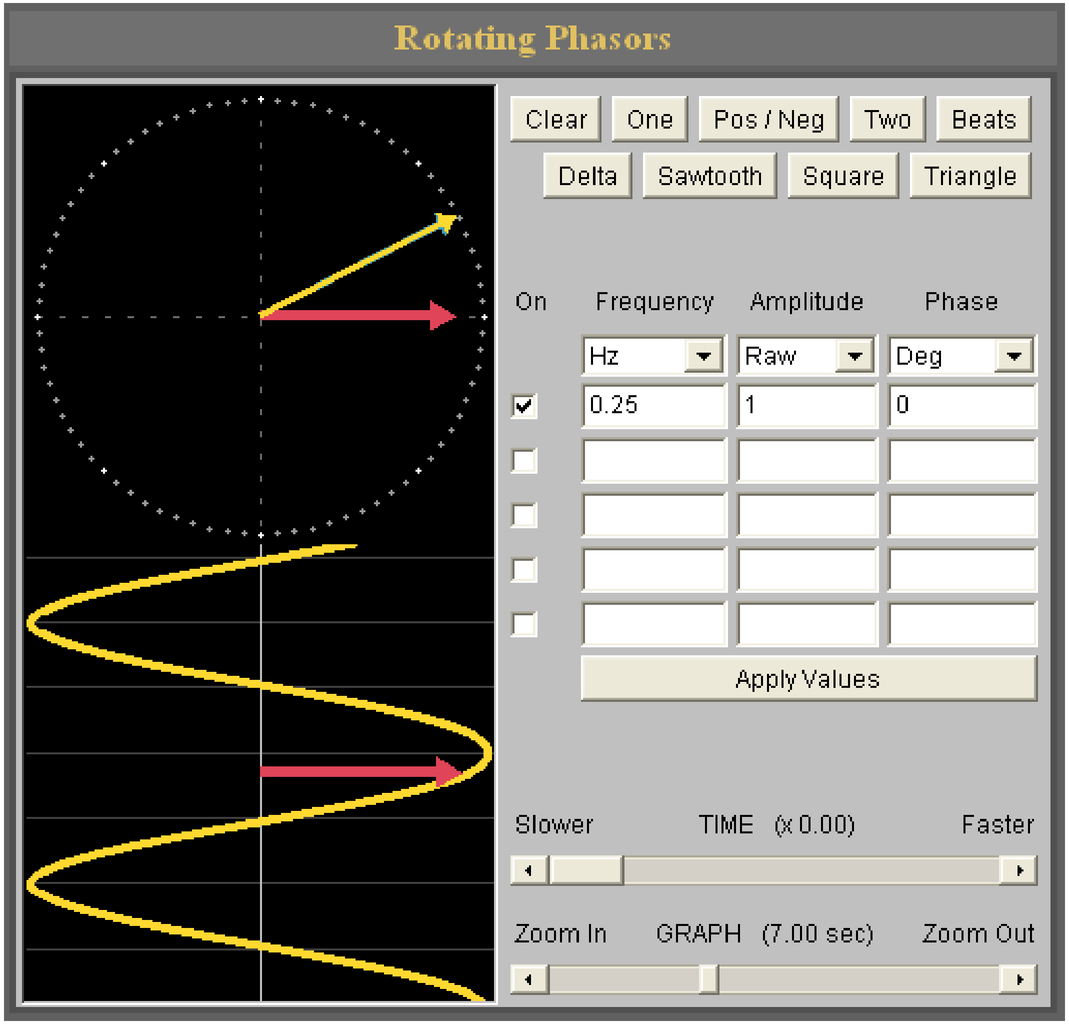
- The real part is our signal.
- The two parts allow us to determine the influence of the phase and amplitude changes mathematically.
- After we manipulate the numbers, we discard the imaginary part.

Phasor References

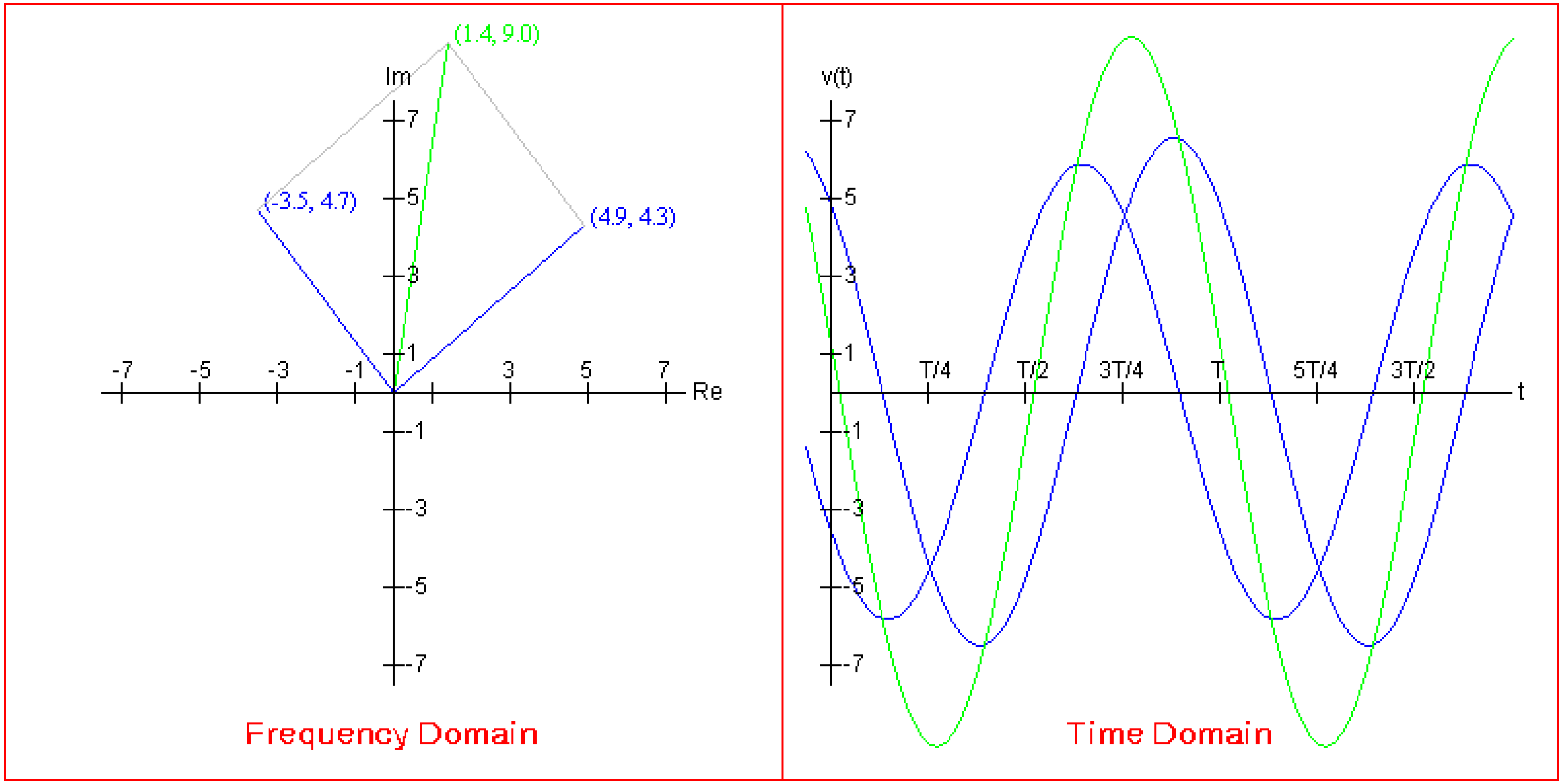
- http://ccrma-www.stanford.edu/~jos/filters/Phasor_Notation.html
- http://www.ligo.caltech.edu/~vsanni/ph3/ExpACCircuits/ACCircuits.pdf
- http://ptolemy.eecs.berkeley.edu/eecs20/berkeley/phasors/demo/phasors.html

Phasor Applet

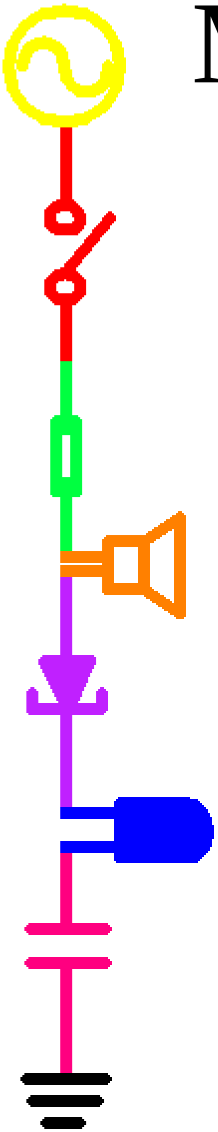




Adding Phasors & Other Applets



Magnitude and Phase

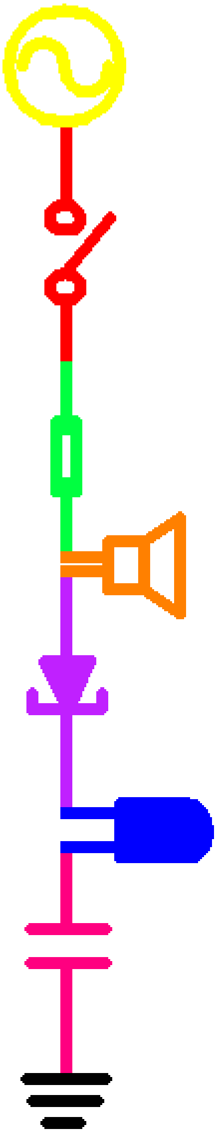


$$\vec{V} \equiv A\cos\phi + jA\sin\phi = x + jy$$

$$\left|\vec{V}\right| \equiv \sqrt{x^2 + y^2} = A \quad \textit{magnitude of } \vec{V}$$

$$\angle\vec{V} = \tan^{-1}\left(\frac{y}{x}\right) = \phi \quad \textit{phase of } \vec{V}$$

- Phasors have a magnitude and a phase derived from polar coordinates rules.



Euler's Formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$

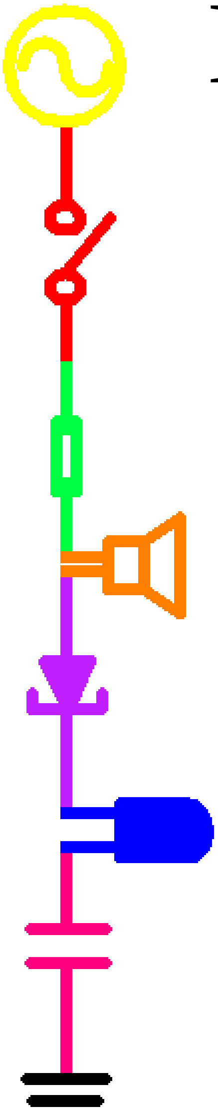
$$if\ \ z = x + jy = r\cos\theta + jr\sin\theta = re^{j\theta}$$

$$then\ z_3 = \frac{z_1}{z_2} = \frac{r_1 e^{j\theta_1}}{r_2 e^{j\theta_2}} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

$$therefore, \quad r_3 = \frac{r_1}{r_2} \quad and \quad \theta_3 = \theta_1 - \theta_2$$

$$and\ \ z_4 = z_1 \cdot z_2 = r_1 e^{j\theta_1} \cdot r_2 e^{j\theta_2} = r_1 \cdot r_2 e^{j(\theta_1 + \theta_2)}$$

$$therefore, \quad r_4 = r_1 \cdot r_2 \quad and \quad \theta_4 = \theta_1 + \theta_2$$



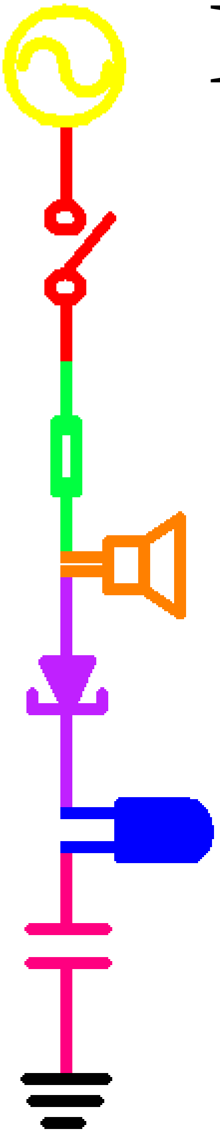
Manipulating Phasors (1)

$$\vec{V} = A\cos(\omega t + \phi) + j\sin(\omega t + \phi) = Ae^{j(\omega t + \phi)}$$

$$\vec{X}_3 = \frac{\vec{V}_1}{\vec{V}_2} = \frac{A_1 e^{j(\omega t + \phi_1)}}{A_2 e^{j(\omega t + \phi_2)}} = \frac{A_1}{A_2}\frac{e^{j\omega t}}{e^{j\omega t}}\frac{e^{j\phi_1}}{e^{\phi_2}} = \frac{A_1}{A_2} e^{j(\phi_1 - \phi_2)}$$

therefore, $\boxed{\left|\vec{X}_3\right| = \frac{A_1}{A_2}}$ *and* $\boxed{\angle \vec{X}_3 = \phi_1 - \phi_2}$

- Note ωt is eliminated by the ratio
  - This gives the phase *change* between signal 1 and signal 2



Manipulating Phasors (2)

$$\vec{V}_1 = x_1 + jy_1 \quad \vec{V}_2 = x_2 + jy_2$$

$$\vec{V}_3 = x_3 + jy_3$$

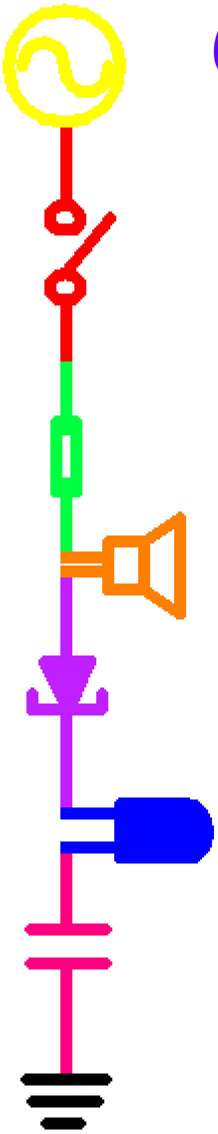
$$\left|\vec{X}_3\right| = \frac{\left|\vec{V}_1\right|}{\left|\vec{V}_2\right|} = \frac{\sqrt{x_1^2 + y_1^2}}{\sqrt{x_2^2 + y_2^2}}$$

$$\angle\vec{X}_3 = \angle\vec{V}_1 - \angle\vec{V}_2 = \tan^{-1}\left(\frac{y_1}{x_1}\right) - \tan^{-1}\left(\frac{y_2}{x_2}\right)$$

Complex Transfer Functions

$$\vec{H}(j\omega) \equiv \frac{\vec{V}_{out}(j\omega)}{\vec{V}_{in}(j\omega)}$$

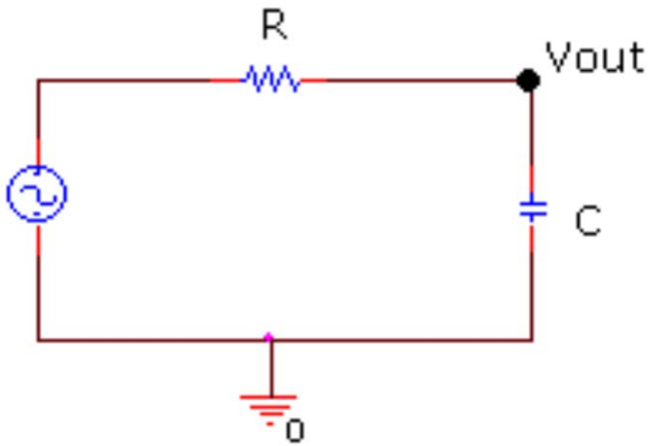
- If we use phasors, we can define H for all circuits in this way.
- If we use complex impedances, we can combine all components the way we combine resistors.
- H and V are now functions of j and ω



Complex Impedance $\vec{V} = \vec{I}Z$

- Z defines the influence of a component on the amplitude and phase of a circuit
  - Resistors: $Z_R = R$
  - Capacitors: $Z_C = 1/j\omega C$
  - Inductors: $Z_L = j\omega L$
- We can use the rules for resistors to analyze circuits with capacitors and inductors if we use phasors and complex impedance.

Simple Example



$$Z_R = R$$

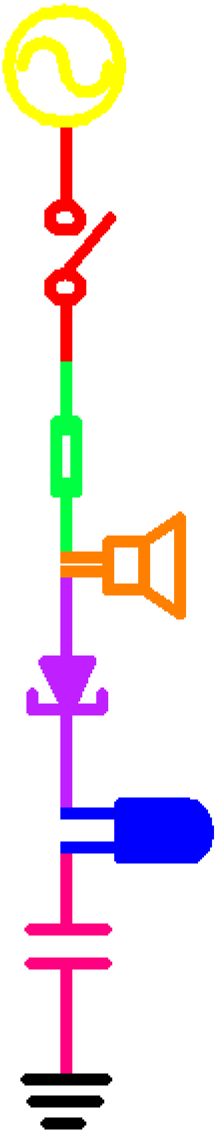
$$Z_C = \frac{1}{j\omega C}$$

$$\vec{H}(j\omega) = \frac{\vec{V}_{out}(j\omega)}{\vec{V}_{in}(j\omega)} = \frac{Z_C \vec{I}}{(Z_R + Z_C)\vec{I}} = \frac{Z_C}{(Z_R + Z_C)}$$

$$\vec{H}(j\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C}$$

$$\boxed{\vec{H}(j\omega) = \frac{1}{j\omega RC + 1}}$$

Simple Example (continued)



$$\vec{H}(j\omega) = \frac{1}{j\omega RC + 1}$$

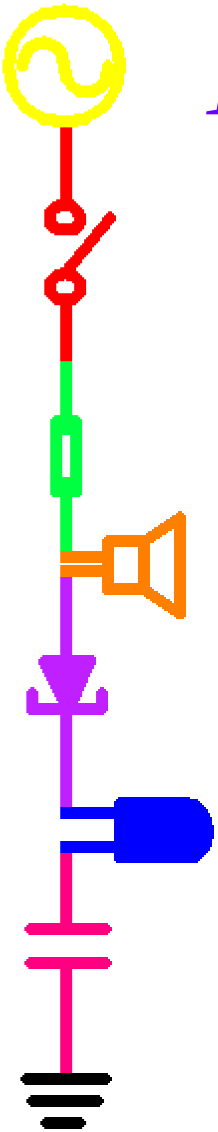
$$\left| H(j\omega) \right| = \frac{\left|1 + j0\right|}{\left|1 + j\omega RC\right|} = \frac{\sqrt{1^2 + 0^2}}{\sqrt{1^2 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\angle H(j\omega) = \angle(1 + j0) - \angle(1 + j\omega RC)$$

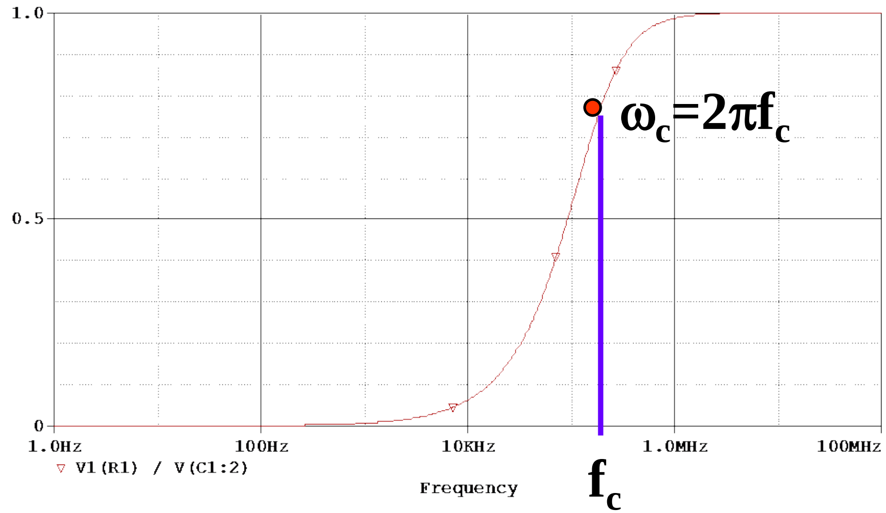
$$\angle H(j\omega) = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega RC}{1}\right) = -\tan^{-1}(\omega RC)$$

$$\left| H(j\omega) \right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}(\omega RC)$$



High and Low Pass Filters

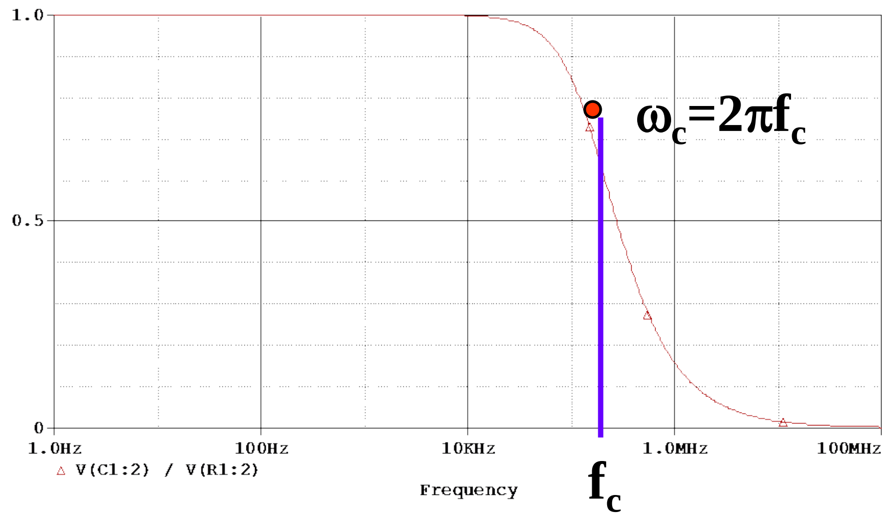


<u>High Pass Filter</u>

$H = 0$ at $\omega \rightarrow 0$

$H = 1$ at $\omega \rightarrow \infty$

$H = 0.707$ at ω_c

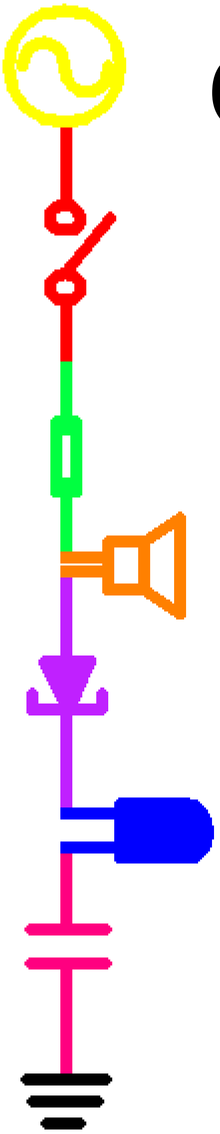


<u>Low Pass Filter</u>

$H = 1$ at $\omega \rightarrow 0$

$H = 0$ at $\omega \rightarrow \infty$

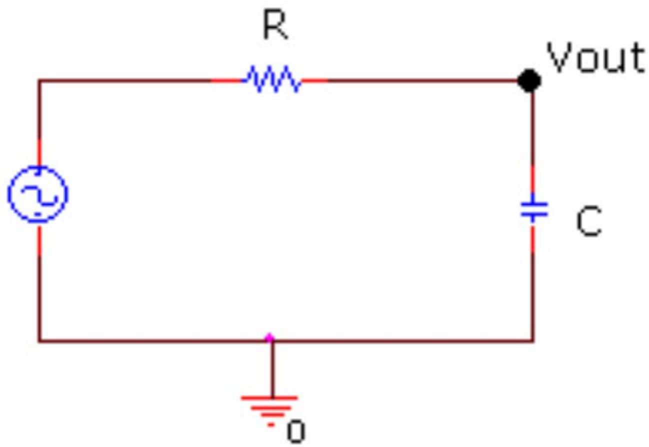
$H = 0.707$ at ω_c



Corner Frequency

- The corner frequency of an RC or RL circuit tells us where it transitions from low to high or visa versa.
- We define it as the place where $\left| H(j\omega_c) \right| = \frac{1}{\sqrt{2}}$
- For RC circuits: $\omega_c = \frac{1}{RC}$
- For RL circuits: $\omega_c = \frac{R}{L}$

Corner Frequency of our example



$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$\left| H(j\omega) \right| = \frac{1}{\sqrt{2}}$$

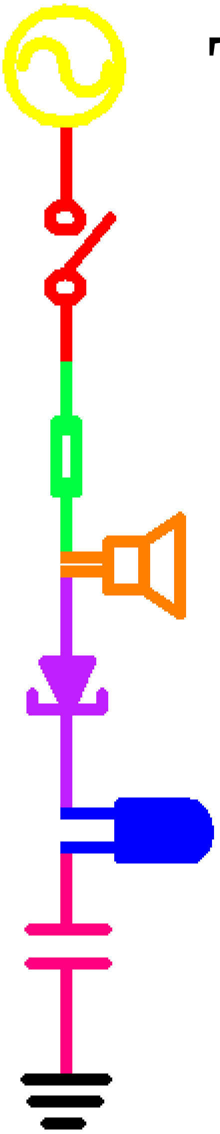
$$\left| H(j\omega) \right| = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{2}} \qquad \frac{1}{1 + (\omega RC)^2} = \frac{1}{2}$$

$$2 = 1 + (\omega RC)^2 \qquad \frac{1}{(RC)^2} = \omega^2 \qquad \omega_c = \frac{1}{RC}$$

H(jω), ω_c, and filters

- We can use the transfer function, H(jω), and the corner frequency, ω_c, to easily determine the characteristics of a filter.
- If we consider the behavior of the transfer function as ω approaches 0 and infinity and look for when H nears 0 and 1, we can identify high and low pass filters.
- The corner frequency gives us the point where the filter changes:

$$f_c = \frac{\omega_c}{2\pi}$$



Taking limits

$$H(j\omega) = \frac{a_2\omega^2 + a_1\omega + a_0}{b_2\omega^2 + b_1\omega + b_0}$$

- At low frequencies, (ie. $\omega = 10^{-3}$), lowest power of ω dominates

$$H(j\omega) = \frac{a_2 10^{-6} + a_1 10^{-3} + a_0 10^{0}}{b_2 10^{-6} + b_1 10^{-3} + b_0 10^{0}} \approx \frac{a_0}{b_0}$$

- At high frequencies (ie. $\omega = 10^{+3}$), highest power of ω dominates

$$H(j\omega) = \frac{a_2 10^{+6} + a_1 10^{+3} + a_0 10^{0}}{b_2 10^{+6} + b_1 10^{+3} + b_0 10^{0}} \approx \frac{a_2}{b_2}$$

Taking limits -- Example

$$H(j\omega) = \frac{9\omega^2 + 15\omega}{3\omega^2 + 2\omega + 5}$$

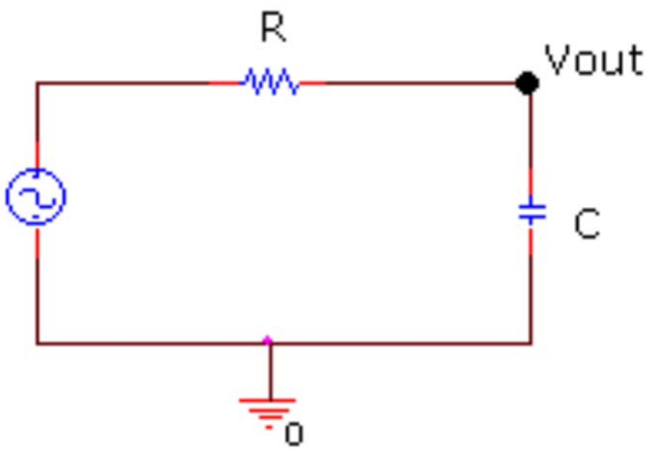
- At low frequencies, (lowest power)

$$H_{LO}(j\omega) = \frac{15\omega}{5} = 3\omega$$

- At high frequencies, (highest power)

$$H_{HI}(j\omega) = \frac{9\omega^2}{3\omega^2} = 3$$

Our example at low frequencies



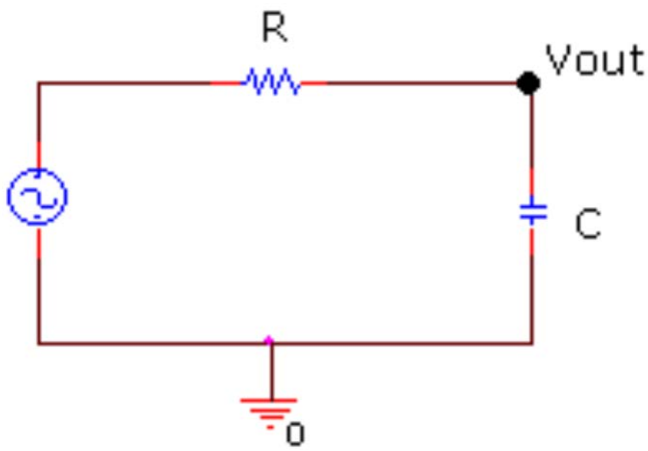
$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$H_{LOW}(j\omega) = \frac{1}{1+0} = 1$$

$$\left| H_{LOW}(j\omega) \right| as\omega \rightarrow 0 = \left| 1 \right| = 1$$

$$\angle H_{LOW}(j\omega) = \tan^{-1}\left(\frac{0}{1}\right) = 0\ (on + x\ axis)$$

Our example at high frequencies



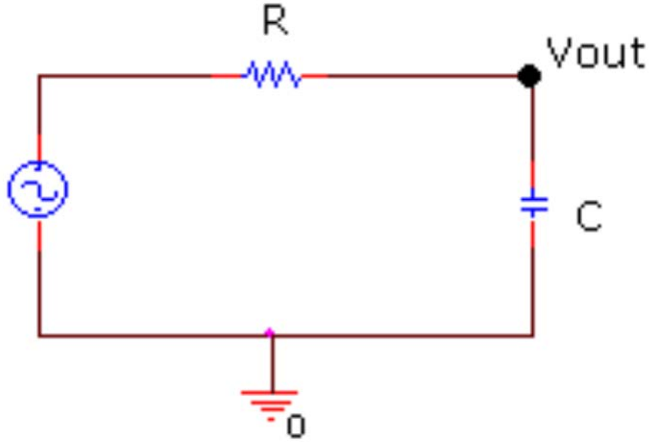
$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$H_{HIGH}(j\omega) = \frac{1}{j\omega RC}$$

$$\left| H_{HIGH}(j\omega) \right| as\, \omega \to \infty = \left| \frac{1}{j\omega RC} \right| = \frac{1}{\infty} = 0$$

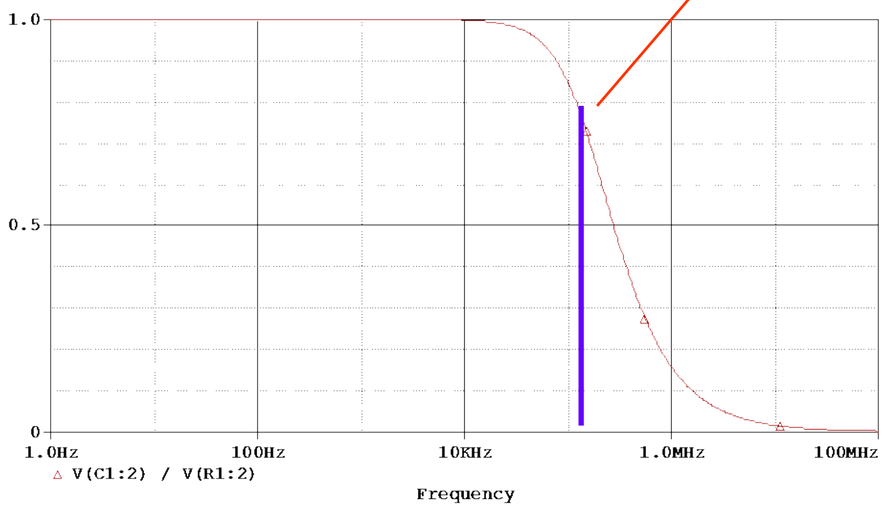
$$\angle H_{HIGH}(j\omega) = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega RC}{0}\right) = 0 - \frac{\pi}{2} = -\frac{\pi}{2}$$

Our example is a low pass filter



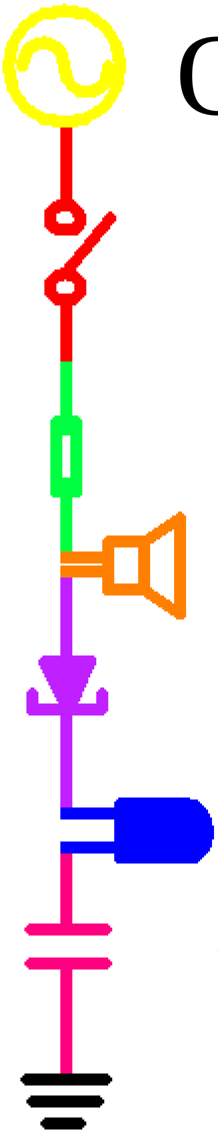
$$|H_{LOW}| = 1 \quad |H_{HIGH}| = 0$$

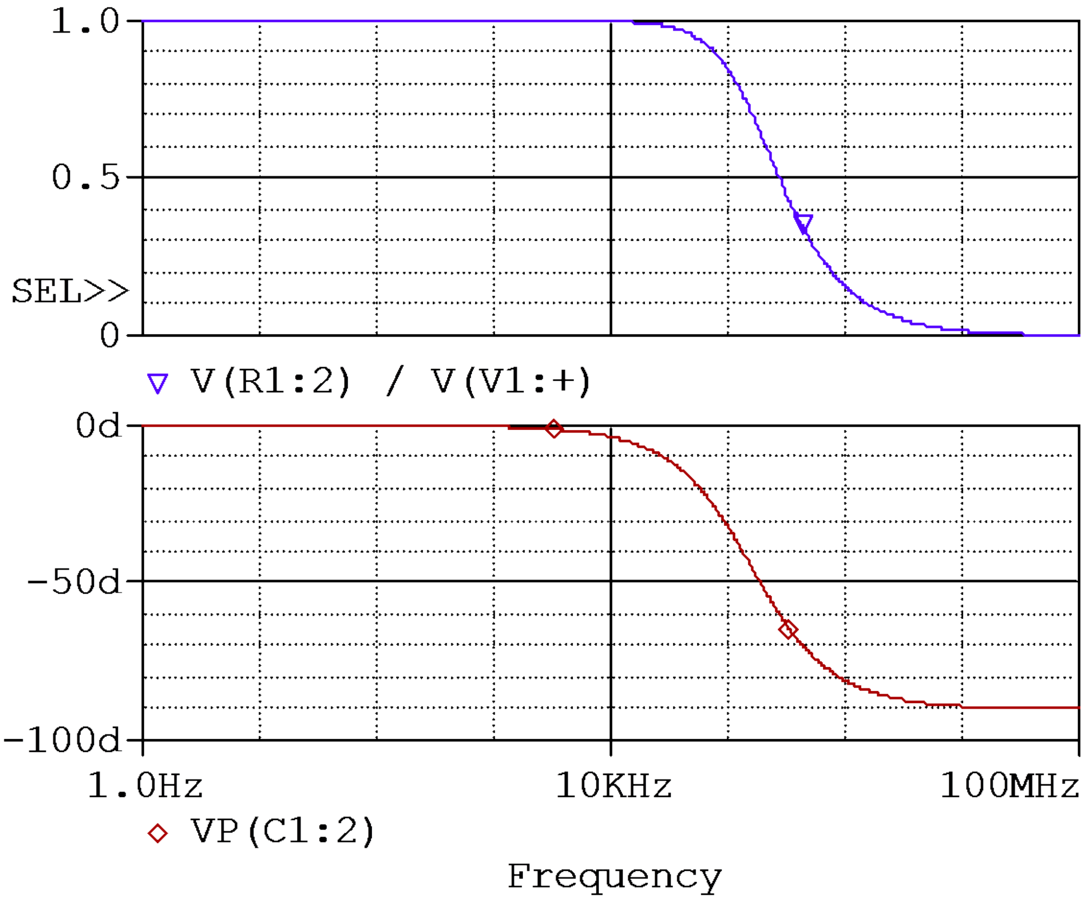
$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi RC}$$



What about the phase?

Our example has a phase shift



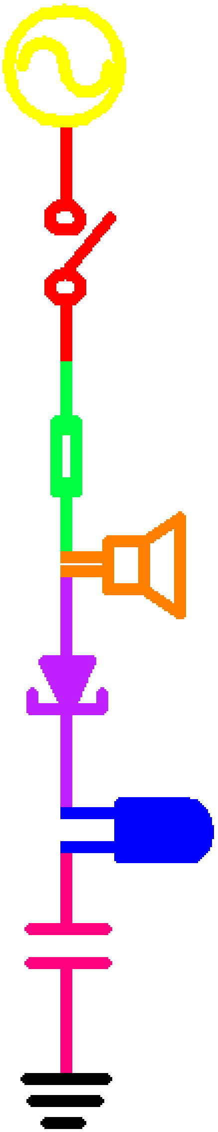


$$|H_{LOW}| = 1$$

$$|H_{HIGH}| = 0$$

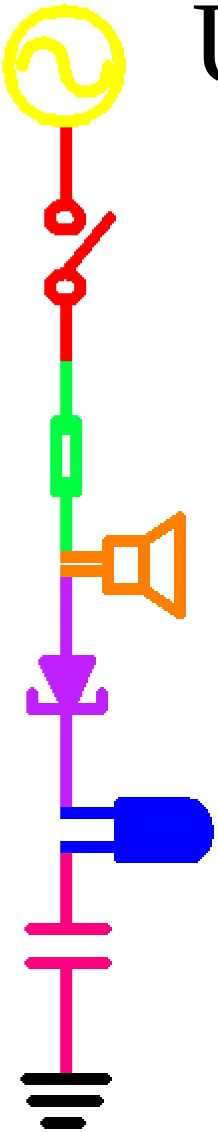
$$\angle H_{LOW}(j\omega) = 0$$

$$\angle H_{HIGH}(j\omega) = -90^\circ$$



Part C

- Using Transfer Functions
- Capacitor Impedance Proof
- More Filters
- Transfer Functions of RLC Circuits



Using H to find V_{out}

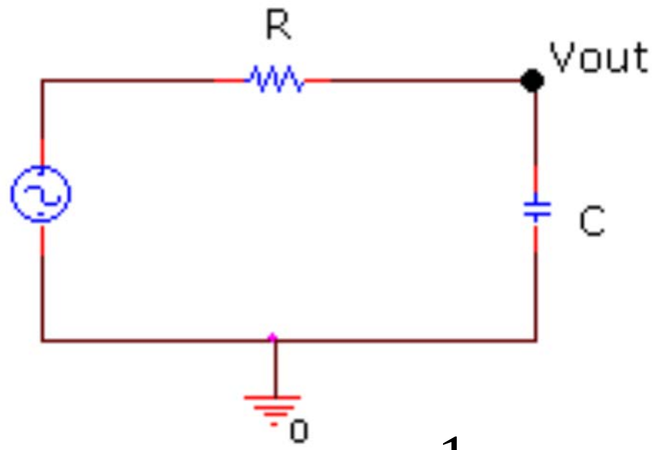
$$H(j\omega) = \frac{\vec{V}_{out}}{\vec{V}_{in}} = \frac{A_{out} e^{j\phi_{out}} e^{j\omega t}}{A_{in} e^{j\phi_{in}} e^{j\omega t}} = \frac{A_{out} e^{j\phi_{out}}}{A_{in} e^{j\phi_{in}}}$$

$$A_{out} e^{j\phi_{out}} = H(j\omega) A_{in} e^{j\phi_{in}}$$

$$A_{out} e^{j\phi_{out}} = \left| H(j\omega) \right| e^{j\angle H(j\omega)} A_{in} e^{j\phi_{in}}$$

$$\boxed{A_{out} = \left| H(j\omega) \right| A_{in}} \qquad \boxed{\phi_{out} = \angle H(j\omega) + \phi_{in}}$$

Simple Example (with numbers)



$$C = 1\mu F \qquad R = 1k\Omega$$

$$V_{in}(t) = 2V\cos(2k\pi t + \frac{\pi}{4})$$

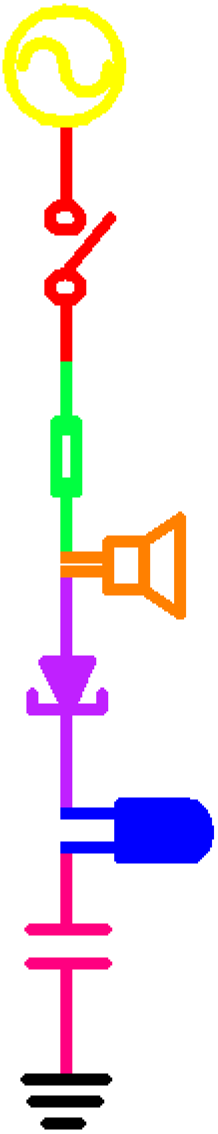
$$\vec{H}(j\omega) = \frac{1}{j\omega RC + 1} = \frac{1}{j2k\pi 1k1\mu + 1} = \frac{1}{2\pi j + 1}$$

$$\left|H(j\omega)\right| = \frac{1^2}{\sqrt{1 + (2\pi)^2}} = 0.157 \qquad \angle H(j\omega) = 0 - \tan^{-1}\left(\frac{2\pi}{1}\right) = -1.41$$

$$V_{out}(t) = 0.157 * 2V\cos(2k\pi t + 0.785 - 1.41)$$

$$\boxed{V_{out}(t) = 0.314V\cos(2k\pi t - 0.625)}$$

Capacitor Impedance Proof



Prove: $$Z_C = \frac{1}{j\omega C}$$

$$I_C(t) = C\frac{dV_C(t)}{dt} \quad and \quad V_C(t) = A\cos(\omega t + \phi)$$

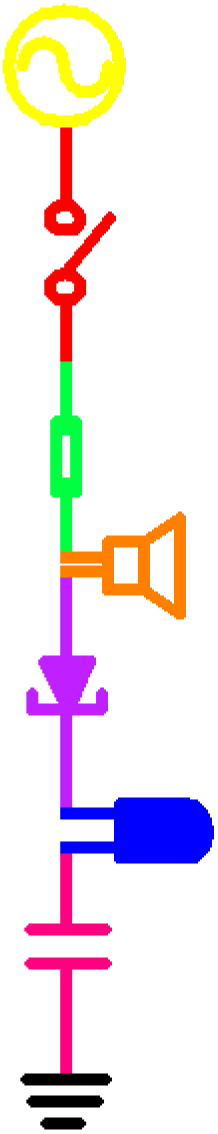
$$\vec{V}_C(j\omega) = A\cos(\omega t + \phi) + jA\sin(\omega t + \phi) = Ae^{j(\omega t + \phi)}$$

$$\frac{d\vec{V}_C(j\omega)}{dt} = \frac{dAe^{j(\omega t + \phi)}}{dt} = Aj\omega e^{j(\omega t + \phi)} = j\omega\vec{V}_C(j\omega)$$

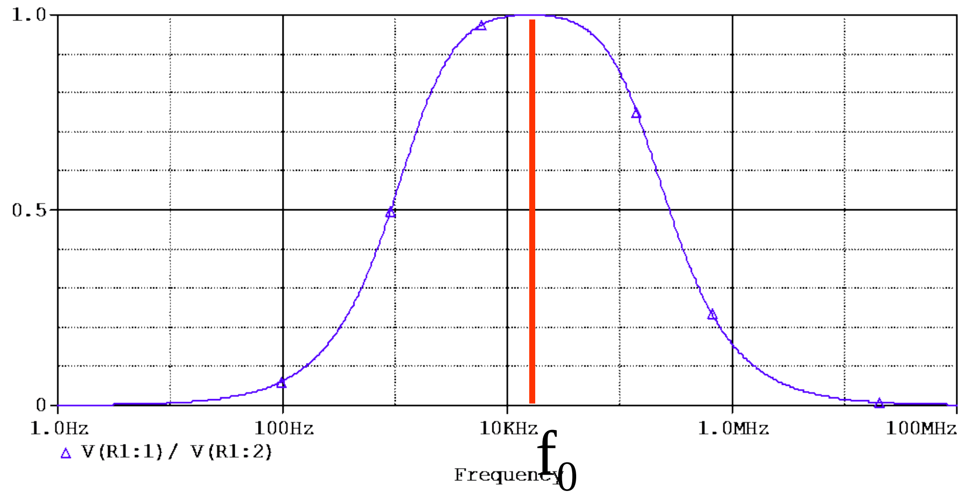
$$\frac{dV_C(t)}{dt} = \mathrm{Re}\left\{\frac{d\vec{V}_C(j\omega)}{dt}\right\} = j\omega A\cos(\omega t + \phi) = j\omega V_C(t)$$

$$I_C(t) = C\frac{dV_C(t)}{dt} = Cj\omega V_C(t)$$

$$\boxed{V_C(t) = \frac{1}{j\omega C} I_C(t)}$$



Band Filters

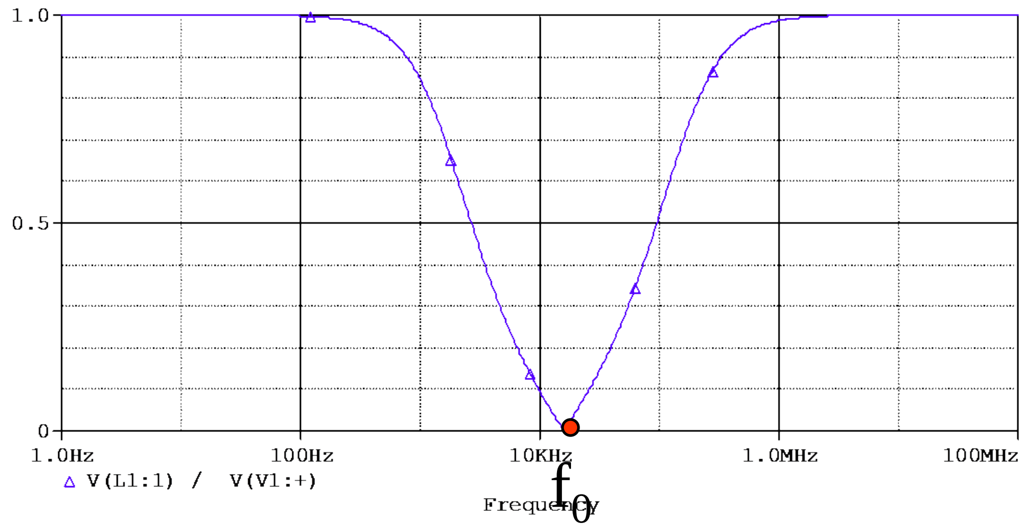


<u>Band Pass Filter</u>

H = 0 at $\omega \rightarrow 0$

H = 0 at $\omega \rightarrow \infty$

H = 1 at $\omega_0 = 2\pi f_0$

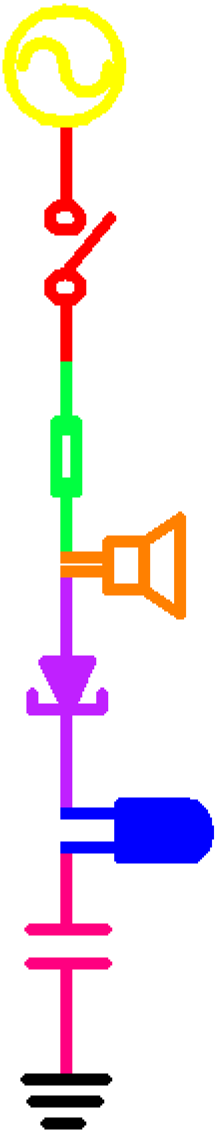


<u>Band Reject Filter</u>

H = 1 at $\omega \rightarrow 0$

H = 1 at $\omega \rightarrow \infty$

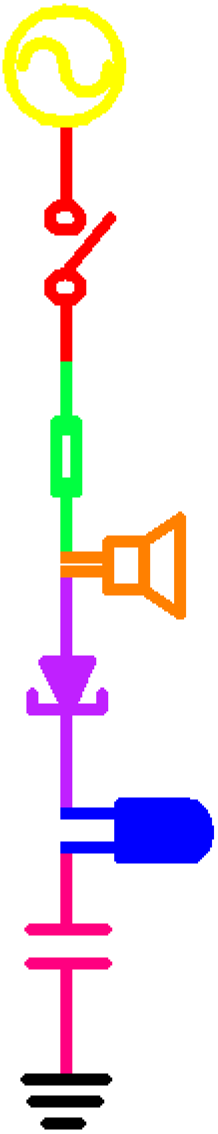
H = 0 at $\omega_0 = 2\pi f_0$



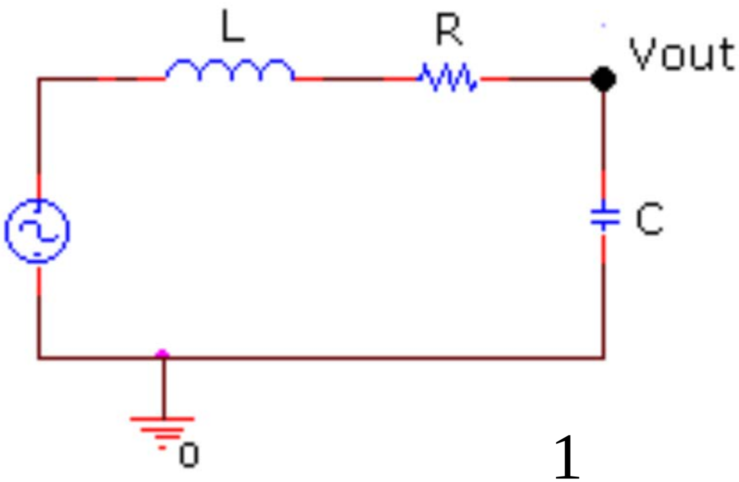
Resonant Frequency

- The resonant frequency of an RLC circuit tells us where it reaches a maximum or minimum.
- This can define the center of the band (on a band filter) or the location of the transition (on a high or low pass filter).
- The equation for the resonant frequency of an RLC circuit is:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



Another Example

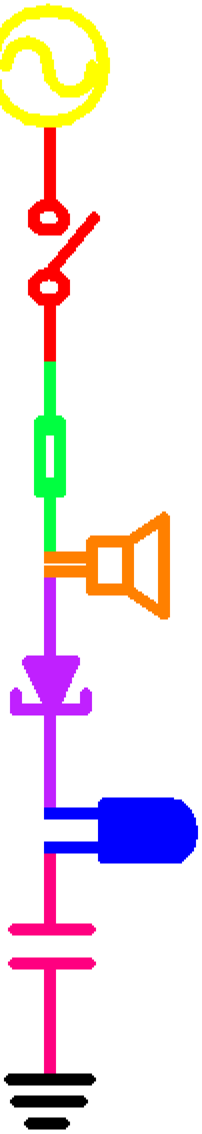


$$Z_R = R \quad Z_L = j\omega L$$

$$Z_C = \frac{1}{j\omega C}$$

$$H(j\omega) = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \frac{1}{j\omega RC + j^2\omega^2 LC + 1}$$

$$H(j\omega) = \frac{1}{(1-\omega^2 LC) + j\omega RC}$$



At Very Low Frequencies

$$H_{LOW}(j\omega) = \frac{1}{1} = 1$$

$$\left| H_{LOW}(j\omega) \right| \omega \to 0 = 1$$

$$\angle H_{LOW}(j\omega) = 0$$

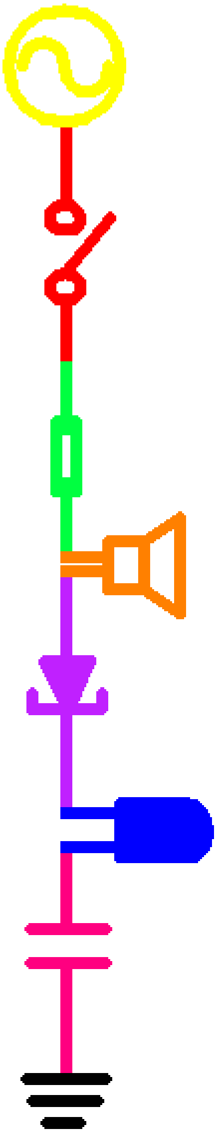
At Very High Frequencies

$$H_{HIGH}(j\omega) = \frac{1}{-\omega^2 LC}$$

$$\left| H_{HIGH}(j\omega) \right| \omega \to \infty = \left| \frac{1}{\infty} \right| = 0$$

$$\angle H_{HIGH}(j\omega) = \pi \ or \ -\pi$$

At the Resonant Frequency



$$H(j\omega)=\frac{1}{(1-\omega^2 LC)+j\omega RC} \qquad \omega_0=\frac{1}{\sqrt{LC}} \qquad f_0=\frac{1}{2\pi\sqrt{LC}}$$

$$H(j\omega_0)=\frac{1}{(1-\left(\frac{1}{\sqrt{LC}}\right)^2 LC)+j\left(\frac{1}{\sqrt{LC}}\right)RC}=\frac{1}{(1-1)+j\left(\frac{RC}{\sqrt{LC}}\right)}$$

$$H(j\omega_0)=-j\frac{\sqrt{LC}}{RC}$$

$$\left|H(j\omega_0)\right|=\frac{\sqrt{LC}}{RC}$$

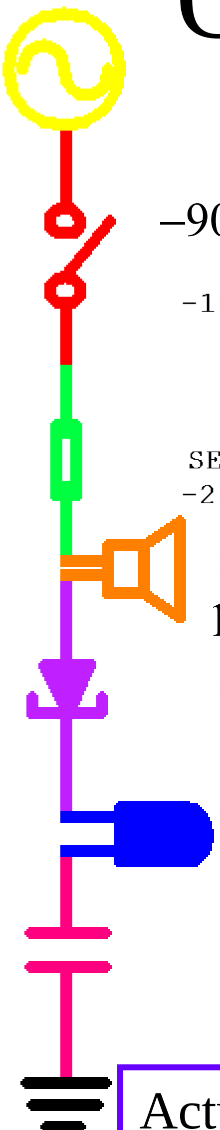
$$\angle H(j\omega_0)=-\frac{\pi}{2}$$

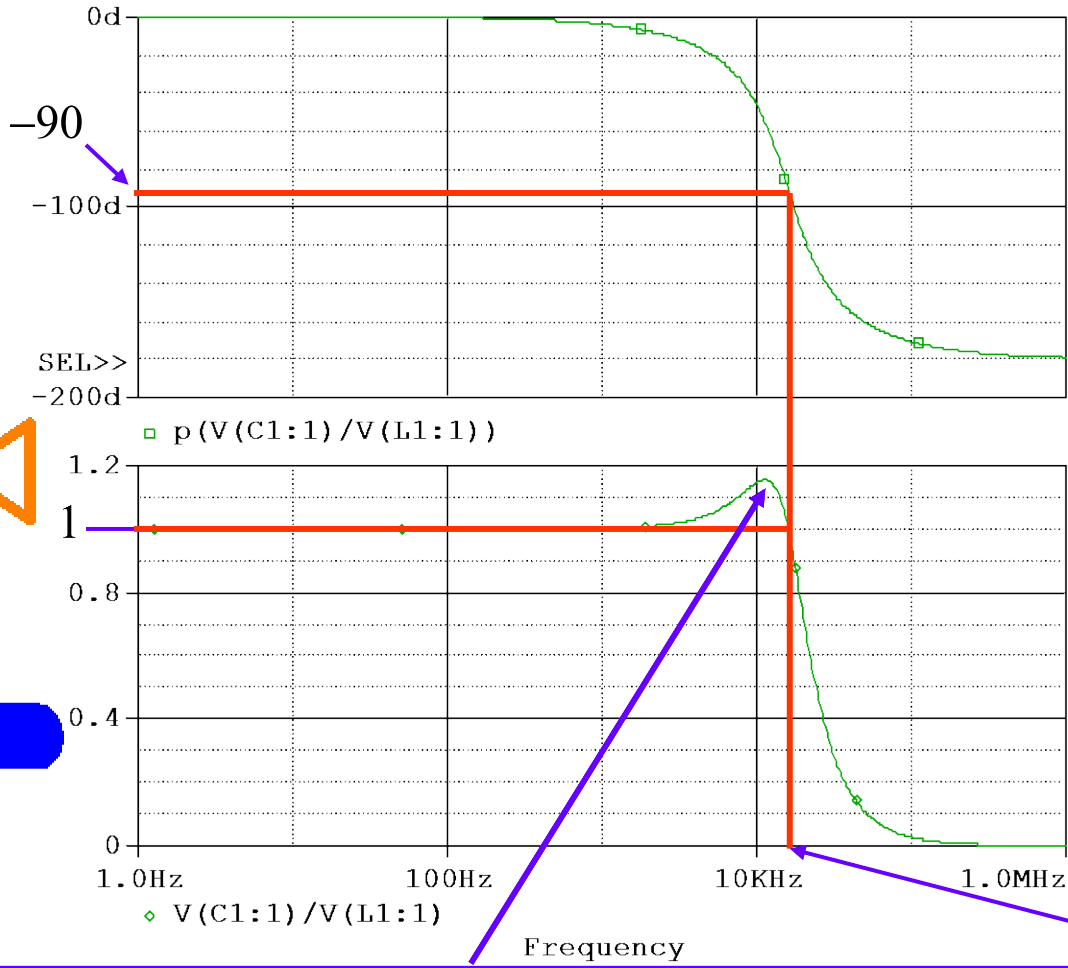
if L=1mH, C=0.1uF and R=100Ω

ω_0=100k rad/sec f_0=16k Hz

$|H_0|=1$ $\angle H=-\frac{\pi}{2}$ *radians*

Our example is a low pass filter





Phase

$\phi = 0$ at $\omega \to 0$

$\phi = -180$ at $\omega \to \infty$

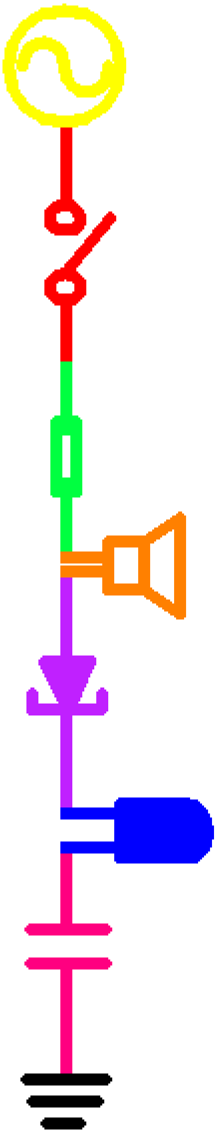
Magnitude

$H = 1$ at $\omega \to 0$

$H = 0$ at $\omega \to \infty$

f_0=16k Hz

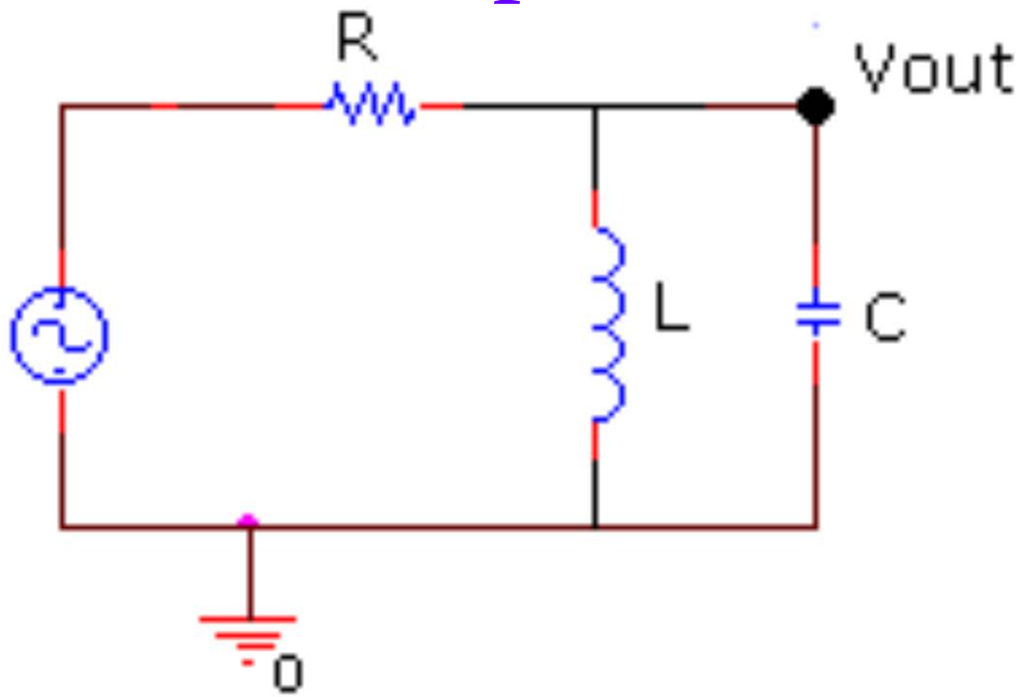
Actual circuit resonance is only at the theoretical resonant frequency, f_0, when there is no resistance.



Part D

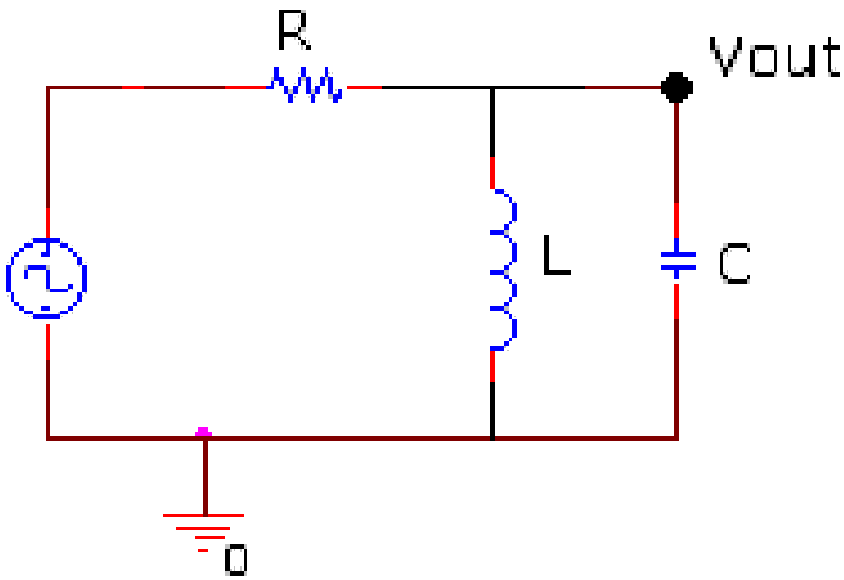
- Equivalent Impedance
- Transfer Functions of More Complex Circuits

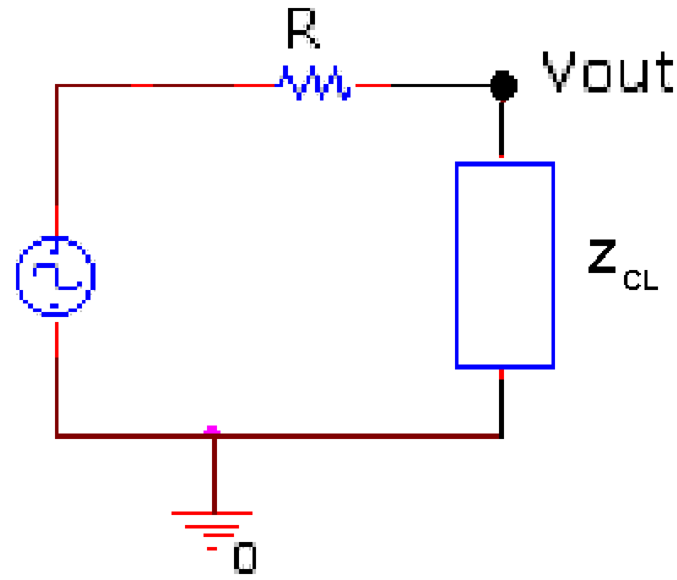
Equivalent Impedance



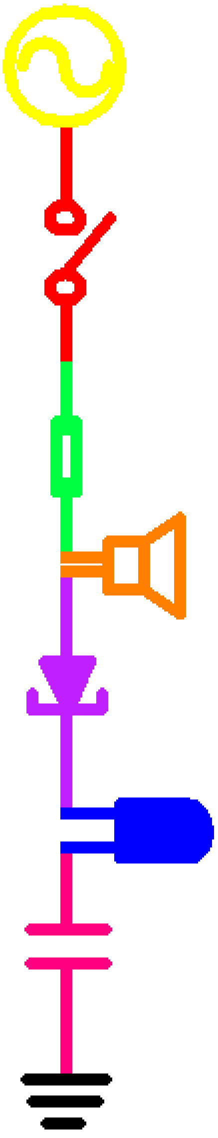
- Even though this filter has parallel components, we can still handle it.
- We can combine complex impedances like resistors to find the equivalent impedance of the components combined.

Equivalent Impedance





$$Z_{CL} = \frac{Z_L \cdot Z_C}{Z_L + Z_C} = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{j^2\omega^2 LC + 1} = \frac{j\omega L}{1 - \omega^2 LC}$$

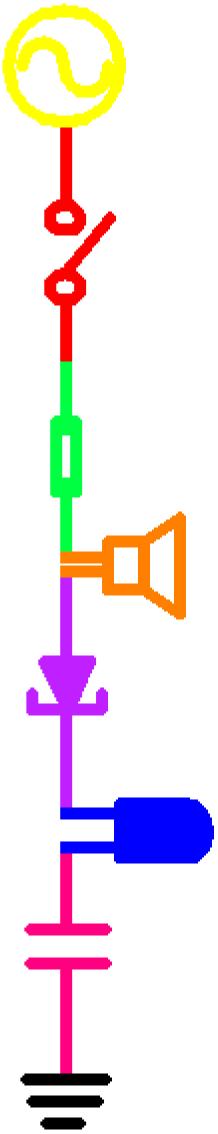


Determine H

$$Z_{CL} = \frac{j\omega L}{1-\omega^2 LC} \qquad H(j\omega) = \frac{Z_{CL}}{R+Z_{CL}}$$

$$H(j\omega) = \frac{\dfrac{j\omega L}{1-\omega^2 LC}}{R+\dfrac{j\omega L}{1-\omega^2 LC}} \quad multiply\, by \frac{1-\omega^2 LC}{1-\omega^2 LC}$$

$$\boxed{H(j\omega) = \frac{j\omega L}{R(1-\omega^2 LC)+j\omega L}}$$



At Very Low Frequencies

$$H_{LOW}(j\omega) = \frac{j\omega L}{R}$$

$$\left| H_{LOW}(j\omega) \right| \omega \to 0 = 0$$

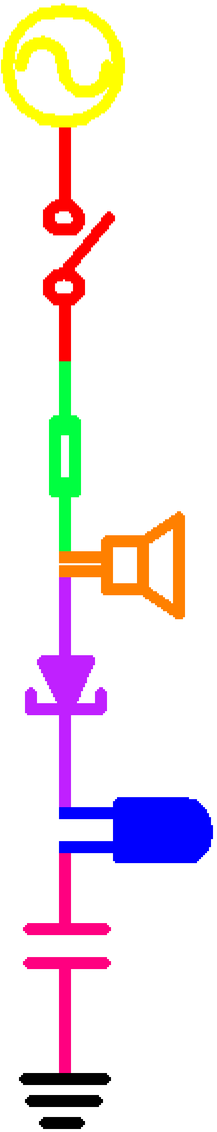
$$\angle H_{LOW}(j\omega) = \frac{\pi}{2}$$

At Very High Frequencies

$$H_{HIGH}(j\omega) = \frac{j\omega L}{-\omega^2 LRC} = \frac{-j}{\omega RC}$$

$$\left| H_{HIGH}(j\omega) \right| \omega \to \infty = \left| \frac{1}{\infty} \right| = 0$$

$$\angle H_{HIGH}(j\omega) = -\frac{\pi}{2}$$



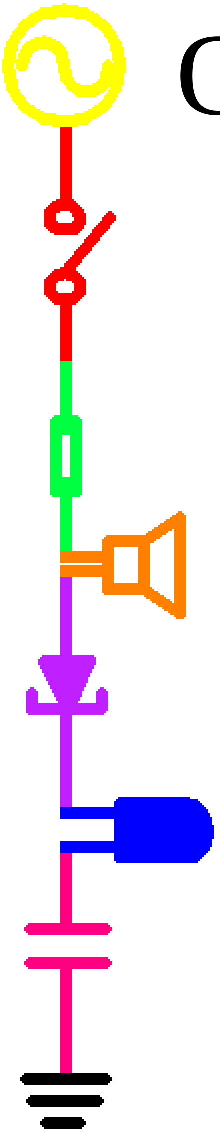
At the Resonant Frequency

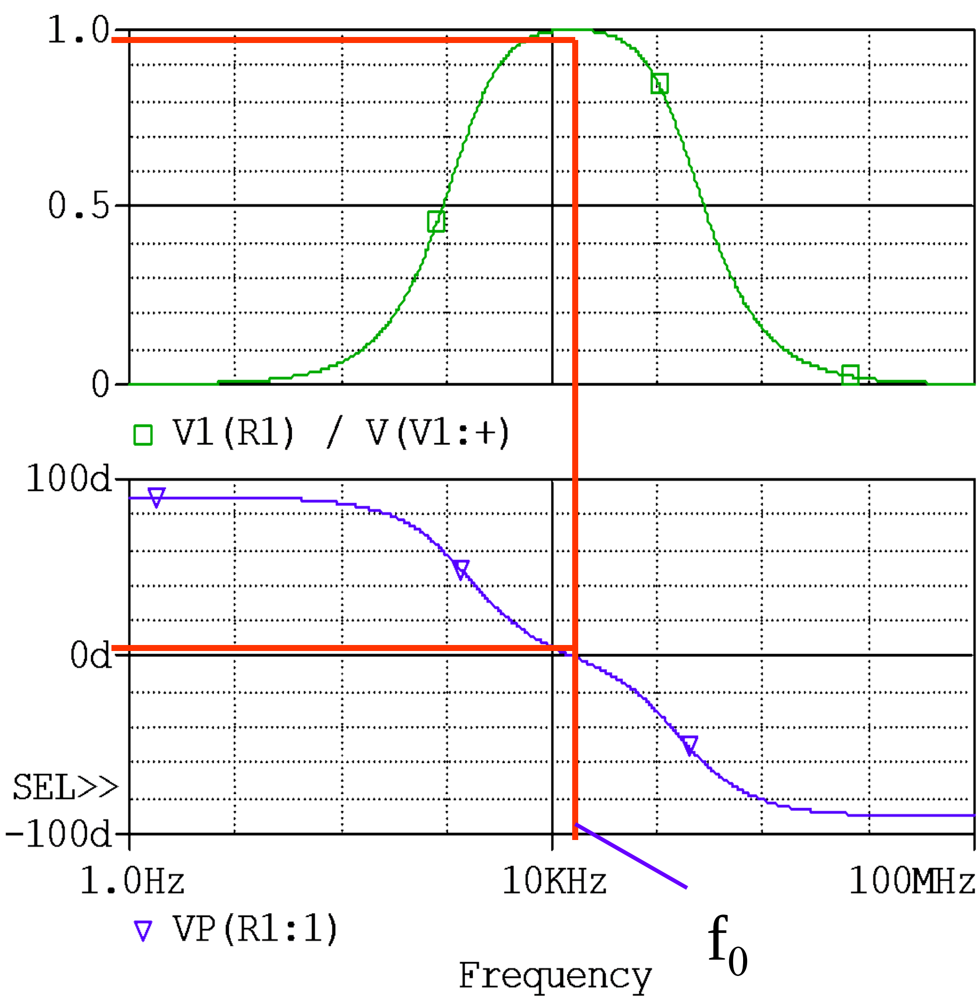
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$H(j\omega_0) = \frac{j\left(\frac{1}{\sqrt{LC}}\right)L}{R\left(1-\left(\frac{1}{\sqrt{LC}}\right)^2 LC\right) + j\left(\frac{1}{\sqrt{LC}}\right)L} = 1$$

$$\left|H(j\omega_0)\right| = 1 \qquad \angle H(j\omega_0) = 0$$

Our example is a band pass filter





Magnitude

$H = 0$ at $\omega \to 0$

$H=1$ at ω_0

$H = 0$ at $\omega \to \infty$

Phase

$\phi = 90$ at $\omega \to 0$

$\phi = 0$ at ω_0

$\phi = -90$ at $\omega \to \infty$