ENGR-2300

## Electronic Instrumentation

Quiz 2
Spring 2016


On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification. Read the entire quiz before answering any questions. Also it may be easier to answer parts of questions out of order.

## A Few Images and Thoughts in Honor of April Fools Day



## From The Engineering Commons Podcast:

- Caltech students pulled off the Great Rose Bowl Hoax during the 1961 Rose Bowl football game, causing the University of Washington's card section to display messages that were altered from their intended configuration. In the final display, captured by network television, the card section spelled out "Caltech," leaving little doubt as to who had pulled off the prank.
- In a similar prank, carried out at the 1984 Rose Bowl game, the Rose Bowl scoreboard was hacked to display the message, "Caltech 38, MIT 9." This stunt gained one of the perpetrators credit in the course, "Experimental Projects in Electrical Circuits."
From XKCD:



Absolute Maximum Ratings (Note 2)
If Military/Aerospace specified devices are required, please contact the National Semiconductor Sales Office/ Distributors for availability and specifications.
(Note 7)

|  | LM741A | LM741 | LM741C |
| :--- | :---: | :---: | :---: |
| Supply Voltage | $\pm 22 \mathrm{~V}$ | $\pm 22 \mathrm{~V}$ | $\pm 18 \mathrm{~V}$ |
| Power Dissipation (Note 3) | 500 mW | 500 mW | 500 mW |
| Differential Input Voltage | $\pm 30 \mathrm{~V}$ | $\pm 30 \mathrm{~V}$ | $\pm 30 \mathrm{~V}$ |
| Input Voltage (Note 4) | $\pm 15 \mathrm{~V}$ | $\pm 15 \mathrm{~V}$ | $\pm 15 \mathrm{~V}$ |
| Output Short Circuit Duration | Continuous | Continuous | Continuous |
| Operating Temperature Range | $-55^{\circ} \mathrm{C}$ to $+125^{\circ} \mathrm{C}$ | $-55^{\circ} \mathrm{C}$ to $+125^{\circ} \mathrm{C}$ | $0^{\circ} \mathrm{C}$ to $+70^{\circ} \mathrm{C}$ |
| Storage Temperature Range | $-65^{\circ} \mathrm{C}$ to $+150^{\circ} \mathrm{C}$ | $-65^{\circ} \mathrm{C}$ to $+150^{\circ} \mathrm{C}$ | $-65^{\circ} \mathrm{C}$ to $+150^{\circ} \mathrm{C}$ |
| Junction Temperature | $150^{\circ} \mathrm{C}$ | $150^{\circ} \mathrm{C}$ | $100^{\circ} \mathrm{C}$ |
| Soldering Information |  |  |  |
| N-Package (10 seconds) | $260^{\circ} \mathrm{C}$ | $260^{\circ} \mathrm{C}$ | $300^{\circ} \mathrm{C}$ |
| J- or H-Package (10 seconds) | $300^{\circ} \mathrm{C}$ |  | $360^{\circ} \mathrm{C}$ |
| M-Package |  | $215^{\circ} \mathrm{C}$ | $300^{\circ} \mathrm{C}$ |
| Vapor Phase ( 60 seconds) | $215^{\circ} \mathrm{C}$ | $215^{\circ} \mathrm{C}$ | $215^{\circ} \mathrm{C}$ |

See AN-450 "Surface Mounting Methods and Their Effect on Product Reliability" for other methods of soldering surface mount devices.
ESD Tolerance (Note 8) 400V 400V 400V

Electrical Characteristics (Note 5)

| Parameter | Conditions | LM741A |  |  | LM741 |  |  | LM741C |  |  | Units |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Min | Typ | Max | Min | Typ | Max | Min | Typ | Max |  |
| Input Offset Voltage | $\begin{aligned} & \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C} \\ & \mathrm{R}_{\mathrm{S}} \leq 10 \mathrm{k} \Omega \\ & \mathrm{R}_{\mathrm{S}} \leq 50 \Omega \end{aligned}$ |  | 0.8 | 3.0 |  | 1.0 | 5.0 |  | 2.0 | 6.0 | $\begin{aligned} & \mathrm{mV} \\ & \mathrm{mV} \end{aligned}$ |
|  | $\begin{aligned} & \mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }} \\ & \mathrm{R}_{\mathrm{S}} \leq 50 \Omega \\ & \mathrm{R}_{\mathrm{S}} \leq 10 \mathrm{k} \Omega \end{aligned}$ |  |  | 4.0 |  |  | 6.0 |  |  | 7.5 | $\begin{aligned} & \mathrm{mV} \\ & \mathrm{mV} \end{aligned}$ |
| Average Input Offset Voltage Drift |  |  |  | 15 |  |  |  |  |  |  | $\mu \mathrm{V} /{ }^{\circ} \mathrm{C}$ |
| Input Offset Voltage Adjustment Range | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}, \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V}$ | $\pm 10$ |  |  |  | $\pm 15$ |  |  | $\pm 15$ |  | mV |
| Input Offset Current | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ |  | 3.0 | 30 |  | 20 | 200 |  | 20 | 200 | nA |
|  | $\mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }}$ |  |  | 70 |  | 85 | 500 |  |  | 300 | nA |
| Average Input Offset Current Drift |  |  |  | 0.5 |  |  |  |  |  |  | $n A /{ }^{\circ} \mathrm{C}$ |
| Input Bias Current | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ |  | 30 | 80 |  | 80 | 500 |  | 80 | 500 | nA |
|  | $\mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }}$ |  |  | 0.210 |  |  | 1.5 |  |  | 0.8 | $\mu \mathrm{A}$ |
| Input Resistance | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}, \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V}$ | 1.0 | 6.0 |  | 0.3 | 2.0 |  | 0.3 | 2.0 |  | $\mathrm{M} \Omega$ |
|  | $\begin{aligned} & \mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }}, \\ & \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \end{aligned}$ | 0.5 |  |  |  |  |  |  |  |  | $\mathrm{M} \Omega$ |
| Input Voltage Range | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ |  |  |  |  |  |  | $\pm 12$ | $\pm 13$ |  | V |
|  | $\mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }}$ |  |  |  | $\pm 12$ | $\pm 13$ |  |  |  |  | V |

Electrical Characteristics (Note 5) (Continued)

| Parameter | Conditions | LM741A |  |  | LM741 |  |  | LM741C |  |  | Units |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Min | Typ | Max | Min | Typ | Max | Min | Typ | Max |  |
| Large Signal Voltage Gain | $\begin{aligned} & \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}, \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega \\ & \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 15 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 15 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 10 \mathrm{~V} \end{aligned}$ | 50 |  |  | 50 | 200 |  | 20 | 200 |  | $\begin{aligned} & \mathrm{V} / \mathrm{mV} \\ & \mathrm{~V} / \mathrm{mV} \end{aligned}$ |
|  | $\mathrm{T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }}$, $\begin{aligned} & \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega, \\ & \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 15 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 15 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 10 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 5 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 2 \mathrm{~V} \end{aligned}$ | $\begin{aligned} & 32 \\ & 10 \end{aligned}$ |  |  | 25 |  |  | 15 |  |  | $\mathrm{V} / \mathrm{mV}$ <br> $\mathrm{V} / \mathrm{mV}$ <br> V/mV |
| Output Voltage Swing | $\begin{aligned} & \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \\ & \mathrm{R}_{\mathrm{L}} \geq 10 \mathrm{k} \Omega \\ & \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega \end{aligned}$ | $\begin{aligned} & \pm 16 \\ & \pm 15 \end{aligned}$ |  |  |  |  |  |  |  |  | $\begin{aligned} & \mathrm{V} \\ & \mathrm{~V} \end{aligned}$ |
|  | $\begin{aligned} & \mathrm{V}_{\mathrm{S}}= \pm 15 \mathrm{~V} \\ & \mathrm{R}_{\mathrm{L}} \geq 10 \mathrm{k} \Omega \\ & \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega \end{aligned}$ |  |  |  | $\begin{aligned} & \pm 12 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 14 \\ & \pm 13 \end{aligned}$ |  | $\begin{aligned} & \pm 12 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 14 \\ & \pm 13 \end{aligned}$ |  | $\begin{aligned} & \mathrm{V} \\ & \mathrm{~V} \end{aligned}$ |
| Output Short Circuit Current | $\begin{aligned} & \hline \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C} \\ & \mathrm{~T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }} \\ & \hline \end{aligned}$ | $\begin{aligned} & 10 \\ & 10 \\ & \hline \end{aligned}$ | 25 | $\begin{aligned} & 35 \\ & 40 \end{aligned}$ |  | 25 |  |  | 25 |  | $\begin{aligned} & \mathrm{mA} \\ & \mathrm{~mA} \end{aligned}$ |
| Common-Mode Rejection Ratio | $\begin{aligned} & \mathrm{T}_{\mathrm{AMIN}} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\mathrm{AMAX}} \\ & \mathrm{R}_{\mathrm{S}} \leq 10 \mathrm{k} \Omega, \mathrm{~V}_{\mathrm{CM}}= \pm 12 \mathrm{~V} \\ & \mathrm{R}_{\mathrm{S}} \leq 50 \Omega, \mathrm{~V}_{\mathrm{CM}}= \pm 12 \mathrm{~V} \\ & \hline \end{aligned}$ | 80 | 95 |  | 70 | 90 |  | 70 | 90 |  | $\begin{aligned} & \mathrm{dB} \\ & \mathrm{~dB} \end{aligned}$ |
| Supply Voltage Rejection Ratio | $\begin{aligned} & \mathrm{T}_{\mathrm{AMIN}} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\mathrm{AMAX}} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \text { to } \mathrm{V}_{\mathrm{S}}= \pm 5 \mathrm{~V} \\ & \mathrm{R}_{\mathrm{S}} \leq 50 \Omega \\ & \mathrm{R}_{\mathrm{S}} \leq 10 \mathrm{k} \Omega \end{aligned}$ | 86 | 96 |  | 77 | 96 |  | 77 | 96 |  | $\begin{aligned} & \mathrm{dB} \\ & \mathrm{~dB} \end{aligned}$ |
| Transient Response <br> Rise Time <br> Overshoot | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$, Unity Gain |  | $\begin{gathered} 0.25 \\ 6.0 \end{gathered}$ | $\begin{gathered} 0.8 \\ 20 \end{gathered}$ |  | $\begin{gathered} 0.3 \\ 5 \\ \hline \end{gathered}$ |  |  | $\begin{gathered} 0.3 \\ 5 \end{gathered}$ |  | $\begin{aligned} & \mu \mathrm{s} \\ & \% \end{aligned}$ |
| Bandwidth (Note 6) | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ | 0.437 | 1.5 |  |  |  |  |  |  |  | MHz |
| Slew Rate | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$, Unity Gain | 0.3 | 0.7 |  |  | 0.5 |  |  | 0.5 |  | $\mathrm{V} / \mathrm{\mu s}$ |
| Supply Current | $\mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}$ |  |  |  |  | 1.7 | 2.8 |  | 1.7 | 2.8 | mA |
| Power ConsumptionLM741A | $\begin{aligned} & \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 15 \mathrm{~V} \end{aligned}$ |  | 80 | 150 |  | 50 | 85 |  | 50 | 85 | $\begin{aligned} & \mathrm{mW} \\ & \mathrm{~mW} \end{aligned}$ |
|  | $\begin{aligned} & \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \\ & \mathrm{~T}_{\mathrm{A}}=\mathrm{T}_{\text {AMIN }} \\ & \mathrm{T}_{\mathrm{A}}=\mathrm{T}_{\text {AMAX }} \end{aligned}$ |  |  | $\begin{aligned} & 165 \\ & 135 \\ & \hline \end{aligned}$ |  |  |  |  |  |  | $\begin{aligned} & \mathrm{mW} \\ & \mathrm{~mW} \end{aligned}$ |
| LM741 | $\begin{aligned} \mathrm{V}_{\mathrm{S}} & = \pm 15 \mathrm{~V} \\ \mathrm{~T}_{\mathrm{A}} & =\mathrm{T}_{\text {AMIN }} \\ \mathrm{T}_{\mathrm{A}} & =\mathrm{T}_{\text {AMAX }} \end{aligned}$ |  |  |  |  | $\begin{aligned} & 60 \\ & 45 \end{aligned}$ | $\begin{gathered} 100 \\ 75 \end{gathered}$ |  |  |  | $\begin{aligned} & \mathrm{mW} \\ & \mathrm{~mW} \end{aligned}$ |

Note 2: "Absolute Maximum Ratings" indicate limits beyond which damage to the device may occur. Operating Ratings indicate conditions for which the device is functional, but do not guarantee specific performance limits.

## 1. Thevenin Equivalent Voltage Source



The Thevenin Equivalent Circuit consists of a voltage source in series with a resistor, which provides a very simple replacement for much more complex circuits. If we have this simple source, analyzing changing loads becomes quite easy.

In this problem, you are to find the Thevenin voltage and resistance for a series of related circuits. While the circuits and their analysis are similar, treat each circuit as a separate problem.

Circuit 1: $\{4 \mathrm{pts}\}$ Find and sketch the Thevenin Equivalent Circuit for the following circuit.


Remove the load, then apply the voltage divider to obtain the open circuit voltage:
$V_{T H}=V_{o c}=\frac{1}{1+2} 150=50 \mathrm{~V}$
Short out the voltage source and then find the parallel combination of R20 and R21
$R_{T H}=\frac{(1 k)(2 k)}{1 k+2 k}=667 \Omega$

Circuit 2: $\{4 \mathrm{pts}\}$ Find and sketch the Thevenin Equivalent Circuit for the following circuit. Note that it is a modification of Circuit 1.


Remove the load, then apply the voltage divider to obtain the open circuit voltage. First find the parallel combination of R13 and R14 as the bottom resistor in the divider. This is also $667 \Omega$.
Then the voltage divider expression gives $V_{T H}=V_{o c}=\frac{667}{667+2000} 150=37.5 \mathrm{~V}$ Note that there is no current through R18, so it does not affect the open circuit voltage. It does play a role in the Thevenin resistance. Short out the voltage source, find the parallel combination of R12, R13, R14 (trivially equal to $500 \Omega$ ) and then add R 18 in series to get $\mathrm{R}_{\mathrm{TH}}=1.5 \mathrm{k} \Omega$

Circuit 3: $\{4 \mathrm{pts}\}$ Find and sketch the Thevenin Equivalent Circuit for the following circuit. Note that it is a modification of the previous circuits.


Open circuit for $\mathrm{V}_{\mathrm{TH}}$ : First find voltage at A by combining all resistors from R 2 on. $\mathrm{R} 9+\mathrm{R} 6=4 \mathrm{k}$, then this combo in parallel with R 5 (also 4 k ) which equals 2 k . That combo in parallel with R 4 (also 2 k ) which equals 1 k . That combo in series with R 8 (also 1 k ) which equals 2 k . That combo in parallel with R3 (also 2 k ) which equals 1 k . That combo in parallel with R2 (also 1 k ) which
equals $500 \Omega$. The voltage divider at A is then $V_{A}=\frac{500}{500+2000} 150=30 \mathrm{~V}$. Then, simple dividers give us each voltage $V_{B}=\frac{1000}{1000+1000} 30=15 \mathrm{~V}$ and $V_{T H}=V_{C}=\frac{2000}{2000+2000} 30=7.5 \mathrm{~V}$ To find the resistance, short out the source V1 and combine resistors left to right. $\mathrm{R} 1||\mathrm{R} 2|| \mathrm{R} 3=500 \Omega$. Then add R 8 in series ( $1.5 \mathrm{k} \Omega$ ) and find the parallel combo with $2 \mathrm{k} \| 4 \mathrm{k}$ or $\frac{1}{R}=\frac{1}{1.5 k}+\frac{1}{2 k}+\frac{1}{4 k}=\frac{8}{12 k}+\frac{6}{12 k}+\frac{3}{12 k}=\frac{17}{12 k}$. The combo is $\mathrm{R}=706 \Omega$. Then add 2 k and find the parallel combo with 2 k or $R_{T H}=\frac{2.7 k(2 k)}{4.7 k}=1.15 k$

Calculations: $\{3 \mathrm{pts}\}$ Determine the voltages at the nodes marked A, B and C in Circuit 3 .
Already determined above. $30 \mathrm{~V}, 15 \mathrm{~V}$ and 7.5 V

Application: \{3 pts \} Using each of your three Thevenin Equivalent Circuits, determine the load voltage and power delivered to the load for a load resistance of $2 \mathrm{k} \Omega$.

Load voltage from voltage divider and power from $V^{2} / R$

| Circuit | V $_{\text {TH }}$ | R $_{\text {TH }}$ | R $_{\text {LOAD }}$ | V LOAD | P $_{\text {LOAD }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | $\mathbf{5 0}$ | $\mathbf{6 6 7}$ | 2k | $\mathbf{3 7 . 5}$ | 703mW |
| $\mathbf{2}$ | $\mathbf{3 7 . 5}$ | $\mathbf{1 . 5 k}$ | 2k | $\mathbf{2 1 . 4}$ | $\mathbf{2 2 9 m W}$ |
| $\mathbf{3}$ | $\mathbf{7 . 5}$ | $\mathbf{1 . 1 5 k}$ | 2k | $\mathbf{4 . 7 6}$ | $\mathbf{1 1 . 3 m W}$ |

All results also checked with PSpice.

Concept: $\{2 \mathrm{pts}\}$ For what load resistance will the power delivered to the load be a maximum for Circuit 2?

Maximum power transfer occurs when the load is equal to the internal resistance of the source so $R_{\text {LOAD }}=1.5 \mathrm{k} \Omega$

## 2. Harmonic Oscillators and Math



We encounter the harmonic oscillator in an unlimited number of circumstances in engineering and science. For example, you have likely done some kind of a pendulum experiment in Physics. As we have seen in class, physical oscillators (damped spring-mass systems like the cantilever beam) and RLC circuits behave the same way, although at different frequencies. The circuit shown here is made by connecting a 1 mH inductor in parallel with a $1 \mu \mathrm{~F}$ capacitor. The resistor in the circuit is the resistance of the inductor and is shown as a separate resistor here. That is, all circuit elements used to model the circuit are ideal, with the R and L used to represent the real inductor. The inductor is made by Bourns, located in Riverside, CA. The capacitor is made by Panasonic, probably at their manufacturing plant in Mexico. Capacitors are very low loss so there is no need to use a resistor to model them, under most conditions. The circuit is charged and then allowed to oscillate down, starting at $\mathrm{t}=0 \mathrm{sec}$, as shown. The figure shows the voltage as a function of time at point A (which is the voltage across the capacitor).

The horizontal scale is time ( 1 ms full scale) and the vertical scale is $\mathrm{V}(-10 \mathrm{~V}$ to $+10 \mathrm{~V})$.
a. Find the decay constant $\alpha$ and the angular frequency $\omega$ for this data. $\{6 \mathrm{pts}\}$

Frequency: 5 periods in 1 ms or $5 \mathrm{kHz} . \omega=2 \pi \mathrm{f}=31.4 \mathrm{k}$
Decay constant: Decays from 10V to about 4V in . 6 ms or $10 e^{-\alpha(.0006)}=4$ or $\alpha \approx 1450$
b. Write the mathematical expression for the voltage at A in one of the forms $V(t)=A e^{-\alpha t} \cos \omega t$ or $V(t)=A e^{-\alpha t} \sin \omega t$, depending on which form fits the data better. Use real values for the constants and provide units where appropriate. $\{4 \mathrm{pts}\}$

The cosine form fits better: $V(t)=10 e^{-1450 t} \cos 31400 t$ Volts The units of $\alpha$ are $s^{-1}$ but there are no units in the exponent, so it is not necessary to include them.
c. Using your expression for the voltage at A, determine the current through the capacitor as a function of time. $\{4 \mathrm{pts}\}$

The current through the capacitor is determined from its current-voltage expression:
$I=C \frac{d V}{d t}=-C A \omega \sin \omega t=-(0.000001)(10)(31400) \sin 31400 t=-0.314 \sin 31400 t$
d. Shown below are the voltages across the resistor and the inductor. Neither is labeled. Determine which is which and label them in the figure. Explain your answer. No credit without an explanation. $\left\{6\right.$ pts \} Current tracks the resistor voltage $(\mathrm{V}=\mathrm{IR})$ so $\mathrm{V}_{\mathrm{R}}$ is the one that looks like a sine function. $\mathrm{V}_{\mathrm{L}}$ is the remaining curve. Note that the sign is wrong because the current in the resistor is in the opposite direction from the capacitor.


## 3. Operational Amplifier Applications

a. For the circuit shown at the right:
i. $\{1 \mathrm{pt}\}$ What type of amplifier is this circuit?
Non-Inverting

$\{1 \mathrm{pt}\}$ Write and expression for Vout as a function of Vin.

$$
A_{v}=1+\frac{R_{f}}{R_{g}}=1+3=4 \quad V_{\text {out }}=4 V_{\text {in }}
$$

ii. $\{2 \mathrm{pts}\}$ Given that Vin is as shown in the plot below, draw Vout on the blank plot. You must label both the y axis and also mark the signal amplitude. For both plots the horizontal scale goes from 0 to 4 ms . For the top plot, the vertical scale goes from 200 mV to +200 mV .


b. For the circuit shown at the right:
i. $\{1 \mathrm{pt}\}$ What type of amplifier is this circuit? Inverting
ii. $\{1 \mathrm{pt}\}$ Write and expression for Vout as a function of Vin.

$A_{v}=-\frac{R_{f}}{R_{g}}=-3 \quad V_{\text {out }}=-3 V_{\text {in }}$
\{2pts \} Given that Vin is as shown in the plot below, draw Vout on the blank plot. You must label both the y axis and also mark the signal amplitude. For both plots the horizontal scale goes from 0 to 4 ms . For the top plot, the vertical scale goes from 200 mV to +200 mV .


c. For the circuit shown at the right:
i. $\{1 \mathrm{pt}\}$ What type of amplifier is this circuit?

Integrator or Active Integrator
\{2pt $\}$ Write and expression for Vout as a function of Vin.


$$
V_{\text {out }}=-\frac{1}{R_{\text {in }} C_{f}} \int V_{\text {in }}(t) d t \quad V_{\text {out }}=-4 \times 10^{4} \int V_{\text {in }}(t) d t
$$

\{2pts \} Given that Vin is as shown in the plot below, draw Vout on the blank plot. You must label both the $y$ axis and also mark the signal amplitude.
At $\mathrm{t}=0$, set Vout to 0 V . For both plots the horizontal scale goes from 0 to 4 ms . For the top plot, the vertical scale goes from -200 mV to +200 mV .
$V_{\text {out }}=-4 \times 10^{4} \int_{0}^{.001} 0.1 d t=-4 \times 10^{4}(0.1)(0.001)=-4$ so while the voltage is high $(0.1 \mathrm{~V})$ the integration drops 4 V . It goes up 4 V when the voltage is low $(-0.1 \mathrm{~V})$.

d. For the circuit shown at the right:
i. $\{1 \mathrm{pt}\}$ What type of amplifier is this circuit?

Differentiator or Active Differentiator
$\{2 \mathrm{pt}\}$ Write and expression for Vout as a function of Vin.

$V_{\text {out }}=-C_{\text {in }} R_{f} \frac{d V_{\text {in }}(t)}{d t}=-2 \times 10^{-3} \frac{d V_{\text {in }}(t)}{d t}$ or phasor form $V_{\text {out }}=-j 2 \times 10^{-3} \omega V_{\text {in }}$
ii. $\quad\{2 \mathrm{pts}\}$ Given that Vin is a sinewave with amplitude $=0.5 \mathrm{~V}$ and frequency $=500 \mathrm{~Hz}$, draw $\operatorname{Vin}(\mathrm{t})$ on the plot below, draw Vout on the blank plot. You must label both the y axis and also mark the signal amplitude. Same time scale as previous plots.

iii. \{2pts\} Draw Vout on the plot below. You must label both the y axis and also mark the signal amplitude. Same time scale as previous plots.

$$
V_{\text {out }}=-2 \times 10^{-3} \omega(0.5) \cos \omega t=-3.14 \cos (1000 \pi t)
$$


$\square \mathrm{V}(\mathrm{U} 4: O U T)$
Time

## 4. Operational Amplifier and Circuit Fundamentals

a. $\{6 \mathrm{pts}\}$ Power draw, related to loading, varies with the op-amp configuration. This can be demonstrated by setting a signal source to a constant voltage and determining where the power is lost (converted to heat.) Analyze the 2 circuits shown and complete the table.



No current from $\mathrm{V}_{\text {sig }}$ on left so no power
from $\mathrm{V}_{\text {sig. }}$ On right $P_{\text {sig }}=\frac{1^{2}}{1000}=1 \mathrm{~mW}$
Power into Ra for both circuits
$P_{R a}=\frac{1^{2}}{1000}=1 m W$ Voltage at neg input on left is $1 V P_{R b}=\frac{4^{2}}{4000}=4 m W$ and on the right the voltage at neg input is 0 V so $P_{R b}=\frac{5^{2}}{5000}=5 m W \mathrm{Op} \mathrm{amp}$ has to make up remaining power so it must provide 5 mW for both circuits.
b. The circuit shown has 2 input signals.
i. $\{2 \mathrm{pts}\}$ Write an expression for Vout in terms of V1 and V2.


$$
\begin{aligned}
& V_{\text {out }}=-\frac{R_{f}}{R_{1}} V_{1}-\frac{R_{f}}{R_{2}} V_{2} \\
& =-2.5 V_{1}-5 V_{2}
\end{aligned}
$$

c. $\{4 \mathrm{pts}\} \mathrm{If} \mathrm{V} 1$ is a dc voltage of $2 \mathrm{~V}, \mathrm{~V} 1=2 \mathrm{Vdc}$, and V 2 is a sinewave, $\mathrm{V} 2=\mathrm{Asin} \omega \mathrm{t}$ what is the largest value of A for which the op-amp can follow your equation in part i. above? State any assumptions you made to calculate this number.
Assume output can go to plus or minus 9 V (will be less in most real op-amps).
Input V1 has a gain of 2.5 so the output will be -5 V . That leaves 4 V for V 2 . With a gain of 5 , A must be $<0.8 \mathrm{~V}$. If assume that output can go to a smaller number, V 2 will be smaller. For output limited by 7 V , $\mathrm{A}<0.4 \mathrm{~V}$

The circuit shown is a practical integrator.
i. $\{4 \mathrm{pts}\}$ Assume $\omega$ is large. What is the magnitude and phase of the output if the input has a magnitude of 0.2 V , a phase of 0 degrees, and a frequency of 10 kHz ?

For $\omega$ large, the feedback resistor can be neglected:
$H(j \omega)=-\frac{1}{j \omega R C} \quad \& \quad V_{\text {out }}=-\frac{1}{j \omega R C} V_{\text {in }}$
$V_{\text {out }}=\frac{1}{j\left(2 \pi 10^{4}\right)(200)\left(25 \times 10^{-9}\right)} 0.2$
$=j 0.64$

Magnitude is 0.64 and phase is $90^{\circ}$

ii. $\{2 \mathrm{pts}\}$ For what range of frequency ( in Hz ) is it valid to make the assumption that $\omega$ is large?
$\omega=2 \pi \mathrm{f} \gg 1 / \mathrm{R}_{\mathrm{f}} \mathrm{C}_{\mathrm{f}}=1 /\left(10^{4}\right)\left(25 \times 10^{-9}\right)$ or $\mathrm{f} \gg 640 \mathrm{~Hz}$. Note that this calculation finds the condition when the feedback capacitor impedance is much smaller than the feedback resistor.
\{2pts \} Give an expression for Vout of this circuit in terms of Vin if the frequency is very very low.

When the frequency is very low, we have the opposite condition from the one above. The feedback resistor is smaller than the capacitor impedance. Then we neglect the capacitor and we have a simple inverting amplifier with $V_{\text {out }}=-\frac{10^{4}}{200} V_{\text {in }}=-50 V_{\text {in }}$

## 5. Concepts, Troubleshooting and Data Analysis

a. Draw lines to connect the batteries to the $741 \mathrm{op}-\mathrm{amp}$ package shown below $\{2 \mathrm{pts}\}$ :

b. The following is from the 741 data sheet:

| Parameter | Conditions | LM741A |  |  | LM741 |  |  | LM741C |  |  | Units |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Min | Typ | Max | Min | Typ | Max | Min | Typ | Max |  |
| Large Signal Voltage Gain | $\begin{aligned} & \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C}, \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega \\ & \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 15 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 15 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 10 \mathrm{~V} \end{aligned}$ | 50 |  |  | 50 | 200 |  | 20 | 200 |  | $\begin{aligned} & \mathrm{V} / \mathrm{mV} \\ & \mathrm{~V} / \mathrm{mV} \end{aligned}$ |
|  | $\begin{aligned} & \mathrm{T}_{\mathrm{AMIN}} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\mathrm{AMAX}}, \\ & \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega, \\ & \mathrm{~V}_{\mathrm{S}}= \pm 20 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 15 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 15 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 10 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{S}}= \pm 5 \mathrm{~V}, \mathrm{~V}_{\mathrm{O}}= \pm 2 \mathrm{~V} \end{aligned}$ | $\begin{aligned} & 32 \\ & 10 \end{aligned}$ |  |  | 25 |  |  | 15 |  |  | V/mV <br> $\mathrm{V} / \mathrm{mV}$ <br> $\mathrm{V} / \mathrm{mV}$ |
| Output Voltage Swing | $\begin{aligned} & \mathrm{V}_{\mathrm{S}}= \pm 20 \mathrm{~V} \\ & \mathrm{R}_{\mathrm{L}} \geq 10 \mathrm{k} \Omega \end{aligned}$ | $\begin{aligned} & \pm 16 \\ & \pm 15 \end{aligned}$ |  |  |  |  |  |  |  |  | $\begin{aligned} & \mathrm{V} \\ & \mathrm{~V} \end{aligned}$ |
|  | $\begin{aligned} & \mathrm{R}_{\mathrm{L}} \geq 10 \mathrm{k} \Omega \\ & \mathrm{R}_{\mathrm{L}} \geq 2 \mathrm{k} \Omega \end{aligned}$ |  |  |  | $\begin{aligned} & \pm 12 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 14 \\ & \pm 13 \end{aligned}$ |  | $\begin{aligned} & \pm 12 \\ & \pm 10 \end{aligned}$ | $\begin{aligned} & \pm 14 \\ & \pm 13 \end{aligned}$ |  | $\begin{aligned} & \mathrm{V} \\ & \mathrm{~V} \end{aligned}$ |
|  | $\begin{aligned} & \mathrm{T}_{\mathrm{A}}=25^{\circ} \mathrm{C} \\ & \mathrm{~T}_{\text {AMIN }} \leq \mathrm{T}_{\mathrm{A}} \leq \mathrm{T}_{\text {AMAX }} \end{aligned}$ | $\begin{aligned} & 10 \\ & 10 \end{aligned}$ | 25 | $\begin{aligned} & 35 \\ & 40 \end{aligned}$ |  | $25$ |  |  | 25 |  | $\begin{aligned} & \mathrm{mA} \\ & \mathrm{~mA} \end{aligned}$ |

Using this data sheet answer the following $\{2 \mathrm{pts}\}$ :
i. If a $100 \Omega$ load is put on the output of a LM741 op-amp, what is the maximum output voltage that one would typically expect to be able to achieve? Current limit of 25 mA . $\mathrm{V}=\mathrm{IR}=0.025(100)=2.5 \mathrm{~V}$ For a current limit of 40 mA , the max voltage is 4 V .

Remember - This was part of an Experiment
ii. If LM741 op-amp is power with a +15 V supply and a -15 V supply, ( $\mathrm{V}_{\mathrm{S}}$ in data sheet), what is the maximum output voltage you would typically expect the op-amp to be able to achieve if the load resistance is $2 \mathrm{k} \Omega$ ?
Max output voltage is 13 V (from spec sheet, as long as the load is at least $2 \mathrm{k} \Omega$ )
c. Which of the following op-amp configurations works best to amplify the signal from a strain gauge bridge circuit? Circle one. $\{2 \mathrm{pts}\}$

| Voltage Follower | Inverting | Non-Inverting |
| :--- | :--- | :--- |
| Adder | Integrator | Differentiator |

This is a good example of looking elsewhere in the quiz ... see the strain gauge circuit on the next page!
d. Which of the following op-amp configurations works best to connect to the output of an accelerometer if it is desired to find the velocity of accelerometer? Circle one. $\{2 \mathrm{pts}\}$
Voltage Follower Inverting
e. Circle the correct answer for the following:

The calibration constant for the ADXL150 accelerometer is given on page 3 of the Project 2 write-up. It is also in the Class 14 lecture slides. It can also be found on the data sheet which is available on the course website. (hint: these are either all true or all false) $\{2 \mathrm{pts}\}$

## True

False
f. The voltage follower op-amp configuration has a gain of 1 . Why bother to use this configuration if your signal voltage is already the desired value? In other words, why not just connect the signal to the load? (20 word limit) $\{2 \mathrm{pts}\}$

Use voltage follower if the load would cause the input voltage to drop if directly coupled. For example, in a voltage divider, if the load resistor is similar in size to the lower divider resistor, the divider will no longer work as designed.
g. In Experiment 5, the strain gauge circuit is configured as shown below:


Figure B-4.

If the +5 V and -5 V power supplies of the Analog Discovery Board were used instead of the two 9 V batteries, would the sensitivity of the system increase, remain the same or decrease? Explain your answer. (20 word max.) (Sensitivity in this case is the amount Vout would change for a given change in the deflection of the beam.) $\{2 \mathrm{pts}\}$

Decrease. From the formula (crib) sheet, the voltage from the strain gauge is proportional to the voltage source that powers it.
h. Using the general harmonic oscillator equation below, determine its natural frequency $\omega$. \{2 pts $\}$
$a \frac{d^{2} H}{d t^{2}}+b H=0$
Assume that the solution is written as $H=A \cos \omega t$.
Then $\ddot{H}=-\omega^{2} A \cos \omega t=-\omega^{2} H$. Plug this into the equation and solve for $\omega=\sqrt{\frac{b}{a}}$ This is something you should have seen in Differential Equations and Dynamics.
i. A very interesting and useful device we will be studying in Experiment 7 is the 555 timer chip. Timer chips make it easy to create sequences of pulses at almost any frequency. Shown below is a typical output for a pulsing circuit made with the 555 timer. The load in this case is $\mathrm{R}_{\mathrm{L}}=1 \mathrm{k} \Omega$. Note that the voltage pulses between $\mathrm{V}_{\text {LOw }}=0 \mathrm{~V}$ and $\mathrm{V}_{\text {HIGH }} \approx 7 \mathrm{~V}$. A series of measurements are made of the high output voltage for load resistors varying from $10 \Omega$ to $1 \mathrm{M} \Omega$. Use this data to determine the Thevenin Equivalent Voltage (for the high voltage) and Resistance for this circuit. As usual, be sure to show all work. Note that, since you are only addressing the high voltage and not the actual time variation of the output signal, you can solve the problem as if it was a DC source. $\{4 \mathrm{pts}\}$


| $\mathrm{R}_{\mathrm{L}}(\Omega)$ | $\mathrm{V}_{\text {HIGH }}(\mathrm{V})$ | $\mathrm{V}_{\text {HIGH }}$ Calc |
| ---: | ---: | ---: |
| 1000 | 6.9215 | 6.92307 |
| 1000000 | 8.995 | 8.99730 |
| 100000 | 8.97 | 8.97308 |
| 10000 | 8.735 | 8.73786 |
| 100 | 2.25 | 2.25 |
| 10 | 0.29 | 0.29032 |
| 300 | 4.5 | 4.5 |
| 3000 | 8.18 | 8.18181 |
| 30000 | 8.9 | 8.91089 |

Peak voltage is about 9 V at $1 \mathrm{M} \Omega$ so that is the source voltage. Half of that voltage is 4.5 , so the resistance is $300 \Omega$. One can check this answer by calculating the load voltage using $V_{L}=9 \frac{R_{L}}{300+R_{L}}$, filling in the additional column \& comparing the results to the measured voltages.

j. A classic puzzle involves an infinite 2D array of resistors (continues out to infinity in both directions), as shown below. All resistors have the same value R. If the resistance is measured at the two points with the large circles, will the resistance measured be $\{2 \mathrm{pts}\}$
i. Greater than R?
ii. Less than R? Since R is in parallel with the other resistors, the combination must be smaller. It is, in fact $R / 2$. This is essentially the same puzzle as in the Nerd Sniping cartoon.
iii. About equal to R ?


Extra Credit: Solve this puzzle exactly \{Up to 4 pts\}
Check the puzzle out online.

