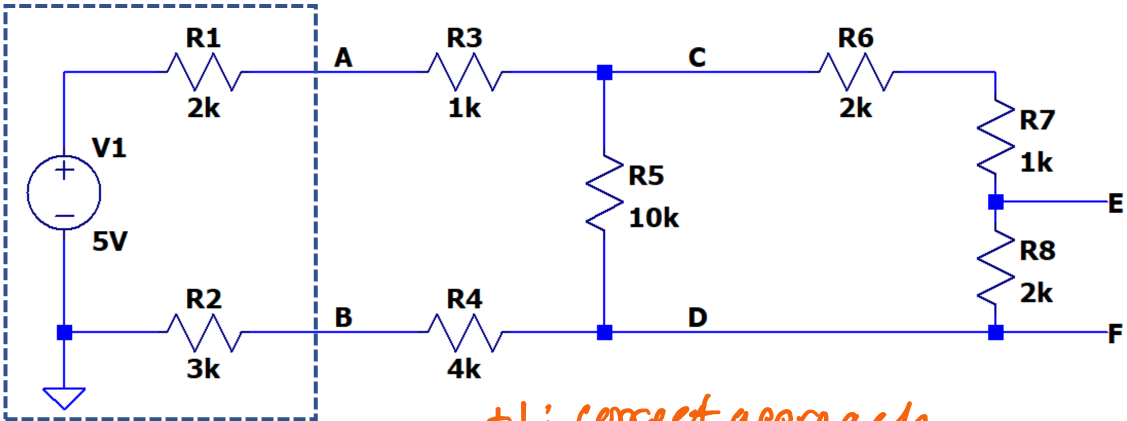
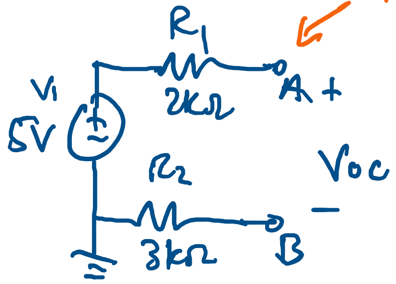
Question 1: Thevenin Equivalent Circuits
[20 Points]

Q1.1) [5 pts] Find the Thevenin Equivalent voltage and resistance between terminals A and B (for the portion of the circuit contained in the box). After finding V_{th} and R_{th}, draw the Thevenin Equivalent circuit.



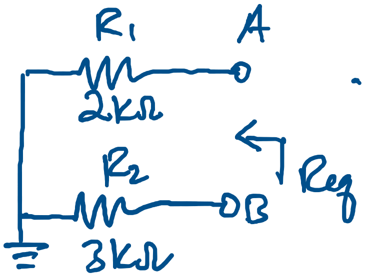
+1: correct approach

V_{th} :



· Since no current flows due to it being an open circuit, $V_{oc} = 5V = V_{th}$ (+1)

R_{th} :

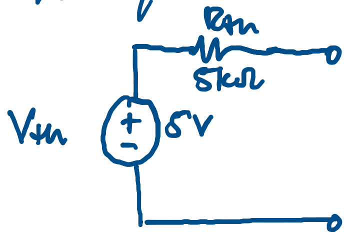


· $R_{eq} = R_1 + R_2 = 5k\Omega = R_{th}$

(+1)

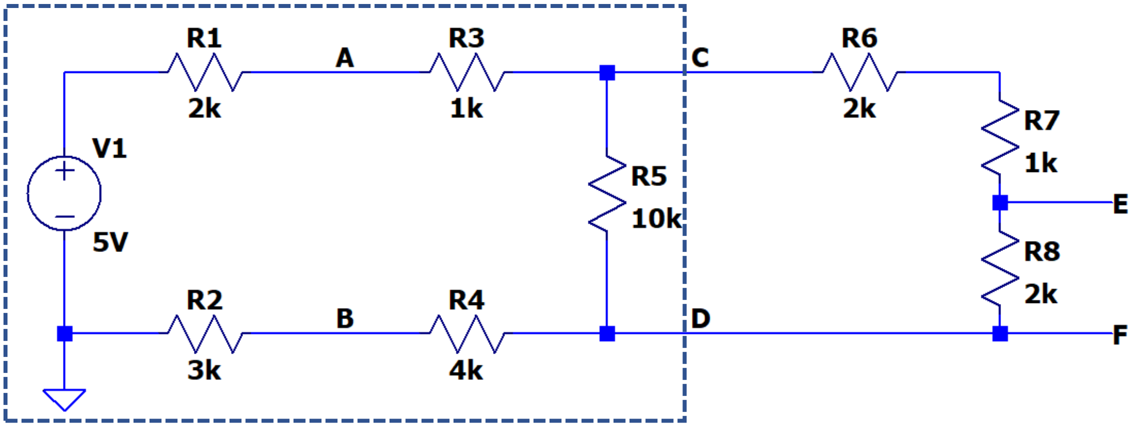
+1: correct approach

Thevenin Equivalent Circuit:

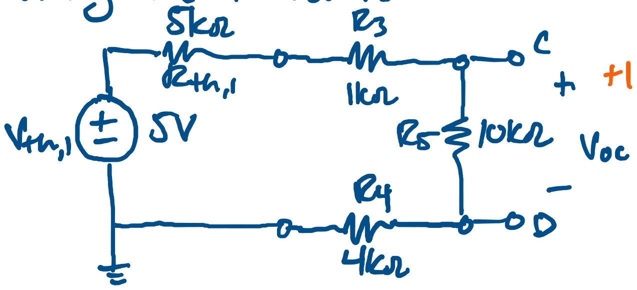


+1: Correct circuit schematic for Thevenin Equivalent

Q1.2) [4 pts] Find the Thevenin Equivalent voltage and resistance between terminals C and D (for the portion of the circuit contained in the box). After finding V_{th} and R_{th}, draw the Thevenin Equivalent circuit.



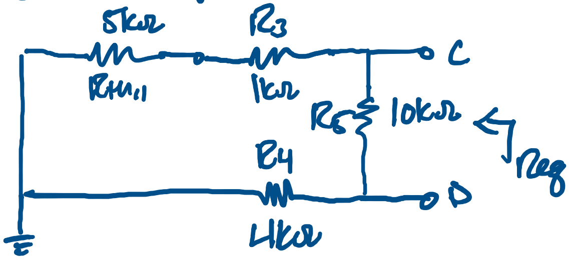
Using the Thevenin from above:



+1: correct approach for both V and Rth

V_{th} : V_{oc} : voltage divider: $V_{oc} = 5V \frac{10k\Omega}{5k\Omega + 1k\Omega + 4k\Omega + 10k\Omega} = 2.5V$ (+1)

R_{th} :

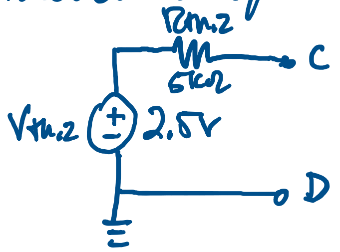


$R_{eq} = (R_{th,1} + R + R_4) \| R_5$

$= 10k\Omega \| 10k\Omega = \underline{5k\Omega} = R_{th,2}$

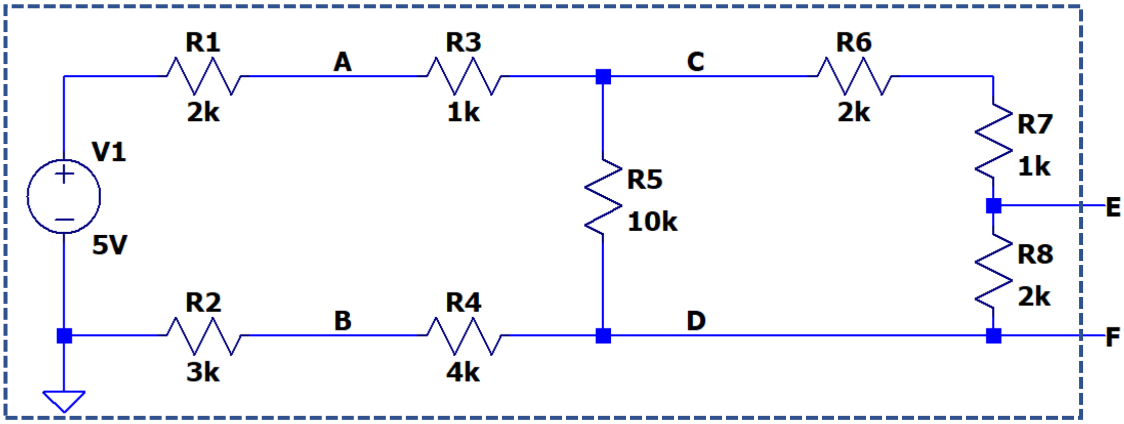
(+1)

Thevenin Equivalent Circuit:

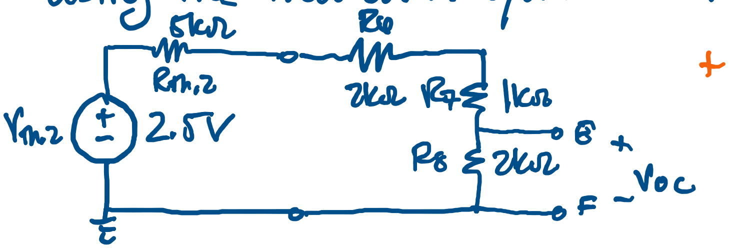


+1: circuit drawn correctly

Q1.3) [4 pts] Find the Thevenin Equivalent voltage and resistance between terminals E and F (for the portion of the circuit contained in the box). After finding V_{th} and R_{th}, draw the Thevenin Equivalent circuit.



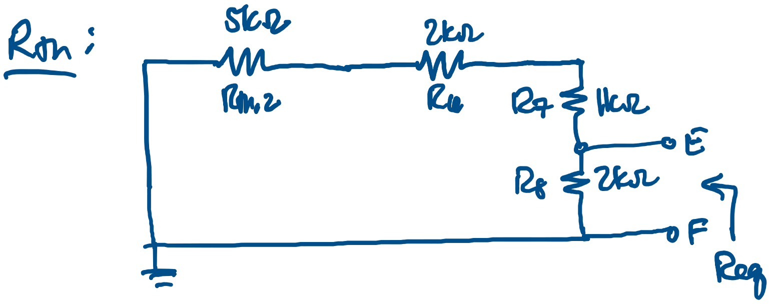
Using the Thevenin Equivalent from above:



+1: correct approach for finding V_{th} and R_{th}

V_{th} : Voltage divider : $V_{oc} = 2.5V \frac{2k\Omega}{5k\Omega + 2k\Omega + 1k\Omega + 2k\Omega} = 0.5V$ (+1)

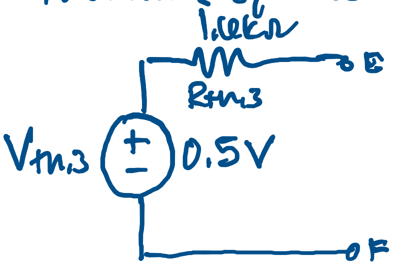
R_{th} :



$R_{th} = (R_{th,2} + R_6 + R_7) \| R_8$ (+1)

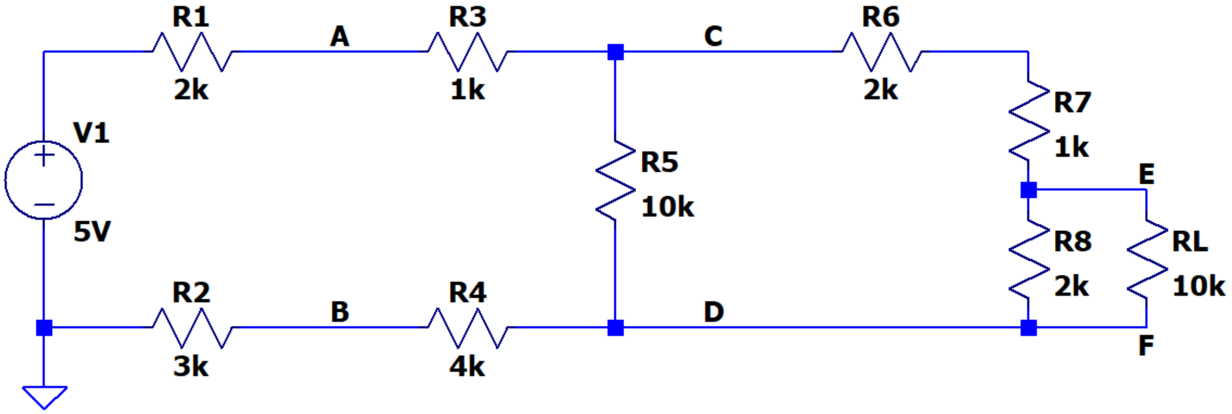
$= 8k\Omega \| 2k\Omega = \frac{16k\Omega}{10k\Omega} = \underline{1.6k\Omega}$

Thevenin Equivalent:



+1 : correctly-drawn thevenin equivalent

Q1.4) [2 pts] If a load resistor with a value RL = 10kΩ is connected across terminals E and F as shown below, how much power is delivered to the resistor?



Using the Thevenin Equivalent from above:

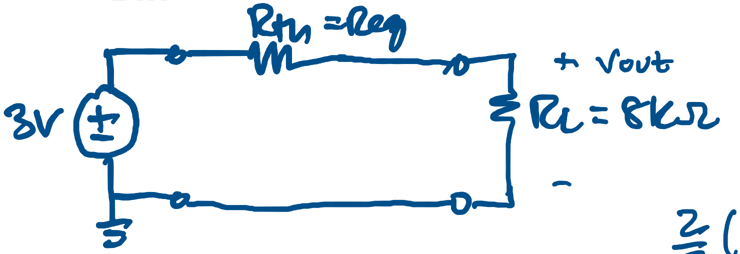
$P = I^2 R$ +1: correct approach

$$I = \frac{0.5V}{11.6k\Omega} = 43.1\mu A$$

$$P = (43.1\times10^{-6}A)^2 \cdot 10\times10^3\Omega = \underline{18.6\mu W}$$

+1: correct calculation

Q1.5) [3 pts] Given an unknown resistive network, you connect a load resistor RL = 8kΩ across the output terminals and apply a voltage Vin = 3V across the input terminals. If you measure a voltage Vout = 2V, what is the equivalent resistance R_{eq} of the resistive network?



+1: correct approach

$$2V = 3V \frac{8k\Omega}{R_{th} + 8k\Omega}$$

$$\frac{2}{3}(R_{th} + 8k\Omega) = 8k\Omega$$

$$R_{th} = 8k\Omega(3/2 - 1) = \underline{4k\Omega}$$

+1: correct calculation

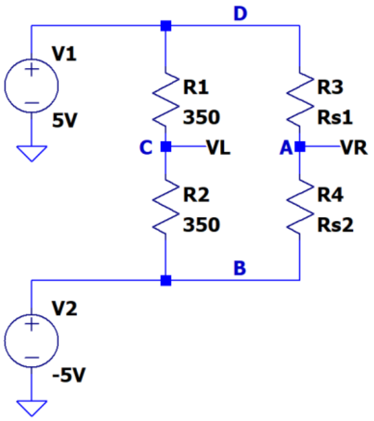
Q1.6) [2 pts] True or False (circle one): only some linear circuits can be expressed as a voltage source in series with an impedance.

True [✓ **False**]

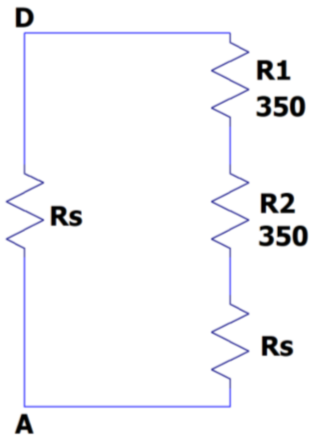
Thevenin's Theorem states that ALL linear circuits can be simplified to a voltage source in series with an equivalent resistance.

Question 2: Bridge Circuits, Strain Gauges, and Harmonic Oscillation [20 Points]

In the circuit below, Rs1 and Rs2 represent the resistances of strain gauges which are attached to the top and bottom of a cantilever beam, as in Experiment 5 and Project 2.



Q2.1) [4 pts] When you measured the resistance of the strain gauges in Experiment 5 (for example between terminals D and A), you were measuring Rs1 in parallel with the series combination of R1, R2, and Rs2, as shown below. If both strain gauges are assumed to have the same resistance Rs, what is the value of Rs if you measured a resistance of 263Ω between terminals A and D? Give your answer to the nearest ohm.



+2: correct approach

$$263\Omega = (350\Omega + 350\Omega + Rs) \parallel Rs$$

$$= \frac{(700\Omega + Rs)Rs}{2Rs + 700\Omega}$$

$$263\Omega(2Rs + 700\Omega) = (700\Omega + Rs)Rs$$

$$526\Omega \cdot Rs + 18410\ \Omega^2 = 700\Omega Rs + Rs^2$$

+1: qu tc equation

$$0 = Rs^2 + 174\Omega Rs - 184100\Omega^2$$

$$\underline{Rs \cong 350\Omega}$$

+1: correct calculation

Q2.2) [3 pts] When the beam is deflected to its maximum extent, Vout = VL – VR = 10mV. Assuming that the strain gauge resistances become Rs1 = Rs + ΔR and Rs2 = Rs – ΔR, what is the value of ΔR? Give your answer to the nearest 0.1Ω.

$V_L = 0$ because $R_1 = R_2 = 350\Omega$ (+1)

$$V_R = -5V + 10V \cdot \frac{R_s - \Delta R}{(R_s + \Delta R) + (R_s - \Delta R)} = -5V + 10\,\frac{R_s - \Delta R}{2R_s}$$

$$= -5V + 10V\left(\frac{1}{2} - \frac{\Delta R}{2R_s}\right) = -5V \cdot \frac{\Delta R}{350} = -\frac{\Delta R}{70}V$$ (+1)

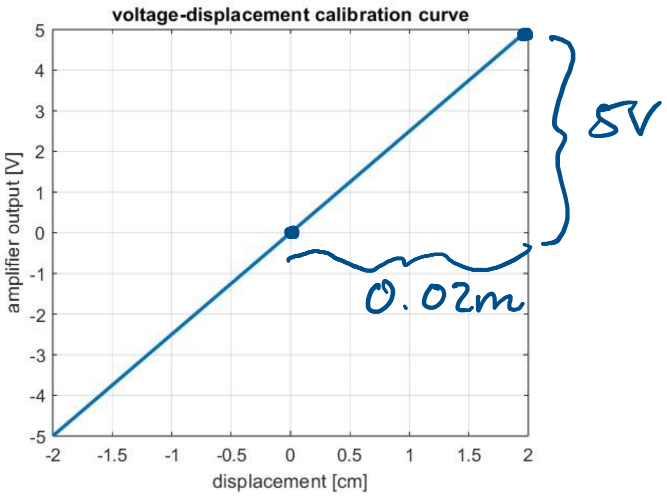
$$V_{out} = V_L - V_R = +\frac{\Delta R}{70}V = 0.01V \rightarrow \underline{\Delta R = 0.7\Omega}$$ (+1)

Q2.3) [2 pts] The output voltage of the strain gauge and bridge circuit is fed into a differential amplifier. Given that Vout = 10mV is the maximum output voltage of the bridge circuit and the op-amp voltage supplies are +5V and -5V, what is the maximum gain of the differential amplifier, such that its output does not saturate?

If $V_{o\,max} = \overset{(+1)}{+5V}$ and $V_{in,max} = 10mV$,

the maximum gain is $\underset{(+1)}{A} = \frac{+5V}{0.01V} = \underline{500}$

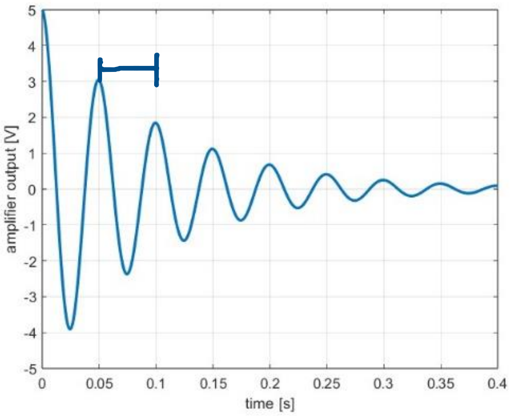
Q2.4) [2 pts] Using the calibration curve below, calculate the proportionality constant that converts the voltage output of the differential amplifier in V to beam displacement in m to the nearest 0.01.



To convert voltage to displacement, we need to find $\frac{\Delta X}{\Delta V} \approx \frac{0.02m}{5V} = \underline{0.004 m/V}$

(+1) (+1)

Q2.5) [3 pts] Given the curve below, calculate the frequency (in Hz) of the beam oscillation and the decay constant to the nearest 0.1.



- $T = 0.05s$, so $f = 1/0.05s = \underline{20Hz}$ (+1)
- $V_1 = V_0 e^{-\alpha(t_1 - t_0)}$ choose $V_1 = 3V$, $V_0 = 5V$
  $t_1 = 0.05s$, $t_0 = 0$

$$\alpha = -\ln\underset{(+1)}{\left(\frac{V_1}{V_0}\right)}\frac{1}{(t_1 - t_0)} = -\ln\left(3/5\right)\frac{1}{0.055} = \underset{(+1)}{\underline{10.2}}$$

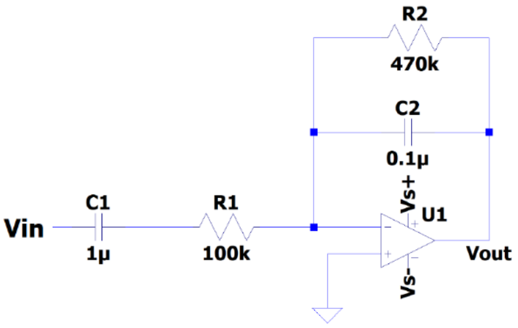
Q2.6) [2 pts] Which type of op-amp circuit would you use to calculate velocity (up to a constant) from the data in Q2.5?

Since we've measured $x(t) \times$ a constant and we want $\frac{dx(t)}{dt}$, we need a differentiator.

Q2.7) [2 pts] If instead you were measuring the voltage signal from an accelerometer, which type of op-amp circuit would you need to calculate velocity (up to a constant)?

Since $V(t) = \int a(t)\,dt$, we would need an integrator.

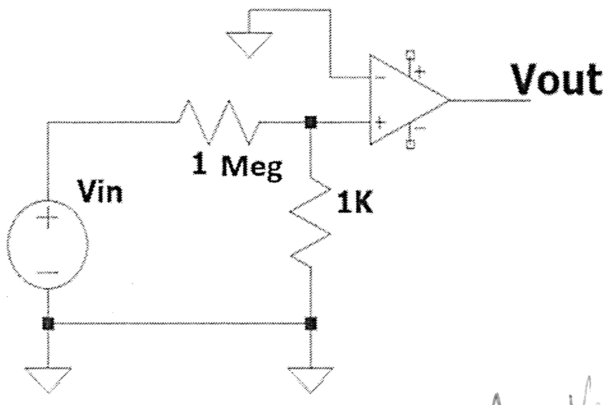
Q2.8) [2 pts] In the Miller Integrator circuit shown below, what is the function of the capacitor C1 at the input?



- The capacitor is a blocking capacitor since it prevents the DC component of a signal from reaching the integrator input. If a DC signal is integrated it may cause the integrator to saturate.

III. Op-Amp Applications and Analysis [26 points]

a) [4 pts] The op-amp circuit below uses an op-amp that is ideal except for having a **finite gain A**. If the measured Vout is 3.5V when Vin is 3.5V, what is the op-amp gain A?
Hint: Vout = A * (V_+ - V_-) when the op-amp is not configured in a feedback loop.



$$V_+ = V_{in} \cdot \frac{1K}{1M + 1K}$$

$$V_- = 0$$

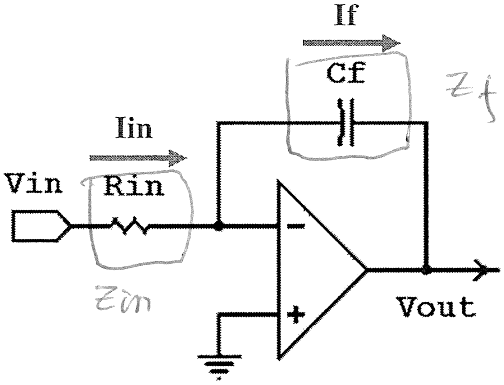
$$V_{out} = A(V_+ - \cancelto{0}{V_-})$$

$$= A \cdot V_{in} \cdot \left(\frac{10^3}{10^6 + 10^3}\right)$$

$$A = \frac{V_{out}}{V_{in}} \cdot \frac{10^6 + 10^3}{10^3} = \frac{3.5V}{3.5V} \cdot (10^3 + 1)$$

$$= \boxed{1001}$$

b) [12 pts] In the ideal integrator as shown below, Rin = 100k ohm and the Cf = 0.1uF. A sine-wave signal is applied to the input Vin.



i) [2 pts] Find Iin and If in terms of Vin, Vout, Rin, and the impedance of Cf. Indicate where you used the Golden Rules of op-amps.

rule 1 : $I_n = I_f$

rule 2 : $V_+ = V_+ = 0$

$$I_{in} = \frac{V_{in} - \cancelto{0}{V_-}}{Z_{in}}$$

$$I_f = \frac{\cancelto{0}{V_-} - V_{out}}{Z_f}$$

Ans: ii) $I_{in} = I_f \Rightarrow \frac{V_{in}}{Z_{in}} = \frac{-V_{out}}{Z_f}$, $\left(\text{now } Z_{in} = R_{in},\ Z_f = \frac{1}{j\omega C_f}\right)$

ii) [2 pts] Find the transfer function of this op-amp circuit: H[jω] = Vout/Vin.

$H(j\omega) = \frac{V_{out}}{V_{in}}$. From $I_{in} = I_f$

$$\frac{V_{in}}{Z_{in}} = \frac{-V_{out}}{Z_f} \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{Z_f}{Z_{in}} = -\frac{\frac{1}{j\omega C_f}}{R_{in}}$$

$$\boxed{= -\frac{1}{j\omega R_{in} C_f}}$$

iii) [4 pts] At what frequency (in Hz) are the input and output signal equal in amplitude?

$$\left|H(j\omega)\right| = \left|\frac{1}{j\omega R_{in} C_f}\right| = 1$$

$\left[\begin{matrix} R_{in} = 100k \\ C_f = 0.1\,\mu F \end{matrix}\right]$

$$\omega R_{in} C_f = 1$$

$$f = \frac{1}{2\pi \cdot R_{in} C_f} = \frac{1}{2\pi \cdot 100 \cdot 10^3 \cdot (0.1 \times 10^{-6})}$$

$$= \underline{\underline{15.9\ Hz}}$$

iv) [2 pts] At that frequency, what is the phase of the output signal relative to the input signal?

$$H(j\omega) = -\frac{1}{j\omega R_{in} C_f} = -\frac{1}{j} \cdot \left(\frac{1}{\omega R_{in} C_f}\right)$$

$$\angle H(j\omega) = \angle -\frac{1}{j} = -90^\circ \text{ or } -\frac{\pi}{4}$$

90° Lagging

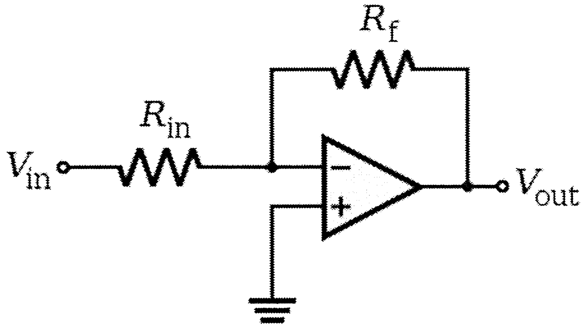
v) [2 pts] If the frequency is lowered by a factor of 10 from that found in iii), by what factor does the output voltage change? In what direction? Choose one: larger or smaller.

$$\angle H(j\omega) = -\frac{1}{j\omega R_{in} C_f}$$

$$\omega \downarrow = 10\times . \Rightarrow |H(j\omega)| \uparrow 10\times .$$

10 times, (Larger)

c) [10 pts] In the following op-amp application circuit the power supply is Vcc = +/- 5V.



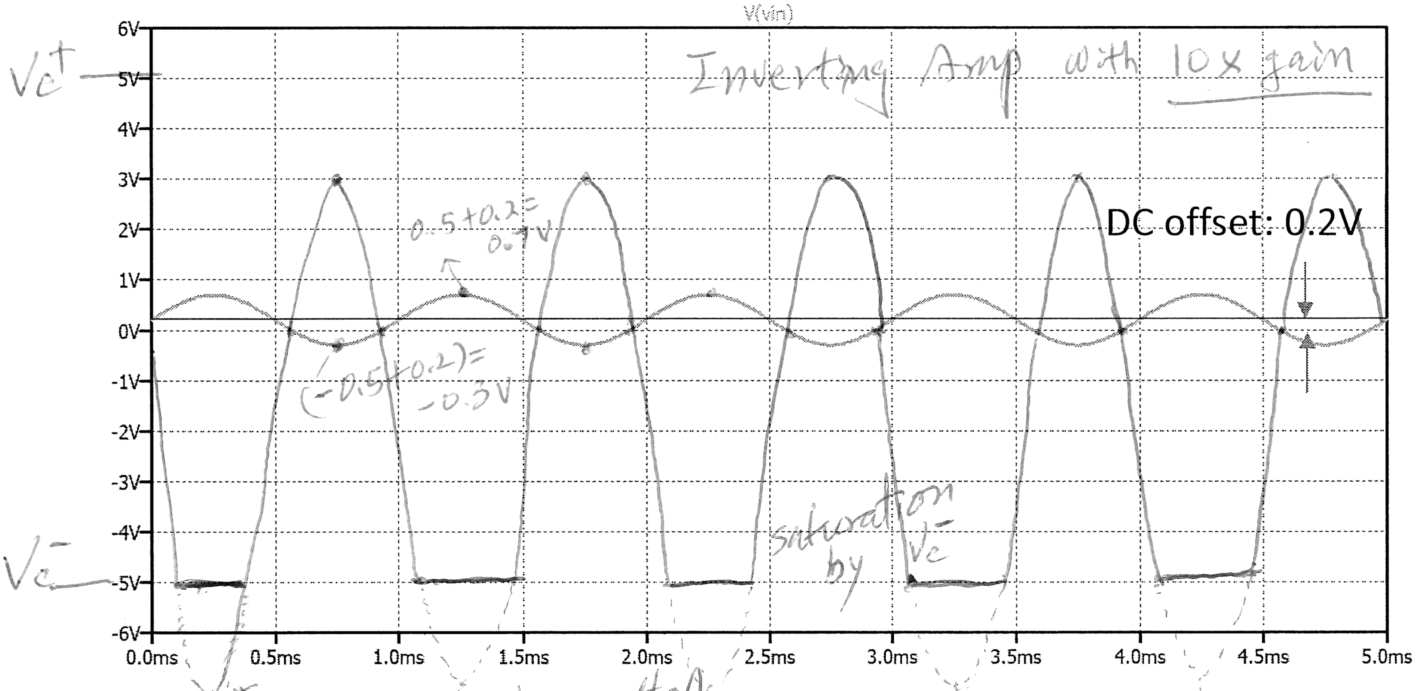
i) [2 pts] If Rin = 10k ohm and we want this amplifier to have a gain of *magnitude*: 10 V/V. What should be the value of the feedback resistor Rf?

Wants gain mag. = 10 V/V

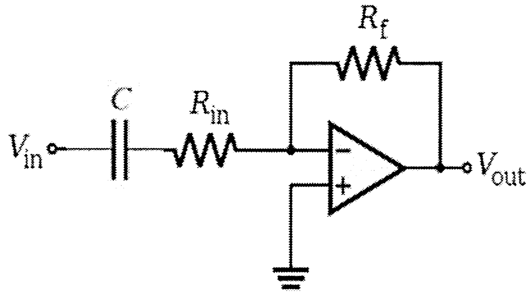
G of this inv. Amp = $\left|\frac{-R_f}{R_{in}}\right| = 10$

$R_f = R_{in} \cdot 10 = \underline{100\ k\Omega}$

ii) [5 pts] If Vin is a sinusoidal wave input of 0.5sin(2π1000t)V on top of DC offset of 0.2V, as drawn below on the V-time plot, draw the output wave form on the graph below. The DC offset is plotted on the graph below as well.



iii) [3 pts] Now we want to remove the effect of the DC offset from the Vin for proper amplification of the input signal. One way is to add a capacitor at the input. What is an appropriate value of the capacitor in the figure below, such that the circuit still operates as an **inverting amplifier**? Hint: what should the relative magnitude of the impedances of Rin and C?



In order to perform as an inv. Amplifier

$Z_c \ll Z_{in}$, just to block low freq.

$$\left|\frac{1}{j\omega C}\right| << R_{in}$$

$$C >> \frac{1}{\omega R_{in}} = \frac{1}{2\pi \cdot (1000) \cdot 10k}$$

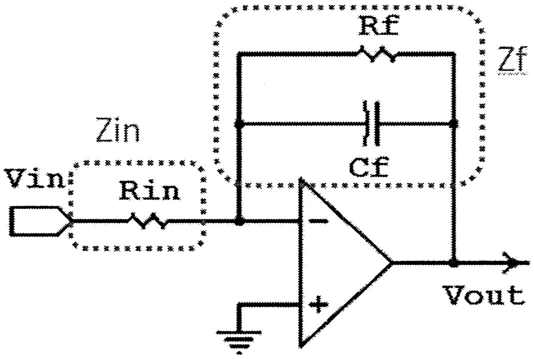
$$= \frac{1}{2\pi} \cdot 10^{-7}$$

$$= 15.9 \text{ nF}$$

$$\underline{C >> 15.9 \text{ nF}}$$

IV. Op-Amp Integrators [14 points]

To improve the performance of the ideal integrator in QIII.b, a feedback resistor Rf is added on the feedback path of Cf.



a) [4 pts] Using the same steps as in QIII.b.i and QIII.b.ii, find the transfer function of this op-amp circuit in terms of Rin, Rf and Cf.

$$H(j\omega) = -\frac{Z_f}{Z_{in}} ,$$

$$Z_f = R_f // C_f = \frac{R_f \cdot \frac{1}{j\omega C_f}}{R_f + \frac{1}{j\omega C_f}} \left(\frac{\times j\omega C_f}{\times j\omega C_f}\right) = \frac{R_f}{1 + j\omega R_f C_f}$$

$$H(j\omega) = -\frac{Z_f}{Z_{in}} = -\frac{\frac{R_f}{1 + j\omega R_f C_f}}{R_{in}} = \boxed{-\frac{R_f}{R_{in}(1 + j\omega R_f C_f)}}$$

b) [3 pts] Given that Rin = 1k ohm, assign values for Rf and Cf such that the DC gain is 100 and the corner frequency (3-dB frequency) is 40Hz.

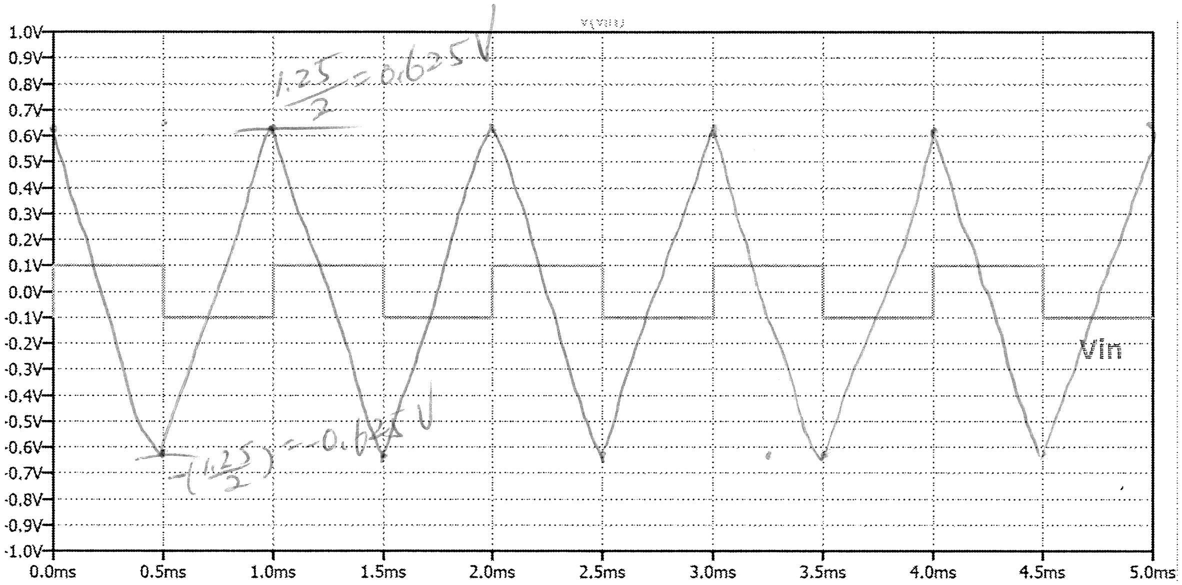
for $R_{in} = 1\,k\Omega$ want DC gain $= 100 = \frac{R_f}{R_{in}}$

$$\boxed{R_f = 100\ k\Omega}$$

$$f_{corner} = \frac{1}{2\pi \cdot R_f \cdot C_f} = 40\ Hz$$

$$C_f = \frac{1}{2\pi \cdot (100k)(40)} = \boxed{0.04\ \mu F}$$

c) [5 pts] For a 0.1V peak-to-peak square wave input with a frequency of 1kHz, draw the output wave form on the voltage-time plot below. Indicate key values from your calculation (such as min and max) on the plot. Hint: since we are in steady-state the DC offset of the output voltage should be 0.



$$V_{out}(t) = \frac{1}{R_{in} C_f} \int_0^t V_{in}(t)\, dt$$

Constant part: $\frac{1}{R_{in} C_f} = \frac{1}{1K \cdot (0.04\ \mu F)} = 2.5 \cdot 10^4$.

integration of Vin. $(0.1\ V)(0.5\ ms) = 0.05 \cdot 10^{-3}$

$V_{out}(t) = 2.5 \times 10^4 \cdot 0.05 \cdot 10^{-3} = 1.25$ V (P-P)

d) [2 pts] At what frequency does the magnitude of the transfer function reduce to unity (=1)?

$$H(j\omega) = \left| -\frac{R_f}{R_{in}(1 + j\omega R_f C_f)} \right| = 1$$

$$R_f = R_{in}(1 + j\omega R_f C_f)$$

$$\frac{\overset{100}{R_f}}{R_{in}} - 1 = |j\omega R_f C_f| = 99$$

$$\omega = 2\pi f = \frac{99}{R_f C_f}$$

$$f = \frac{99}{2\pi \cdot R_f C_f} = \frac{99}{2\pi \cdot (100K)(0.04\ \mu F)} \cong \underline{4 KHz}$$