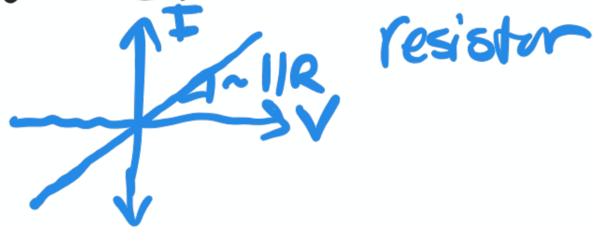
Intro to ECSE Lecture Notes 3/17/23

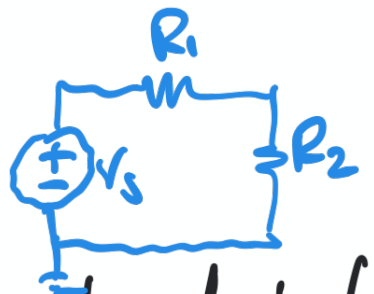
Properties of Linearity : Devices → Circuits → Systems

1) What have we said that linearity is so far?

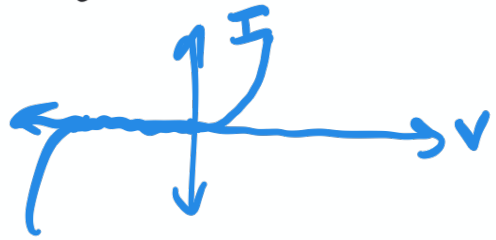
1) circuit elements : IV characteristics are linear



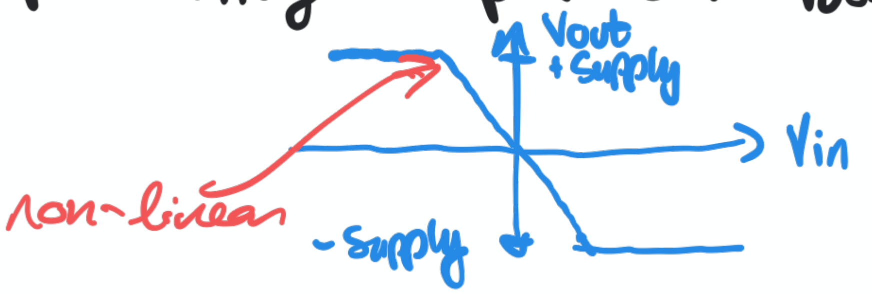
2) Circuits : only consists of linear elements



- What non-linear circuit elements have you encountered so far?
    - diode



- (transistors)
- op-amps : voltage amplifiers : Vout vs. Vin

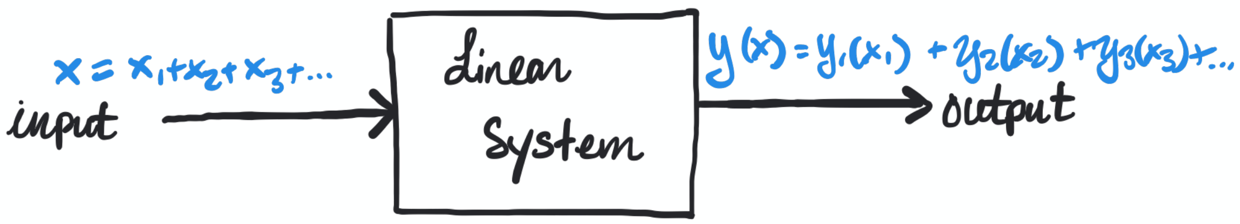


· What does linearity mean for Systems?

· A system is linear if it satisfies the:

Superposition Principle:

+ if you have a sum of inputs: $x = x_1 + x_2 + x_3 + \ldots$



· output is the sum of the outputs ($x \to y$) for the individual inputs

· Linear Systems are described by linear functions.

· So, what are linear functions? A function $f(x)$ satisfies superposition via 2 simpler properties:

1) Additivity: $f(x_1 + x_2) = f(x_1) + f(x_2)$

multiplication: $f \to$ multiplies input by 2

$x_1 = 2;\ x_2 = 3 \to f(x_1 + x_2) = 2 \cdot (2+3) = 10$

$f(x_1) = 2 \cdot 2 = 4;\ f(x_2) = 2 \cdot 3 = 6$

$f(x_1) + f(x_2) = 10$

2) Homogeneity: $F(cx) = cF(x)$ (c: constant)

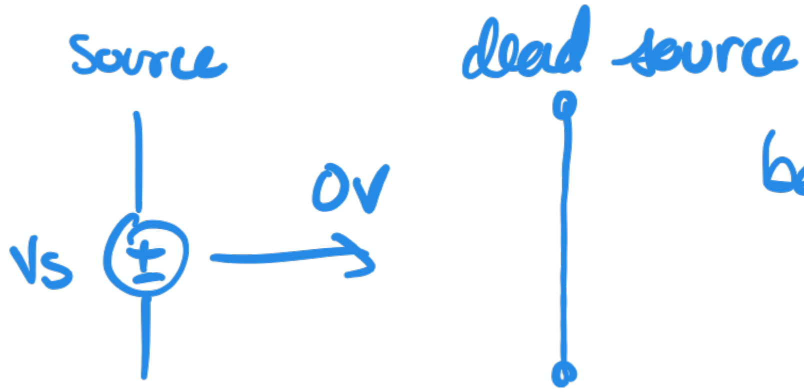
multiplication: same as above

$f(cx) = 2(3 \cdot 2) = 12;\ cf(x) = 3(2 \cdot 2) = 12$ ($c = 3$)

· So, if we said that a circuit is a linear system, how does it follow Superposition?

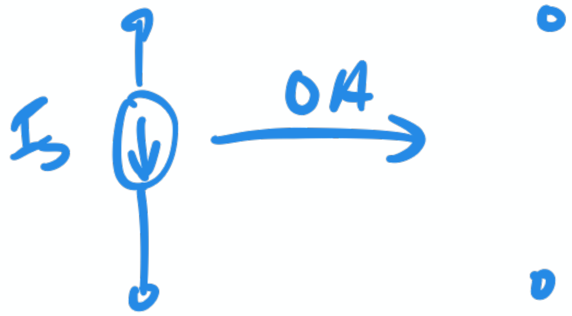
2) How do we use superposition in a circuit?

- Find the output due to each independent source, with all other independent sources set to zero, then add all of the individual outputs together to find the solution.
  - A source set to 0 is called a dead source (you "kill" them)
  - Dead Voltage source:



becomes a short circuit (no voltage drop)

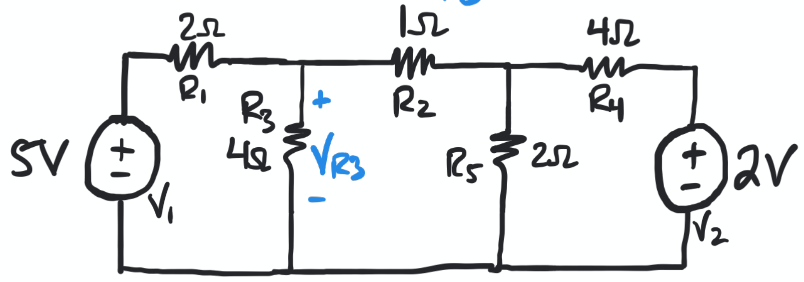
  - Dead Current Source:



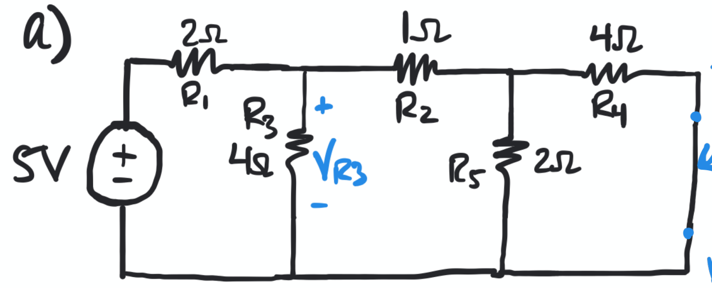
becomes an open circuit (no current flows)

Using Superposition to Solve a circuit

Find V_{R3}



a)

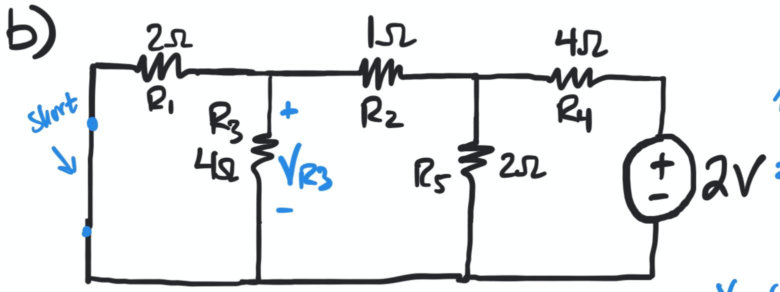


1) we have killed V_2 in this circuit

2) Solve for V_{R3} due to V_1

$V_{R3}(V_1) = \underline{2.12V}$

b)



1) We have "killed" V_1

2) Solve for V_{R3} due to V_2

$V_{R3}(V_2) = 0.242V$

V_{R3} due to V_1 and 2

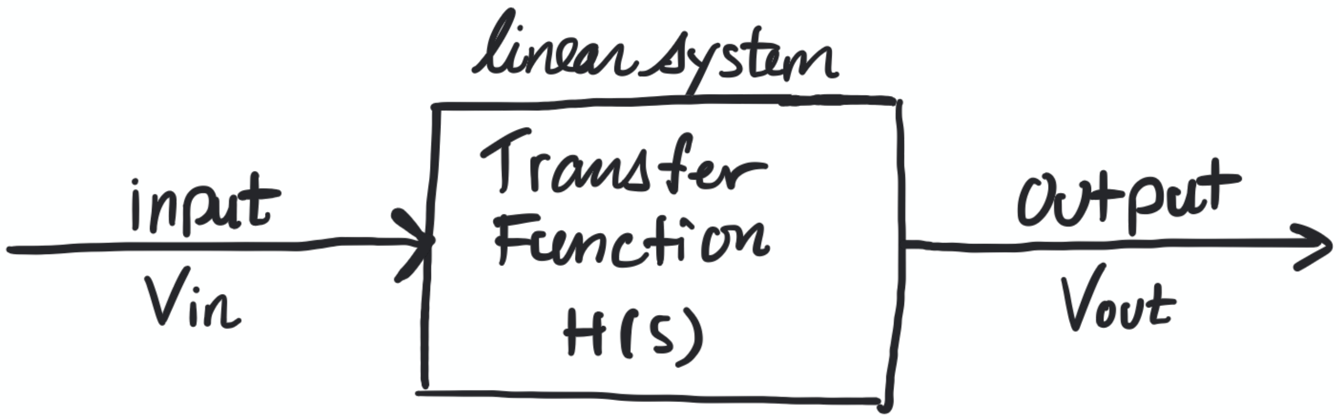
Final answer $V_{R3}(V_1 + V_2) = V_{R3}(V_1) + V_{R3}(V_2)$

$= 2.12V + 0.24V = \underline{2.36V}$

· If you solve the original circuit with both sources on, you should get $\underline{V_{R3} = 2.36V}$

3) Transfer Functions

- If a system is linear, we can derive a transfer function for it



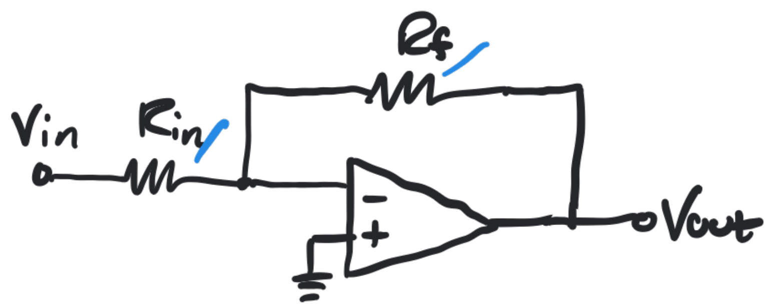
- Definition: $H = \frac{V_{out}}{V_{in}} = \frac{\text{output}}{\text{input}}$ ratio

- How do we find Vout for a given Vin?

$$V_{out} = H \cdot V_{in} \quad \text{for any } V_{in}$$

- Once we have the transfer function, we know Vout for all possible values of Vin → H characterizes the system

Example: Inverting Amplifier:



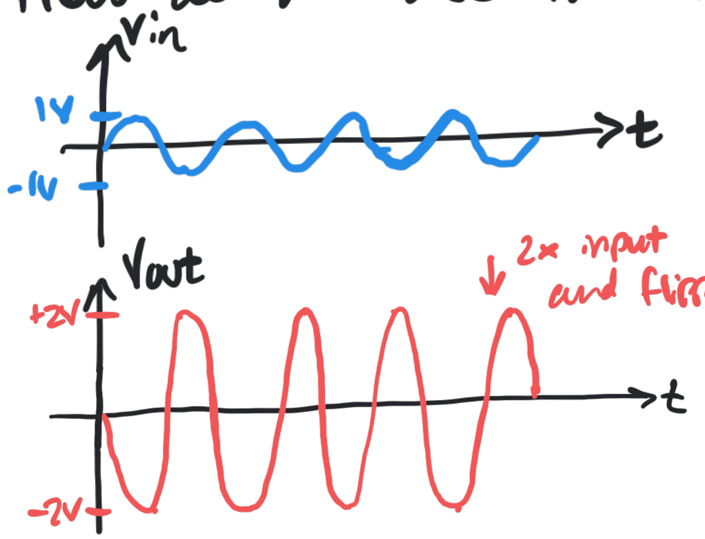
Recall: • $V_{out} = -\frac{R_f}{R_{in}} V_{in}$

- Knowing $V_{out}(V_{in})$, what is the transfer function for the inverting amplifier?

$$H = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

if $R_f = 2k\Omega$, $R_{in} = 1k\Omega$ } $H = -2$

- How do we use the transfer function? $H = -2$

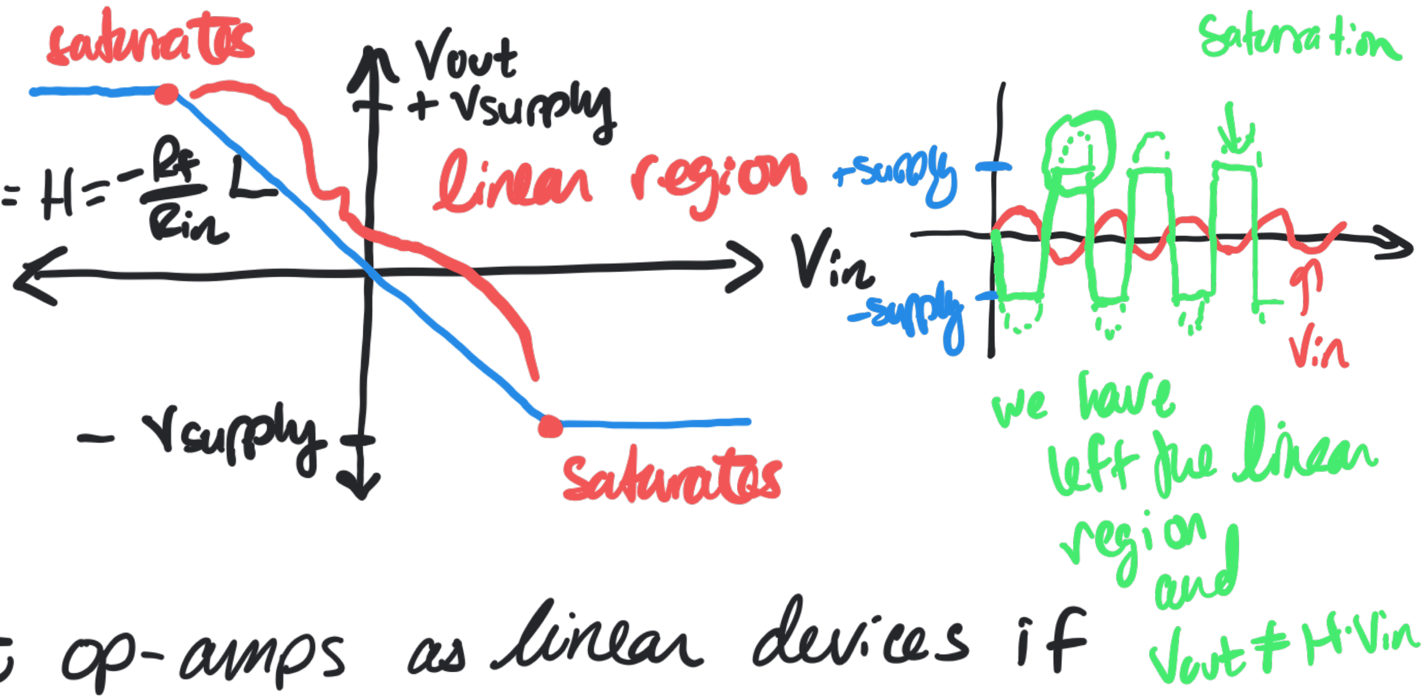


$V_{out} = H \cdot V_{in} = -2 V_{in}$

- Almost all op-amp circuits are characterized in terms of their transfer functions, because we can determine Vout for <u>any</u> Vin.

· STOP! Didn't we say that op-amp circuits were non-linear because of their non-linear transfer characteristic?!

Inverting Amplifier: Slope $= H = -\frac{R_f}{R_{in}}$



· We can treat op-amps as linear devices if we operate them in the region where the transfer characteristic is linear. This is also where $V_{out} = H \cdot V_{in}$.