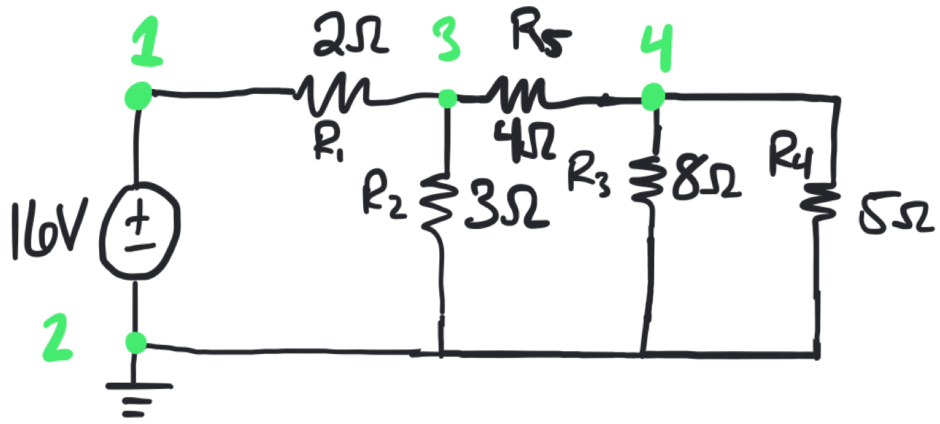
Intro to ECSE: Nodal Analysis Example Problems

1) Multiple Voltage Sources



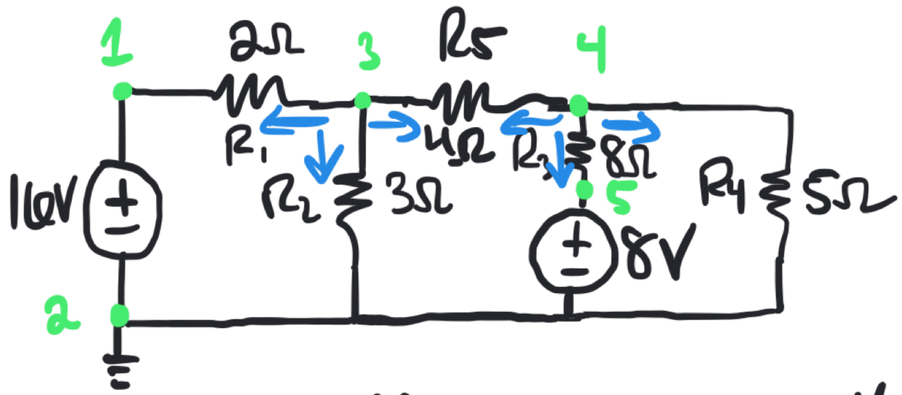
How many unknowns?

\# nodes total: 4

\# known nodes: 2 : node 1: 16V, node 2: 0V

\# unknown nodes: 2 : node 3, node 4

What if we add another voltage source?



node 3: $$\frac{V_3 - 16}{R_1} + \frac{V_3}{R_2} + \frac{V_3 - V_4}{R_5} = 0$$

$$V_3\left(1/R_1 + 1/R_2 + 1/R_5\right) + V_4\left(-1/R_5\right) = \frac{16}{R_1}$$

node 4: $$\frac{V_4 - V_3}{R_5} + \frac{V_4 - 8}{R_3} + \frac{V_4}{R_4}$$

$$V_3\left(-1/R_5\right) + V_4\left(1/R_5 + \ldots\right)$$

How many unknowns?

\# nodes total: 5

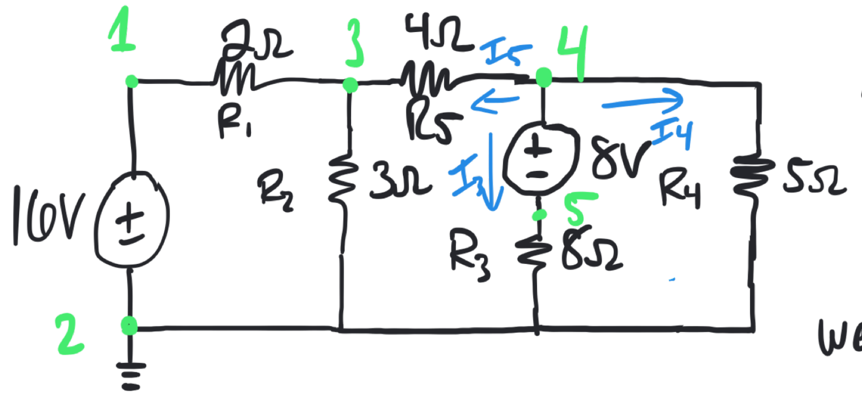
\# known nodes: 3 : node 1, node 2, node 5: 8V

\# unknown nodes: 2 : node 3, node 4

$$\begin{pmatrix} (1/R_1 + 1/R_2 + 1/R_3) & (-1/R_5) \\ (-1/R_5) & (1/R_5 + 1/R_3 + 1/R_4) \end{pmatrix} \begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 16/R_1 \\ 8/R_3 \end{bmatrix}$$

$$\begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 8.054 \\ 5.502 \end{bmatrix}$$

What if we place a voltage such that it's not grounded?



which are the unknown nodes?

known nodes are: 1: 16V
2: 0V

we also know: $V_4 - V_5 = 8V$

replace all V_5 with $V_5 = V_4 - 8V$ in your equations, then solve as usual

node 3: $\dfrac{V_3 - 16V}{2\Omega} + \dfrac{V_3}{3\Omega} + \dfrac{V_3 - V_4}{4\Omega} = 0$

$$V_3\left(1/2\Omega + 1/3\Omega + 1/4\Omega\right) + V_4\left(-1/4\Omega\right) = \frac{16V}{2\Omega}$$

node 4: $\underbrace{\dfrac{V_4 - V_3}{4\Omega}}_{I_5} + \underbrace{\dfrac{V_5}{8\Omega}}_{I_3} + \underbrace{\dfrac{V_4}{5\Omega}}_{I_4} = 0$ $\qquad \dfrac{V_4 - V_3}{4\Omega} + \dfrac{V_4 - 8V}{8\Omega} + \dfrac{V_4}{5\Omega} = 0$

$V_3\left(-1/4\Omega\right) + V_4\left(1/4\Omega + 1/5\Omega + 1/8\Omega\right) = \dfrac{8V}{8\Omega}$

current flowing from 4 to 5 must be the same as from 5 to 2 (series)

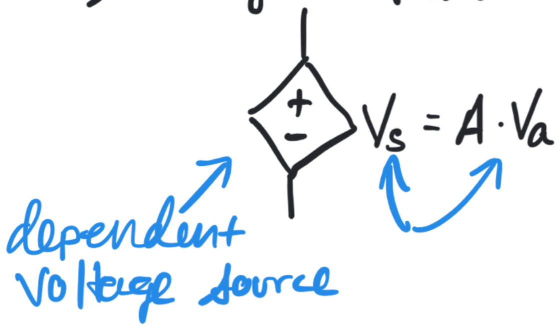
so $I_3 = \dfrac{V_5}{8\Omega} = \dfrac{V_4 - 8V}{8\Omega}$

node 5: $V_4 - 8V = V_5$

$$\begin{bmatrix} (1/2\Omega + 1/3\Omega + 1/4\Omega) & (-1/4\Omega) \\ (-1/4\Omega) & (1/4\Omega + 1/8\Omega + 1/5\Omega) \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 16V/2\Omega \\ 8V/8\Omega \end{bmatrix}$$

$$\begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 8.654 \\ 5.502 \end{bmatrix}$$

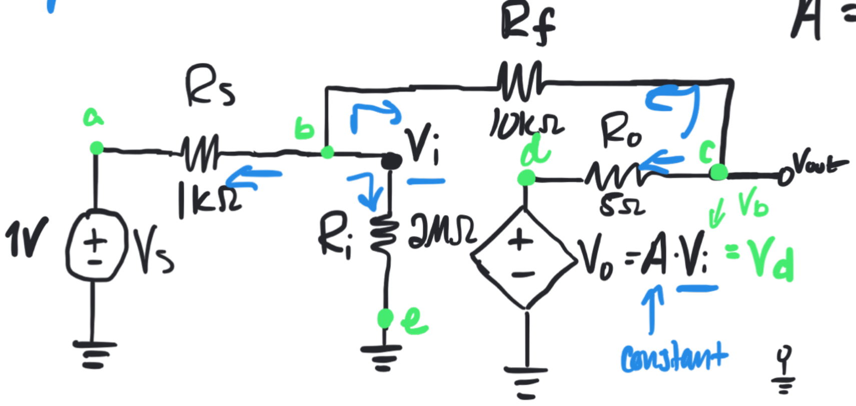
2) Voltage-dependent Voltage Sources



Voltage supplied by source depends on a voltage somewhere else in the circuit

→ op-amps are modeled this way

$A = 10^5$



• How many nodes?
5 total

known: $V_a = 1V$
$V_e = 0V$
$V_d = AV_i = AV_b$ ✓ $A = 10^5$

unknown: V_b and V_c

at node b: $\dfrac{V_b - 1V}{1k\Omega} + \dfrac{V_b}{2M\Omega} + \dfrac{V_b - V_c}{10k\Omega} = 0$

$$V_b\left(\tfrac{1}{1k\Omega} + \tfrac{1}{2M\Omega} + \tfrac{1}{10k\Omega}\right) + V_c\left(-\tfrac{1}{10k\Omega}\right) = \tfrac{1}{1k\Omega}$$

at node c: $\dfrac{V_c - \overset{AV_b}{V_d}}{5\Omega} + \dfrac{V_c - V_b}{10k\Omega} = 0$

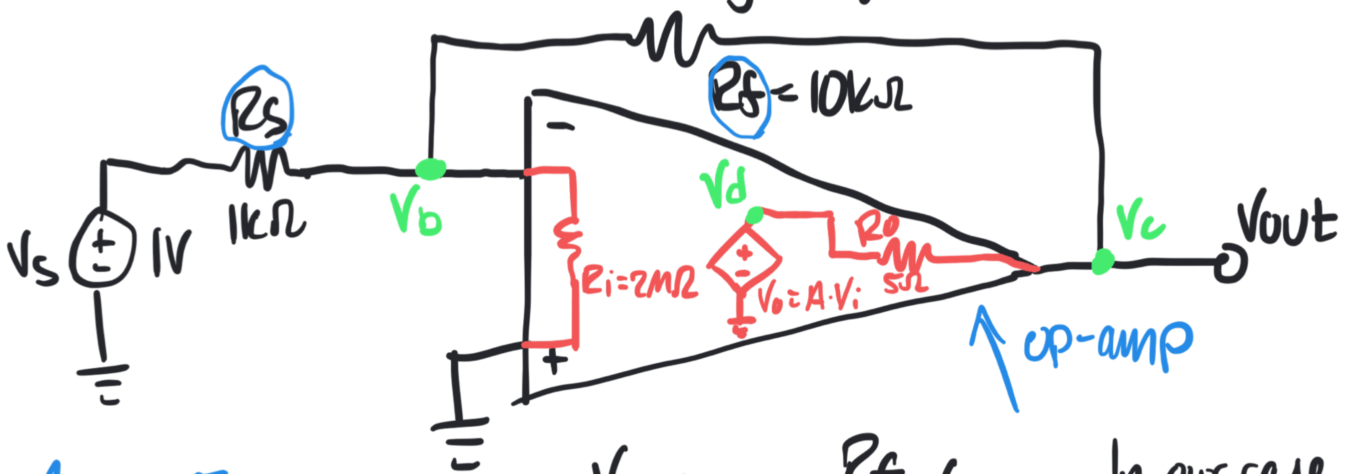
$$V_b\left(-\tfrac{1}{10k\Omega} - \tfrac{A}{5\Omega}\right) + V_c\left(\tfrac{1}{5\Omega} + \tfrac{1}{10k\Omega}\right) = 0$$

$$\begin{bmatrix} \left(\frac{1}{1k\Omega} + \frac{1}{2M\Omega} + \frac{1}{10k\Omega}\right) & \left(-\frac{1}{10k\Omega}\right) \\ \left(-\frac{1}{10k\Omega} - \frac{10^5}{5\Omega}\right) & \left(\frac{1}{5\Omega} + \frac{1}{10k\Omega}\right) \end{bmatrix} \begin{bmatrix} V_b \\ V_c \end{bmatrix} = \begin{bmatrix} \frac{1}{1k\Omega} \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} V_b \\ V_c \end{bmatrix} = \begin{bmatrix} \overset{\sim 0}{0} \\ -10 \end{bmatrix}$$

$V_c = V_{out}$
$V_s = V_{in}$

What does this circuit do? $V_{in} = 1V$
$V_{out} = -10V$

op-amp circuit: inverting amplifier



$A = 10^5$
intrinsic gain of the op-amp

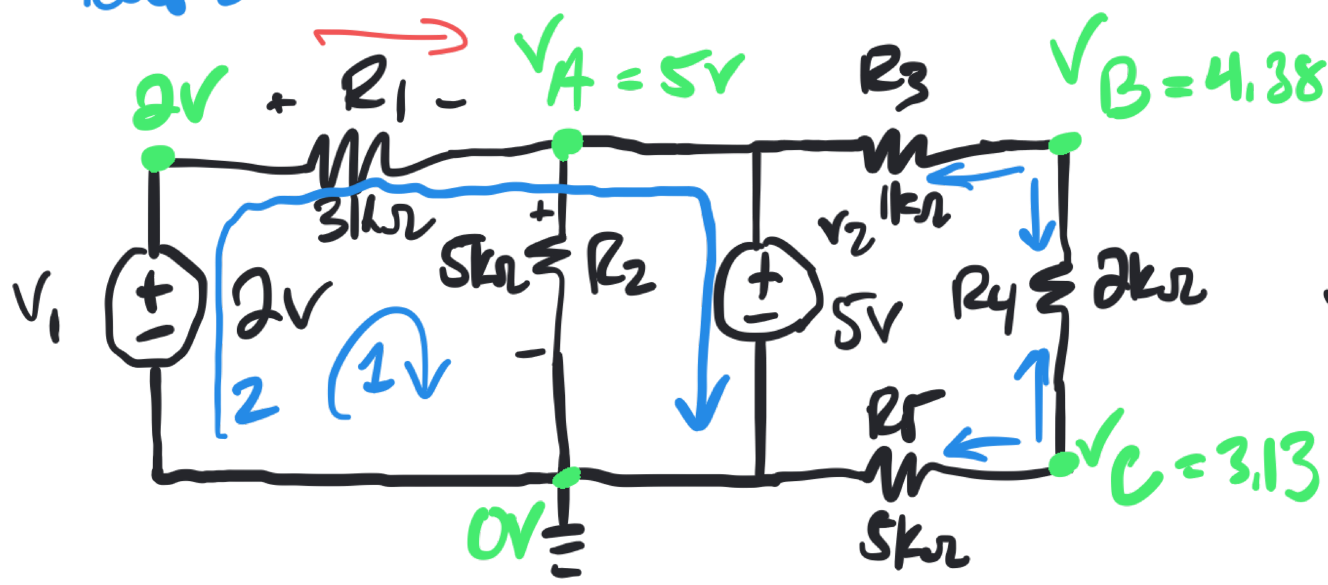
$$V_{out} = -\frac{R_f}{R_s} V_{in}$$

In our case

$V_{out} = -\dfrac{10k\Omega}{1k\Omega} \cdot 1V$
$= \underline{-10V}$

KVL: loop 1: $-2V - 3V + 5V = 0$

loop 2: $-2V - 3V + 5V = 0$



$V_A = 5V$

$V_B = ?$

$V_C = ?$

at node B: $$\frac{V_B - 5V}{1k\Omega} + \frac{V_B - V_C}{2k\Omega} = 0$$

$$V_B\left(\frac{1}{1k\Omega} + \frac{1}{2k\Omega}\right) + V_C\left(-\frac{1}{2k\Omega}\right) = \frac{5V}{1k\Omega}$$

at node C: $$\frac{V_C - V_B}{2k\Omega} + \frac{V_C - 0V}{5k\Omega} = 0$$

$$V_B\left(-\frac{1}{2k\Omega}\right) + V_C\left(\frac{1}{2k\Omega} + \frac{1}{5k\Omega}\right) = 0$$

$$\begin{bmatrix} \left(\frac{1}{1k\Omega} + \frac{1}{2k\Omega}\right) & \left(-\frac{1}{2k\Omega}\right) \\ \left(-\frac{1}{2k\Omega}\right) & \left(\frac{1}{2k\Omega} + \frac{1}{5k\Omega}\right) \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 5V/1k\Omega \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 4.38 \\ 3.13 \end{bmatrix}$$