# Intro to ECSE : Nodal Analysis 2/17/23

## Circuit Analysis Method #3: Nodal Analysis

→ How is it different from KCL/KVL/Ohm's Law?

1) More "efficient": no "guessing" which equations to use
   <u>and</u> usually fewer equations needed

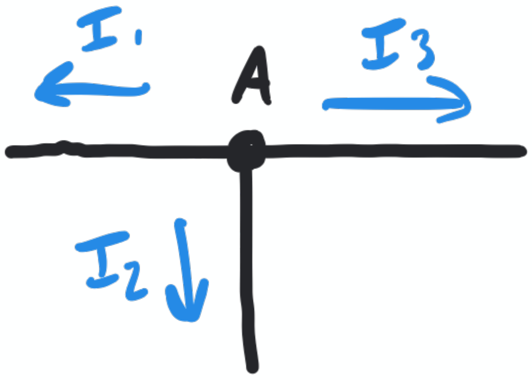
2) Covers more of the circuit: can handle voltage sources

3) Has an equation to determine # of equations needed

# of unknowns → # of linearly independent equations needed = total # of nodes − $\underbrace{\text{\# of voltage sources}}_{\text{known nodal voltages}}$ − 1 (ground)

4) Passive Sign Convention (current flow)
   → set direction of current flow first

   my convention: KCL sum of all outgoing currents is 0




$I_1 + I_2 + I_3 = 0$

KCL

# Nodal Analysis Example

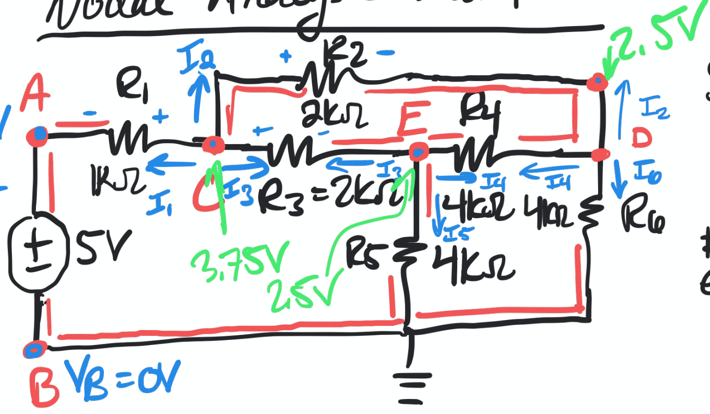
must express in terms of nodal voltages

$V_A = 5V$

$I_1 = \frac{\Delta V_R}{R_1} = \frac{V_C - V_A}{R_1}$

$I_2 = \frac{V_C - V_D}{R_2}$

$I_3 = \frac{V_C - V_E}{R_3}$




Step 1: How many equations do we need?

$$\#\text{ of eqns} = \underset{\text{total \# nodes}}{5} - \underset{\text{\# of V. sources}}{1} - \underset{\text{gnd}}{1} = 3$$

(would need 6 for KVL/KCL)

Step #2: label the nodes and the known nodal voltages

node A voltage

$V_A = 5V$ ← voltage source

$V_B = 0V$ ← ground

Step #3: Do KCL at each node

Node C: $I_1 + I_2 + I_3 = 0$

$$\frac{V_C - V_A}{R_1} + \frac{V_C - V_D}{R_2} + \frac{V_C - V_E}{R_3} = 0 \leftarrow \text{KCL eqn. for node C}$$

Node D: $I_2 + I_4 + I_6 = 0$

$$\frac{V_D - V_C}{R_2} + \frac{V_D - V_E}{R_4} + \frac{V_D - V_B}{R_6} = 0$$

Node E: $I_3 + I_4 + I_5 = 0$

$$\frac{V_E - V_C}{R_3} + \frac{V_E - V_D}{R_4} + \frac{V_E - V_B}{R_5} = 0$$

Step #4: Put in standard form & simplify

constants belong on right side

Node C: $\underbrace{\boxed{V_A}}_{\text{source = constant}}^{5V}\left(-\frac{1}{R_1}\right) + \cancel{V_B}^{0\text{ gnd}}(0) + V_C\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + V_D(-1/R_2) + V_E(-1/R_3) = 0$ 5V

$\rightarrow V_C(1/R_1 + 1/R_2 + 1/R_3) + V_D(-1/R_2) + V_E(-1/R_3) = V_A/R_1$

Node D: $\cancel{V_B(-1/R_6)}^{0} + V_C(-1/R_2) + V_D(1/R_2 + 1/R_4 + 1/R_6) + V_E(-1/R_4) = 0$

Node E: $\cancel{V_B(-1/R_5)}^{0} + V_C(-1/R_3) + V_D(-1/R_4) + V_E(1/R_3 + 1/R_4 + 1/R_5) = 0$

Step #5: Put in matrix form

Constants

$V_A = 5V$ = constant

$$\begin{array}{c|ccc}
 & A & x & b \\
\begin{array}{c}\text{node C}\\ \text{node D}\\ \text{node E}\end{array} & \begin{bmatrix} 1/R_1 + 1/R_2 + 1/R_3 & -1/R_2 & -1/R_3 \\ -1/R_2 & 1/R_2 + 1/R_4 + 1/R_6 & -1/R_4 \\ -1/R_3 & -1/R_4 & 1/R_3 + 1/R_4 + 1/R_5 \end{bmatrix} & \begin{bmatrix} V_C \\ V_D \\ V_E \end{bmatrix} = & \begin{bmatrix} V_A/R_1 \\ 0 \\ 0 \end{bmatrix} \\
 & 3\times 3 & 3\times 1 & 3\times 1
\end{array}$$

$$\begin{bmatrix} (1/1000 + 1/2000 + 1/2000) & (-1/2000) & (-1/2000) \\ (-1/2000) & (1/2000 + 1/4000 + 1/4000) & (-1/4000) \\ (-1/2000) & (-1/4000) & (1/2000 + 1/4k + 1/4k) \end{bmatrix} \begin{bmatrix} V_C \\ V_D \\ V_E \end{bmatrix} = \begin{bmatrix} 5/1000 \\ 0 \\ 0 \end{bmatrix}$$

$$A^{-1}b = \begin{bmatrix} V_C \\ V_D \\ V_E \end{bmatrix} = \begin{bmatrix} 3.75 \\ 2.5 \\ 2.5 \end{bmatrix}$$

$(\mathrm{inv}(A) \cdot b)$

Matlab