

Name SOLUTIONS

1. _____ (9 pts)
2. _____ (12 pts)
3. _____ (10 pts)
4. _____ (10 pts)
5. _____ (8 pts)
6. _____ (10 pts)
7. _____ (16 pts)

Total _____

For partial credit on some questions, you may want to re-draw/label circuit diagrams to clarify your answers.

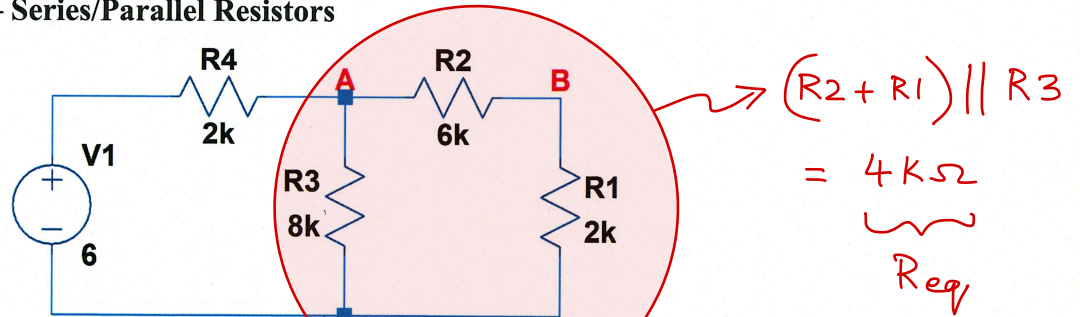
Show all of your work. Use the backs of pages if there is not enough room on the front.

Many problems can be solved using more than one method. Check your answers by using a second method.

At least skim through the entire quiz before you begin and then start with the problems you know best.

The proctor will only answer clarification questions where wording is unclear or where there may be errors/typos. No other questions will be responded to.

Problem 1 (9 Points) – Series/Parallel Resistors



a) Use any method to find the voltages at nodes A and B. (6 pts)

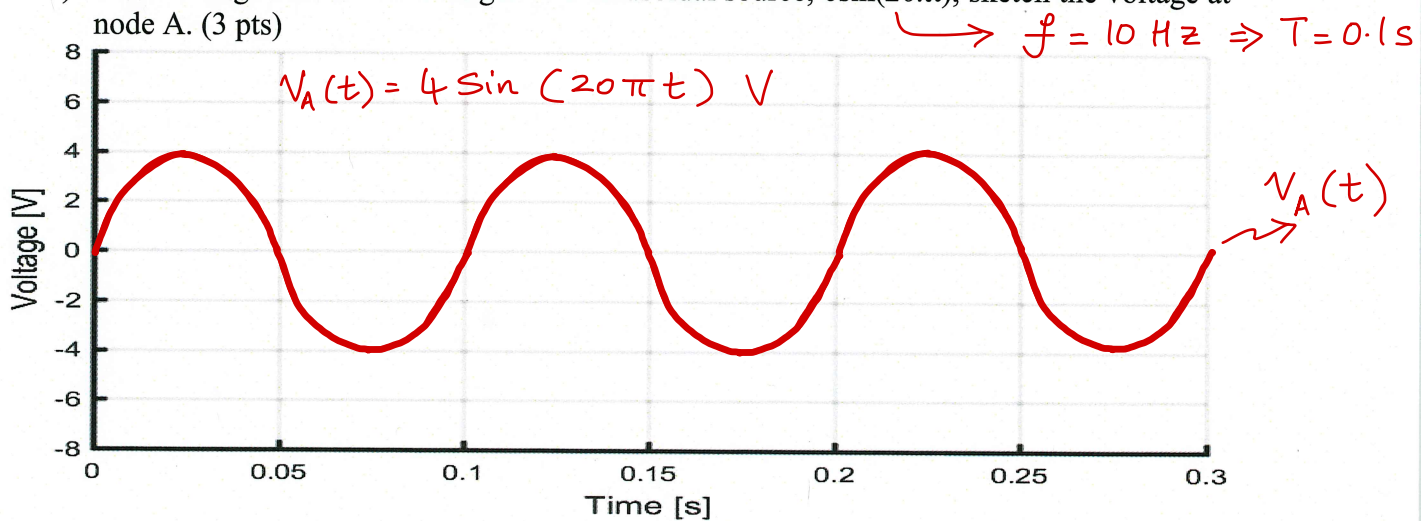
$$V_A = V_1 \left(\frac{R_{eq}}{R_4 + R_{eq}} \right) = 6 \left(\frac{4k}{6k} \right) = 4V$$

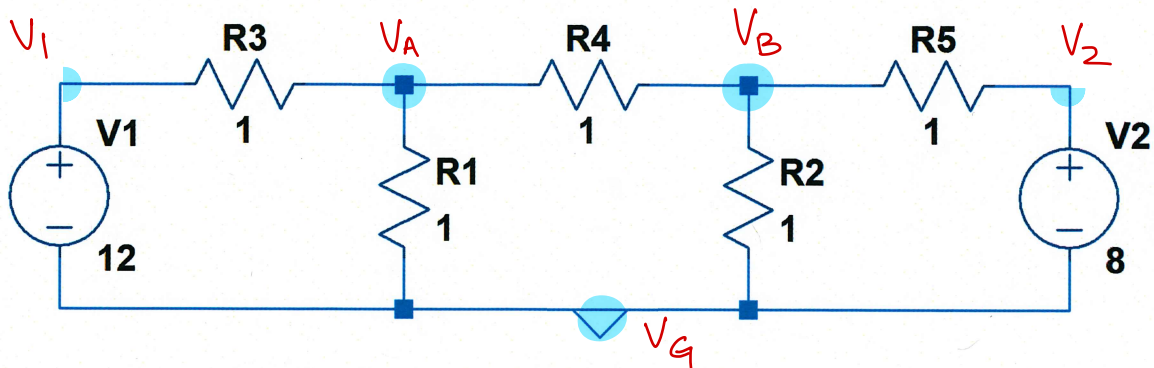
$$V_B = V_A \left(\frac{R_1}{R_1 + R_2} \right) = 4 \left(\frac{2k}{8k} \right) = 1V$$

Other methods possible

V_A	4	[V]
V_B	1	[V]

b) If the voltage source was changed to a sinusoidal source, $6\sin(20\pi t)$, sketch the voltage at node A. (3 pts)



Problem 2 (12 Points) – Nodal Analysis


- a) Label all the nodes in the above circuit. (2 pts)
- b) Which labelled nodes have an unknown voltage? V_A, V_B (1 pt)
- c) Apply KCL and determine the linear system of equations associated with the unknown nodes. (Do not simplify the circuit using parallel and series relationships for resistors.) (3 pts)

KCL @ node A

$$\frac{V_A - 12}{1} + \frac{V_A}{1} + \frac{V_A - V_B}{1} = 0$$

$$3V_A - V_B = 12$$

KCL @ node B

$$\frac{V_B - V_A}{1} + \frac{V_B}{1} + \frac{V_B - 8}{1} = 0$$

$$\Rightarrow -V_A + 3V_B = 8$$

Dependent equations are also acceptable

d) Based on your part c equations, build the matrix expression $Ax = b$. (2 pts)

$$\underbrace{\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}}_A \underbrace{\begin{bmatrix} V_A \\ V_B \end{bmatrix}}_x = \underbrace{\begin{bmatrix} 12 \\ 8 \end{bmatrix}}_b$$

e) Using any method, find the inverse of the A matrix in the above expression. (2 pts)

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}^{-1} = \frac{1}{8} \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix}$$

f) Use matrix mathematics to determine the voltage at the unknown nodes. Circle your voltage results. (2 pts)

$$x = A^{-1} b = \begin{bmatrix} 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix} \begin{bmatrix} 12 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 5.5 \\ 4.5 \end{bmatrix}$$

$$V_A = 5.5 \text{ V}$$

$$V_B = 4.5 \text{ V}$$

Problem 3 (10 Points) – Data and Curve Fitting

The following data was obtained from some circuit. The source voltages, V_A and V_B were varied and an output voltage was measured.

V_A	V_B	V_{out}
0	1	0
1	1	1
2	2	2
2	3	3

Using **matrix mathematics**, find the coefficients, a and b , that give the best fit relationship between the two inputs and the input in the following expression,

$$aV_A + bV_B = V_{out}$$

You must show your work to receive credit. Hint: This problem involves almost all the matrix manipulations we have utilized when preparing for exam 3.

matrix formulation:

$$x = (A^T A)^{-1} A^T b$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 2 \\ 2 & 3 \end{bmatrix}_{4 \times 2} \begin{bmatrix} a \\ b \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}_{4 \times 1}$$

A x b

$$\Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 2 \\ 2 & 3 \end{bmatrix}^T \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 2 \\ 2 & 3 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 2 \\ 2 & 3 \end{bmatrix}^T \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

(next page left blank)

$$\begin{bmatrix} a \\ b \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 2 \\ 2 & 3 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

2×4
 4×2
 2×4
 4×1

$$= \left(\begin{bmatrix} 9 & 11 \\ 11 & 15 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

2×4
 4×1

$$= \begin{bmatrix} 1.0714 & -0.7857 \\ -0.7857 & 0.6429 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

2×2
 2×4
 4×1

$$= \begin{bmatrix} -0.7857 & 0.2857 & 0.5714 & -0.2143 \\ 0.6429 & -0.1429 & -0.2857 & 0.3571 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

2×4
 4×1

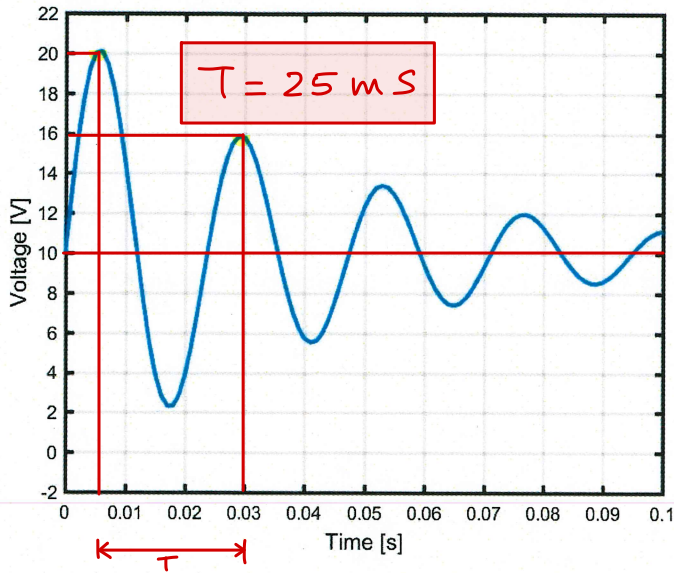
$$= \begin{bmatrix} 0.7857 \\ 0.3571 \end{bmatrix}$$

2×1

$$a = 0.7857$$

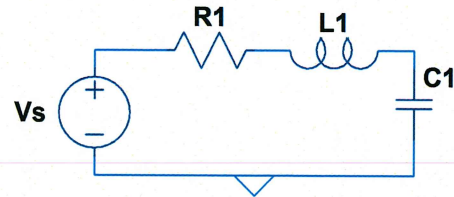
$$b = 0.3571$$

Problem 4 (10 Points) – First Order Circuit



$$\beta = \frac{2\pi}{T} = 251.33 \text{ rad/s}$$

Detailed solution available: Quiz# 4 Problem 6. (with more accurate values)



The above plot was measured on the series RLC circuit shown to the right, when the source is a pulsed source. Determine values for R1, L1 and C1. **Note**, there are an infinite number of solutions, which means you can pick a value for one of the components.

$$\alpha = - \frac{\ln \left(\frac{V_2 - V_{\text{offset}}}{V_1 - V_{\text{offset}}} \right)}{t_2 - t_1} = 20.43 \text{ rad/s} = \frac{R}{2L}$$

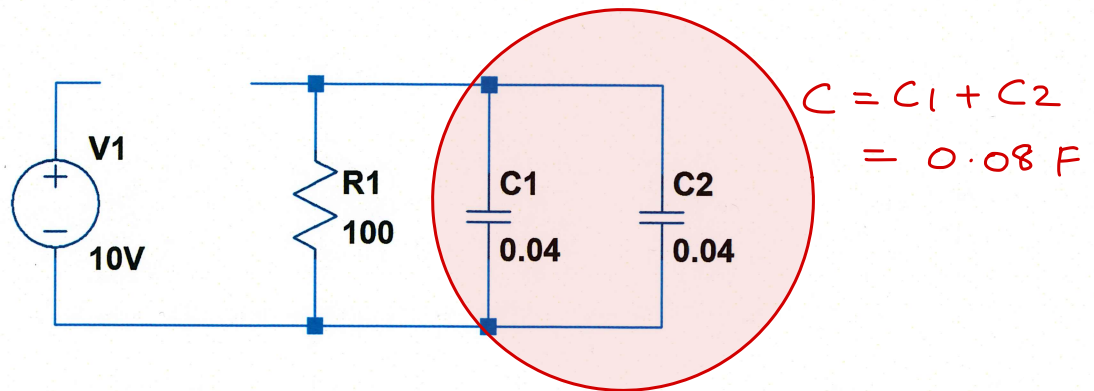
$$\omega_0 = \sqrt{\beta^2 + \alpha^2} = \sqrt{251.33^2 + 20.43^2} = 252.16 \text{ rad/s} = \frac{1}{\sqrt{LC}}$$

Choose $L = 100 \text{ mH}$, then $R = 2L\alpha = 4.09 \Omega$

and $C = \frac{1}{L\omega_0^2} = 0.1573 \text{ mF}$

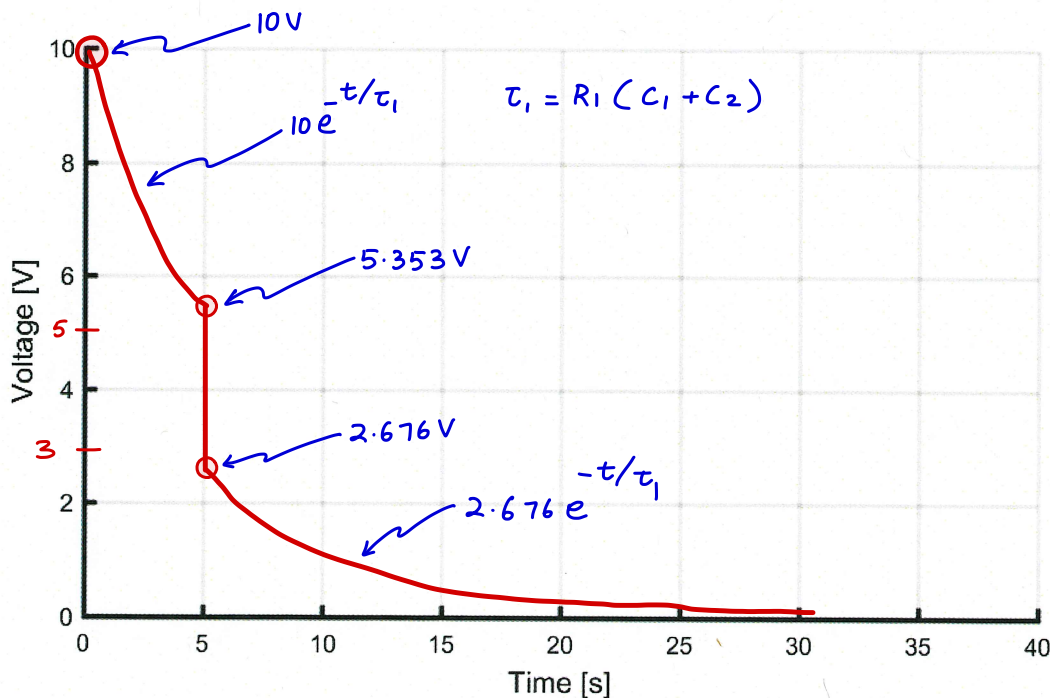
R1	4.09	[Ω]
L1	100m	[H]
C1	0.1573m	[F]

Problem 5 (8 Points) – RC Circuits

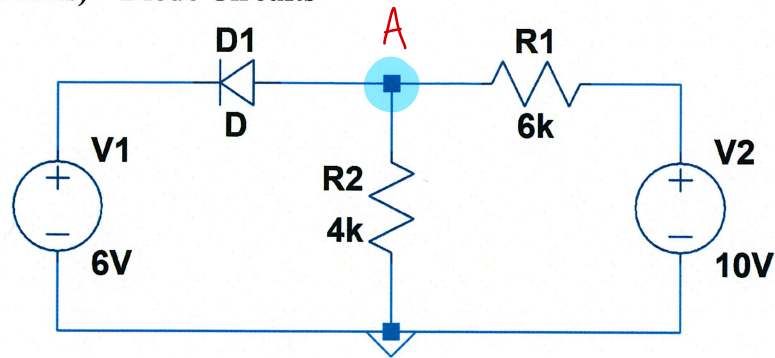


In the above circuit, the voltage source is connected to the circuit and then disconnected at $t = 0\text{s}$ (time equals zero). Additionally, at $t = 5\text{s}$, capacitor C_2 is disconnected from the circuit, discharged by shorting the leads and then immediately reconnected to the circuit (all happening very quickly). To the best of your artistic ability, sketch the voltage across capacitor C_1 as a function of time.

For first 5 seconds $\tau_1 = R_1 C = 8 \text{ s}$
 $\Rightarrow V_0(t) = 10 e^{-t/\tau_1}$ @ 5s $V_0(5) = 10 e^{-5/8} = 5.353 \text{ V}$
 C_2 discharged @ $t = 5 \Rightarrow V_0(t)$ is halved.
 $\Rightarrow V_0(5^+) = 2.676 \text{ V}$

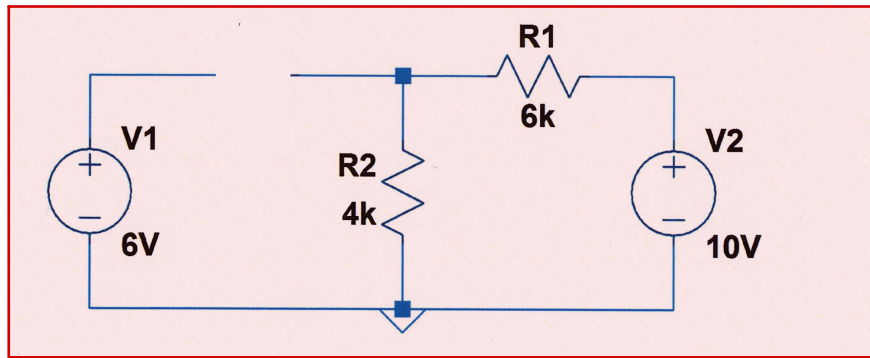


Problem 6 (10 Points) – Diode Circuits



The diode in the above circuit is ideal, with $V_{Turn-on} = 1V$.

- a) Assume the diode is off and redraw the circuit. (1 pt)



- b) Find the voltage difference across the diode (anode to cathode). (2 pts)

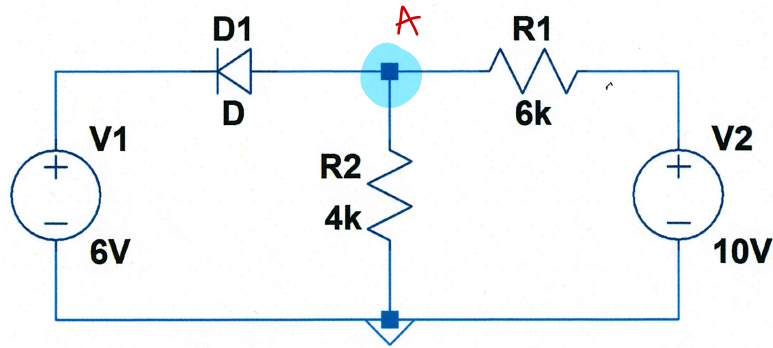
$$\begin{aligned} \text{Voltage @ node A, } V_A &= V_2 \left(\frac{R_2}{R_2 + R_1} \right) \\ &= 10 \left(\frac{4k}{10k} \right) = 4V \end{aligned}$$

$$\begin{aligned} \text{Voltage across diode} &= V_A - V_1 = 4 - 6 \\ &= \boxed{-2V} \end{aligned}$$

- c) Based on your above answer, is the assumption that the diode is off correct? Why? (2 pts)

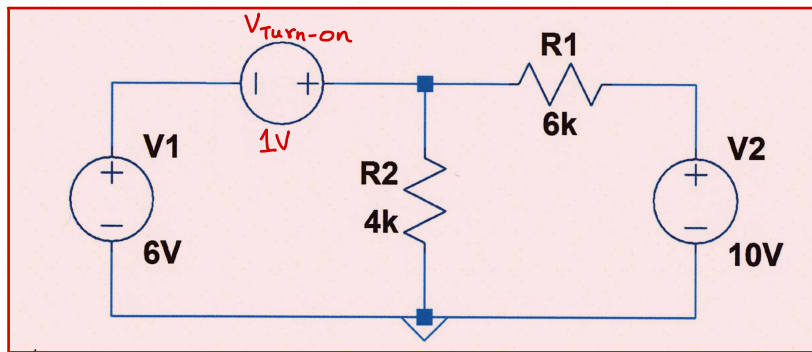
The assumption that diode is off is CORRECT.

Reason: Voltage across diode $<$ $V_{Turn-on}$
 \downarrow \rightarrow
 $-2V$ $1V$



The diode in the above circuit is ideal, with $V_{Turn-on} = 1V$. (Same circuit as the previous page.)

d) Assume the diode is on and redraw the circuit. (1 pt)



e) Find the current through the diode (from the anode to the cathode). (2 pts)

Voltage @ node A, $V_A = V1 + V_{Turn-on} = 7V$

Current through resistor $R1 = I_{R1} = \frac{V2 - V_A}{6k}$

$= 0.5 \text{ mA}$ into node A

Current through resistor $R2 = I_{R2} = \frac{V_A}{4k} = 1.75 \text{ mA}$ out of node A

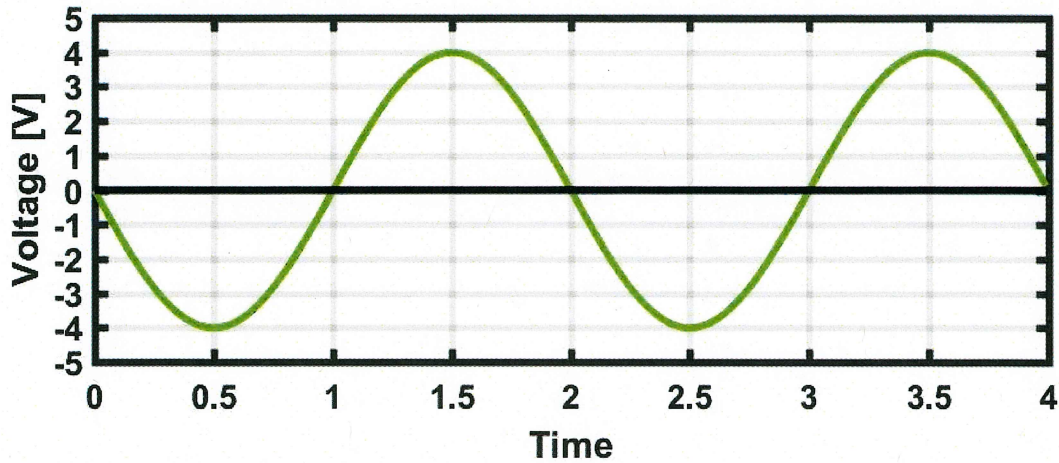
current through diode (from anode to cathode) = $I_{R2} - I_{R1}$
 $= -1.25 \text{ mA}$

f) Based on your above answer, is the assumption that the diode is on correct? Why? (2 pts)

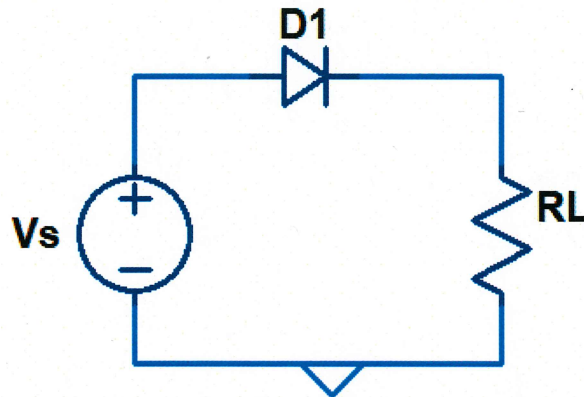
The assumption that diode is on is INCORRECT.

Reason: Current through diode (anode to cathode) when on, should be a positive quantity.

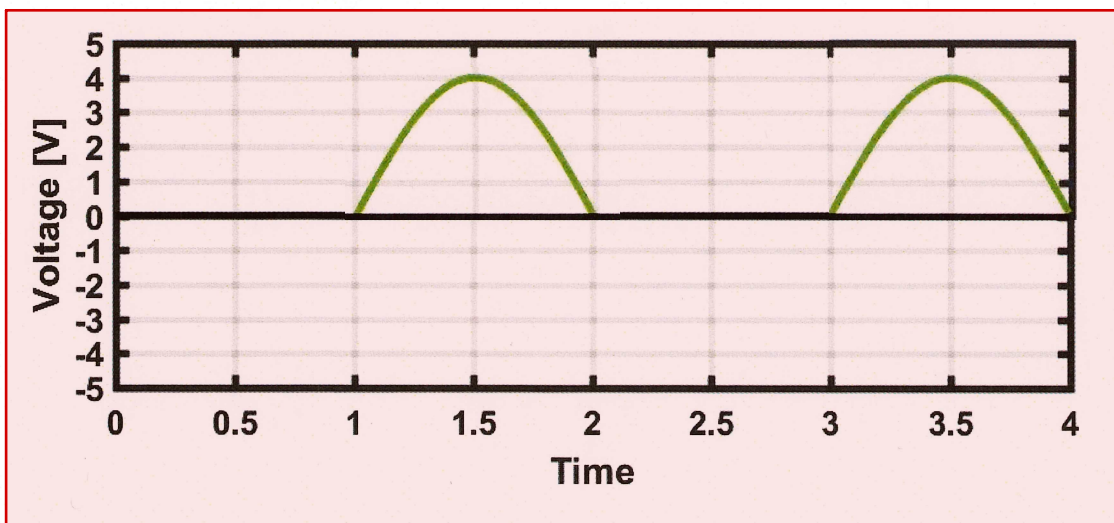
Problem 7 (16 Points) – Rectifiers



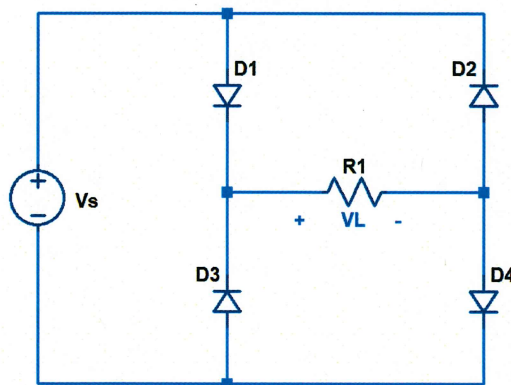
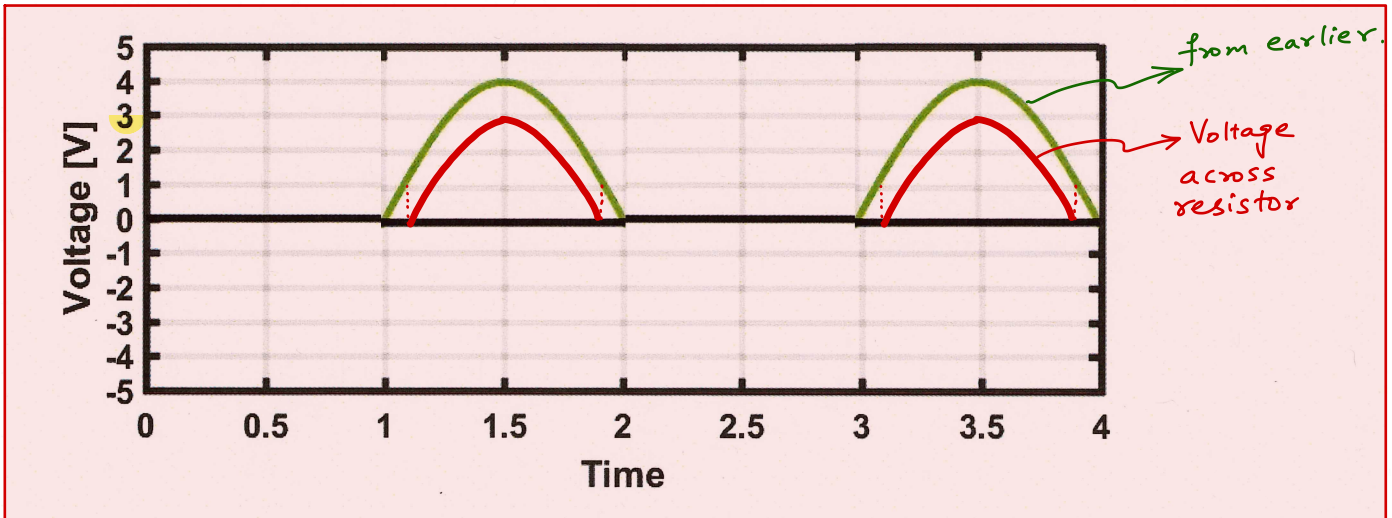
The above sinusoidal voltage represents the voltage source in all circuits for this problem.



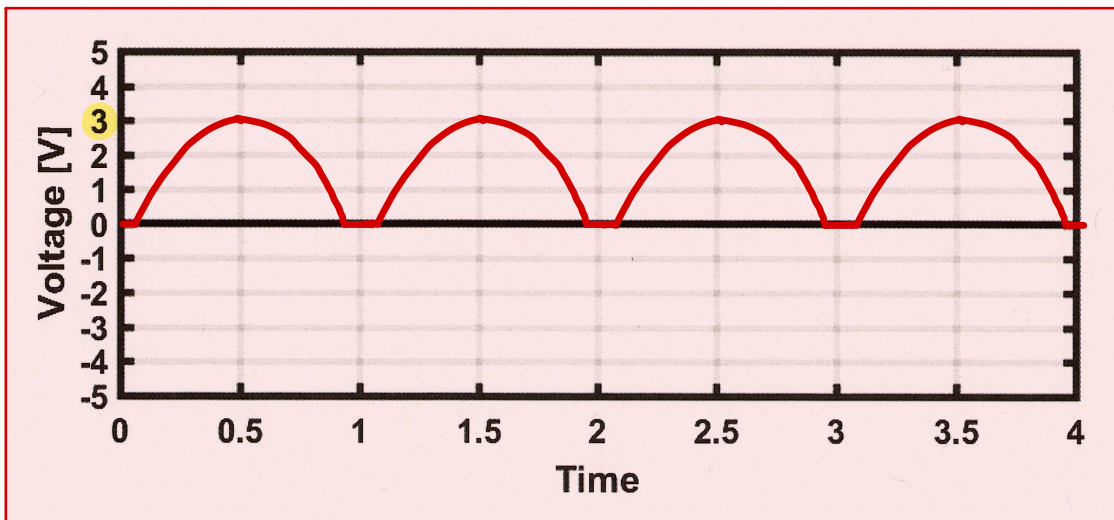
- a) For the above circuit, assuming the diode is ideal with $V_{Turn-on} = 0V$, sketch the voltage across the resistor. (3 pts)

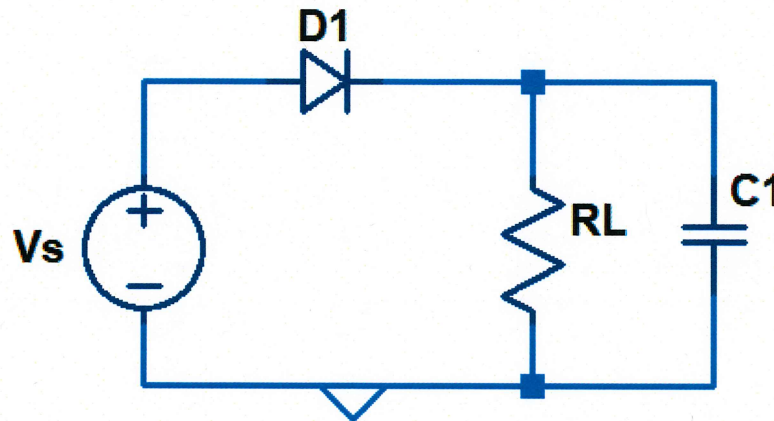


- b) For the same circuit on the previous page (part a circuit), assuming the diode is ideal with $V_{Turn-on} = 1V$, sketch the voltage across the resistor. (3 pts)



- c) For the above circuit, assuming the diodes are ideal with $V_{Turn-on} = 1V$, sketch the voltage across the resistor. (5 pts)





- d) Two tests are implemented on the above circuit, with different capacitors added in parallel with the load resistor. One of the capacitors has a much larger capacitance. Sketch the approximate voltage across the load resistor for both tests on the same axes (you should have two sketches). Indicate which plot corresponds to the smaller capacitor and which plot corresponds to the larger capacitor. Assume $V_{Turn-on} = 0V$ for the diode. The source voltage is included for reference, with more periods shown. (5 pts)

