Name SOLUTIONS

1. ________ (4 pts)

2. ________ (12 pts)

3. ________ (12 pts)

4. ________ (12 pts)

5. ________ (10 pts)

Total ________________

For partial credit on some questions, you may want to re-draw/label circuit diagrams to clarify your answers.

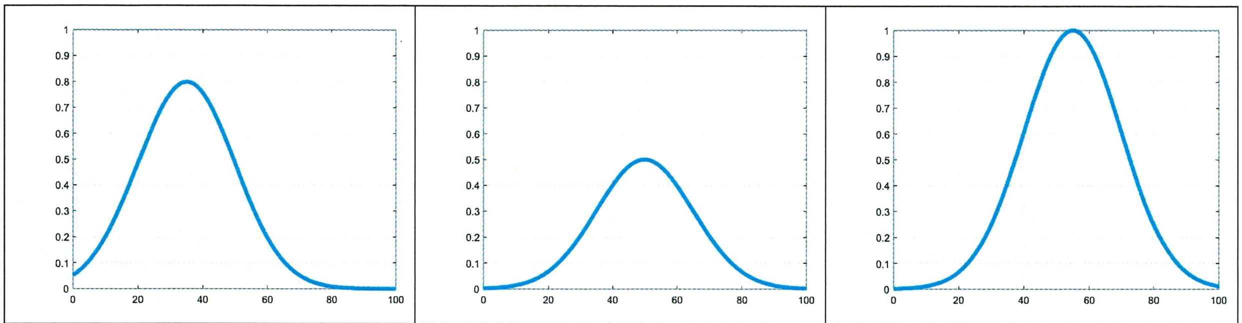
Show all of your work. Use the backs of pages if there is not enough room on the front.

Many problems can be solved using more than one method. Check your answers by using a second method.

At least skim through the entire quiz before you begin and then start with the problems you know best.

The proctor will only answer clarification questions where wording is unclear or where there may be errors/typos. No other questions will be responded to.

Problem 1 (12 Points) – Concept Questions



Plot A | Plot B | Plot C

Which of the above Guassian distributions has the largest standard deviation? The horizontal and vertical axis are the same for each figure. (Circle one answer).

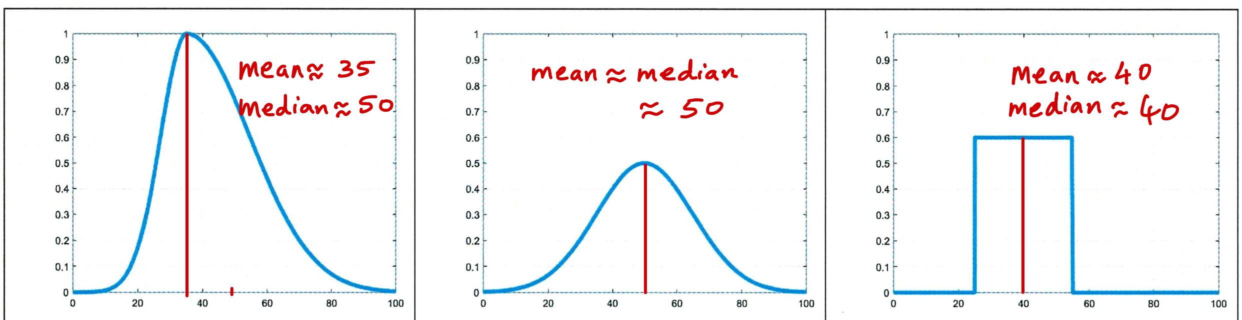
a) Plot A

b) Plot B

c) Plot C

d) They are about the same

both accepted.



Plot A | Plot B | Plot C

Which of the above plots have a mean and median that are approximately the same? (Circle all correct answers).

a) Plot A

b) Plot B

c) Plot C

d) None of the above (and aside)

Problem 2 (12 Points) – Determinants

Find the determinant of the following matrices. You must show your work to receive credit.

a) (2 pts) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A$

$\det(A) = 1 - 0 = \boxed{1}$

b) (2 pts) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = B$

$\det(B) = 4 - 4 = \boxed{0}$

c) (3 pts) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} = C$

$\det(C) = 1(2) - 0(1) + 1(1) = \boxed{3}$

d) (3 pts) $\begin{bmatrix} 1 & 3 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} = D$

$\det(D) = 1(0) - 3(0) + 0(1-2) = \boxed{0}$

e) Based on your above answers, which of the above matrices have an inverse? (Circle all correct answers.) (2 pts)

Part a Part b Part c Part d

Problem 3 (12 Points) – Matrix Solutions

let $Z = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right]$

a) Using techniques applied in the laboratories, determine the inverse of the above matrix. You must show your work to receive credit. There is more than one method and you can use any method. (4 pts)

$$Z^{-1} = \frac{1}{\det(Z)} \operatorname{adj}(Z)$$

$$\det(Z) = 1 - 4 = -3$$

$$\operatorname{adj}(Z) = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow Z^{-1} = -\frac{1}{3}\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -0.33 & 0.667 \\ 0.667 & -0.33 \end{bmatrix}}$$

Problem 3 (12 Points) – Matrix Solutions

Alternate Solution

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right]$$

a) Using techniques applied in the laboratories, determine the inverse of the above matrix. You must show your work to receive credit. There is more than one method and you can use any method. (4 pts)

Row 2 = Row 2 − 2x Row 1

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -3 & -2 & 1 \end{array}\right]$$

Row 3 = Row 3 / −3

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 2/3 & -1/3 \end{array}\right]$$

Row 1 = Row 1 − 2xRow 2

$$\left[\begin{array}{cc|cc} 1 & 0 & -1/3 & 2/3 \\ 0 & 1 & 2/3 & -1/3 \end{array}\right]$$

$$\Rightarrow \boxed{Z^{-1} = \begin{bmatrix} -0.33 & 0.667 \\ 0.667 & -0.33 \end{bmatrix}}$$

let

$$Z = \left[\begin{array}{ccc|ccc} {}^{a}1 & {}^{b}2 & {}^{c}0 & 1 & 0 & 0 \\ {}^{d}-1 & {}^{e}0 & {}^{f}1 & 0 & 1 & 0 \\ {}^{g}0 & {}^{h}1 & {}^{i}1 & 0 & 0 & 1 \end{array}\right]$$

b) Using techniques applied in the laboratories, determine the inverse of the above matrix. You must show your work to receive credit. There is more than one method and you can use any method. (8 pts)

Matrix of cofactors: $\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}$

$A = -1$ $E = 1$ $I = 2$

$B = -1 \times -1$ $F = 1 \times -1$

$C = -1$ $G = 2$

$D = 2 \times -1$ $H = 1 \times -1$

$$= \begin{bmatrix} -1 & 1 & -1 \\ -2 & 1 & -1 \\ 2 & -1 & 2 \end{bmatrix}$$

$$\text{adj}(Z) = \text{Transpose of matrix of Cofactors} = \begin{bmatrix} -1 & 1 & -1 \\ -2 & 1 & -1 \\ 2 & -1 & 2 \end{bmatrix}^T = \begin{bmatrix} -1 & -2 & 2 \\ 1 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$\det(Z) = aA + bB + cC = -1 + 2 + 0 = 1$$

$$Z^{-1} = \frac{1}{\det(Z)} \text{adj}(Z) = \boxed{\begin{bmatrix} -1 & -2 & 2 \\ 1 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Alternate Solution

b) Using techniques applied in the laboratories, determine the inverse of the above matrix. You must show your work to receive credit. There is more than one method and you can use any method. (8 pts)

$R2 = R2 + R1$ (1)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R3 = R3 - \frac{1}{2} R2$ (2)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0.5 & -0.5 & -0.5 & 1 \end{array}\right]$$

$R2 = R2 - 2R3$ (3)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 & 2 & -2 \\ 0 & 0 & 0.5 & -0.5 & -0.5 & 1 \end{array}\right]$$

$R1 = R1 - R2$ (4)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -2 & 2 \\ 0 & 2 & 0 & 2 & 2 & -2 \\ 0 & 0 & 0.5 & -0.5 & -0.5 & 1 \end{array}\right]$$

$R2 = R2/2$ and $R3 = R3 \times 2$ (5)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -2 & 2 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 2 \end{array}\right]$$

$$\boxed{Z^{-1} = \begin{bmatrix} -1 & -2 & 2 \\ 1 & 1 & -1 \\ -1 & -1 & 2 \end{bmatrix}}$$

Problem 4 (12 Points) – Linear Regression

a) (3 pts) Using linear regression analysis, find the best fit line for the points (1,2), (3,3), (5,4)
Express your answer in the form $y = mx + b$

$x_1 = 1$, $x_2 = 3$, $x_3 = 5$ $\Rightarrow$ $\bar{x} = \frac{1}{3}\sum_{i=1}^{3} x_i = \boxed{3}$

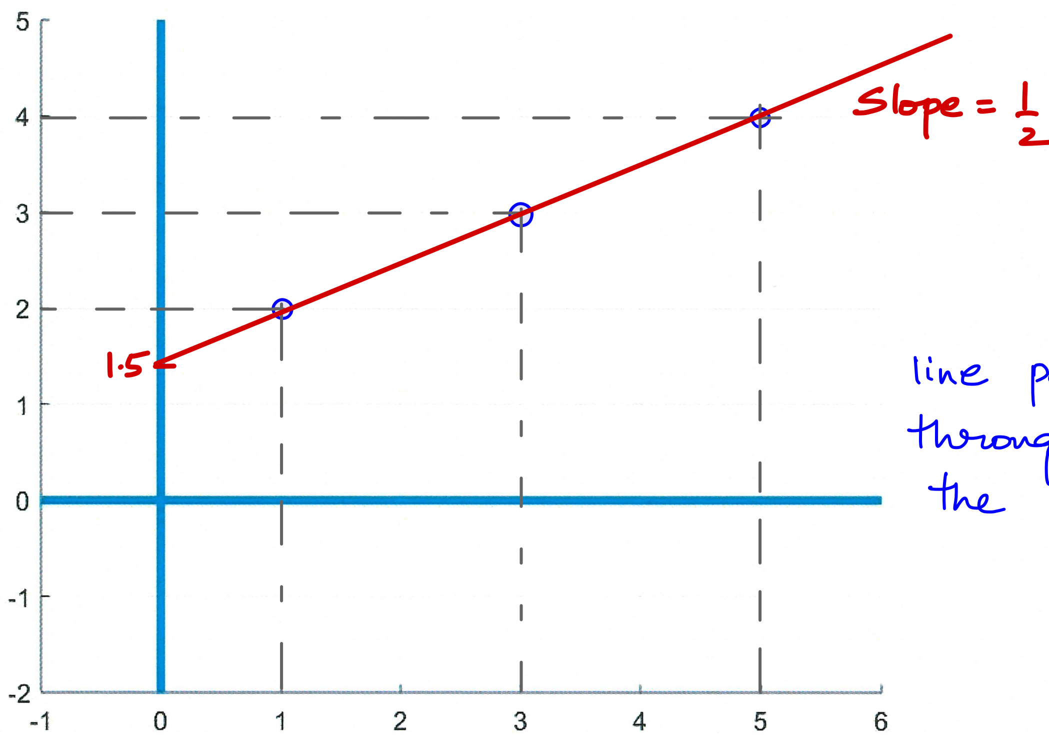
$y_1 = 2$, $y_2 = 3$, $y_3 = 4$ $\Rightarrow$ $\bar{y} = \frac{1}{3}\sum_{i=1}^{3} y_i = \boxed{3}$

$$m = \frac{\sum_{i=1}^{3}(x_i y_i - x_i \bar{y})}{\sum_{i=1}^{3}(x_i^2 - x_i \bar{x})} = \boxed{\frac{1}{2}}$$

$$\boxed{y = \frac{1}{2}x + \frac{3}{2}}$$

$$b = \bar{y} - m\bar{x} = \boxed{\frac{3}{2}}$$

(2 pts) On the following figure, plot the data points and your best fit line.



line passes through the points!

b) (3 pts) Using linear regression analysis, find the best fit line for the points (1,1), (3,3), (3,5)
Express your answer in the form $y = mx + b$

$$x_1 = 1,\quad x_2 = 3,\quad x_3 = 3 \Rightarrow \bar{x} = \frac{1}{3}\sum_{i=1}^{3} x_i = \boxed{2.33}$$

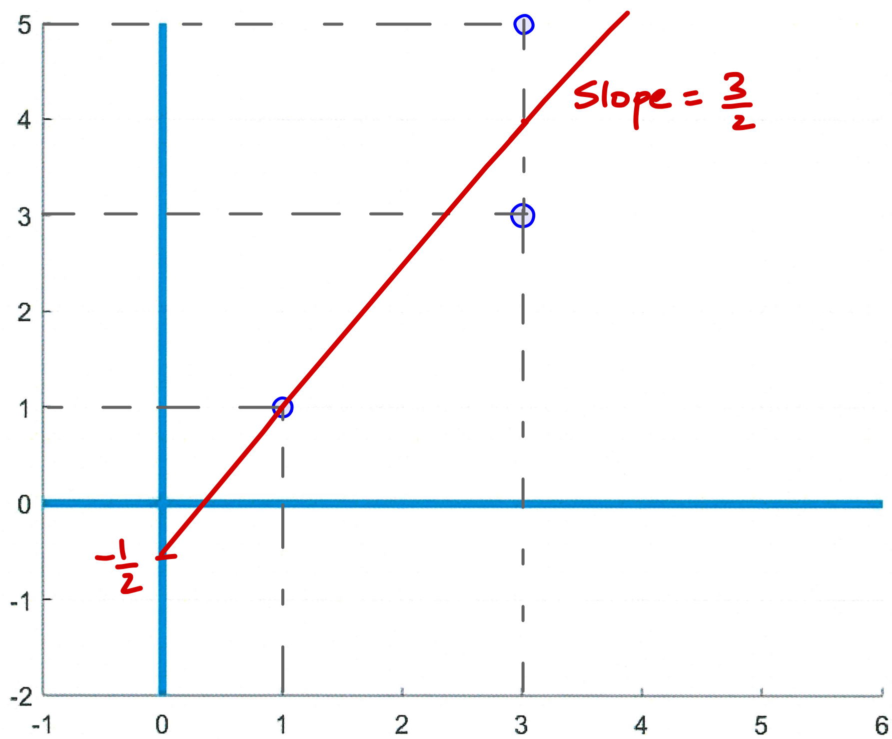
$$y_1 = 1,\quad y_2 = 3,\quad y_3 = 5 \Rightarrow \bar{y} = \frac{1}{3}\sum_{i=1}^{3} y_i = \boxed{3}$$

$$m = \frac{\sum_{i=1}^{3}(x_i y_i - x_i \bar{y})}{\sum_{i=1}^{3}(x_i^2 - x_i \bar{x})} = \boxed{\frac{3}{2}}$$

$$\boxed{y = \frac{3}{2}x - \frac{1}{2}}$$

$$b = \bar{y} - m\bar{x} = \boxed{\frac{-1}{2}}$$

(2 pts) On the following figure, plot the data points and your best fit line.



c) (2 pts) When considering the parts a and b, which linear regression line (best fit line) has the largest correlation coefficient? (Circle one)

Part a Part b

What is the correlation coefficient for your circled answer? 1.0

Perfect fit!

Problem 5 (10 Points) – Data and Curve Fitting

The following data was obtained from some circuit. The source voltages, VA and VB were varied and an output voltage was measured.

V_A	V_B		V_{out}
0	1		1
1	1		1.5
1	2		2
2	3		3

Using **matrix mathematics**, find the coefficients, *a* and *b*, that give the best fit relationship between the two inputs and the input in the following expression,

$$aV_A + bV_B = V_{out}$$

You must show your work to receive credit. Hint: This problem involves almost all the matrix manipulations we have utilized in recent laboratories.

matrix formulation:

$$\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}_{4\times 2}}_{A} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}_{2\times 1}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \end{bmatrix}_{4\times 1}}_{b}$$

$$x = (A^T A)^{-1} A^T b$$

$$\Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}^T \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}^T \begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \end{bmatrix}$$

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$$\begin{bmatrix} a \\ b \end{bmatrix} = \left(\underset{2\times4}{\begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 2 & 3 \end{bmatrix}} \underset{4\times2}{\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}} \right)^{-1} \underset{2\times4}{\begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 2 & 3 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \end{bmatrix}}$$

$$= \left(\begin{bmatrix} 6 & 9 \\ 9 & 15 \end{bmatrix} \right)^{-1} \underset{2\times4}{\begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 2 & 3 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \end{bmatrix}}$$

$$= \underset{2\times2}{\begin{bmatrix} 1.667 & -1 \\ -1 & 0.667 \end{bmatrix}} \underset{2\times4}{\begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 1 & 2 & 3 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \end{bmatrix}}$$

$$= \underset{2\times4}{\begin{bmatrix} -1 & 0.667 & -0.33 & 0.33 \\ 0.667 & -0.33 & 0.33 & 0 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \end{bmatrix}}$$

$$= \underset{2\times1}{\begin{bmatrix} 0.33 \\ 0.83 \end{bmatrix}}$$

$$\boxed{\begin{aligned} a &= 0.33 \\ b &= 0.83 \end{aligned}}$$