

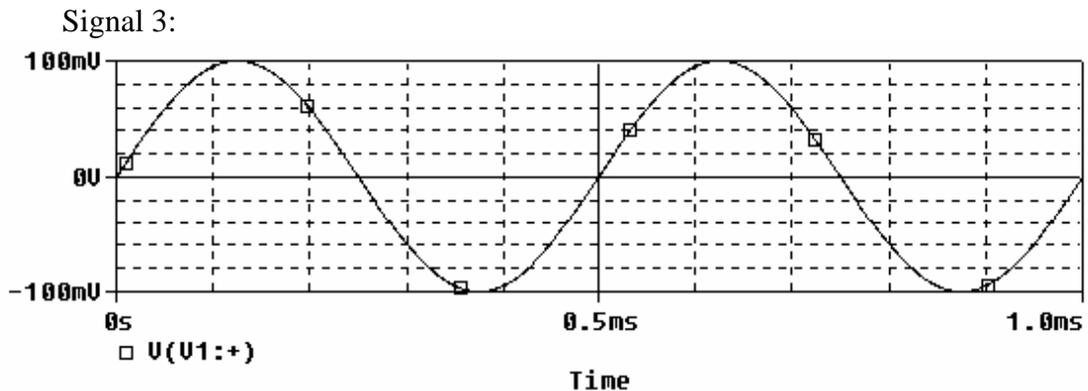
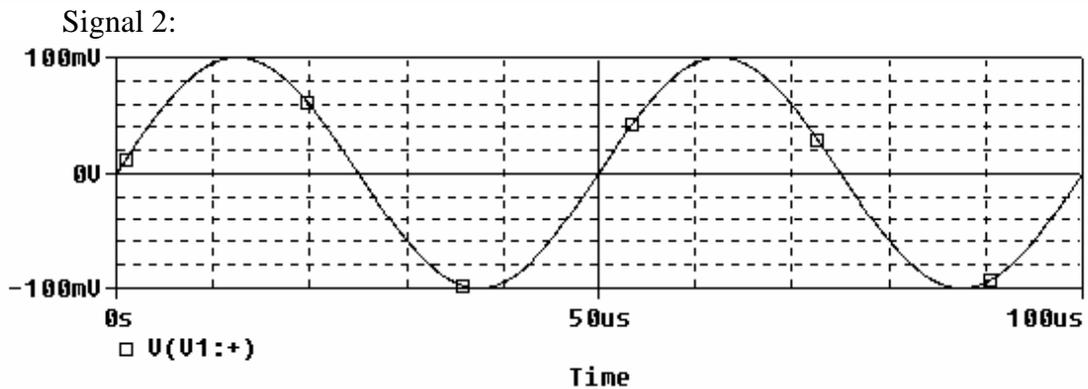
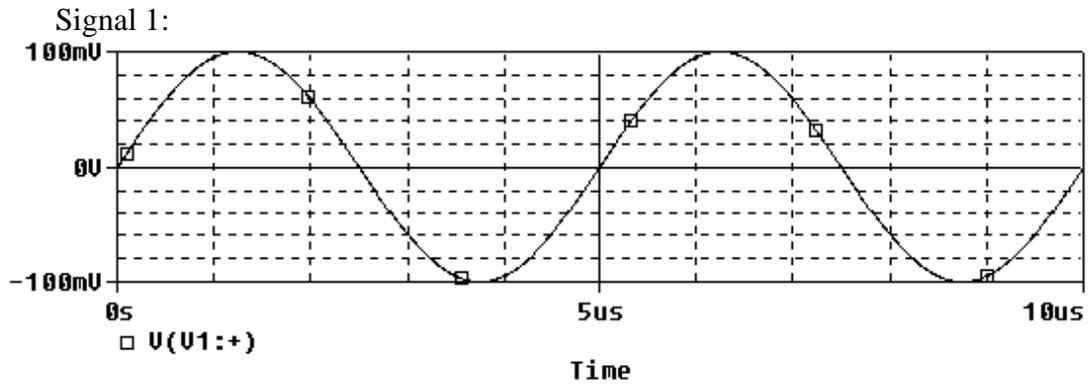
Questions on Filters

These should help prepare you for question 2 of quiz 1

Fall 2004 (See Fall 2004 question I in Transfer Functions for first part)

Question II – Filters (16 points)

Consider the following three input signals:



1. What is the frequency of each of the signals above (in Hertz)? (6 points)

Signal 1:

Signal 2:

Signal 3:

2. What type of filter is circuit A from question I ? (2 point)

3. What type of filter is circuit B from question I ? (2 point)

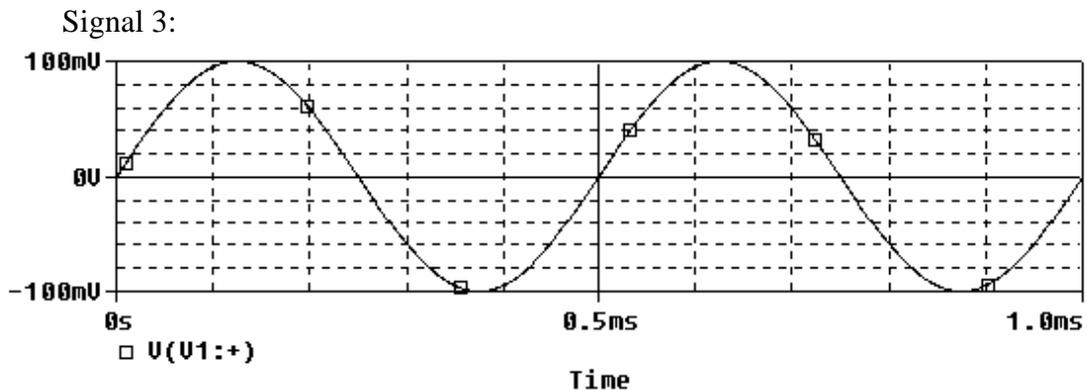
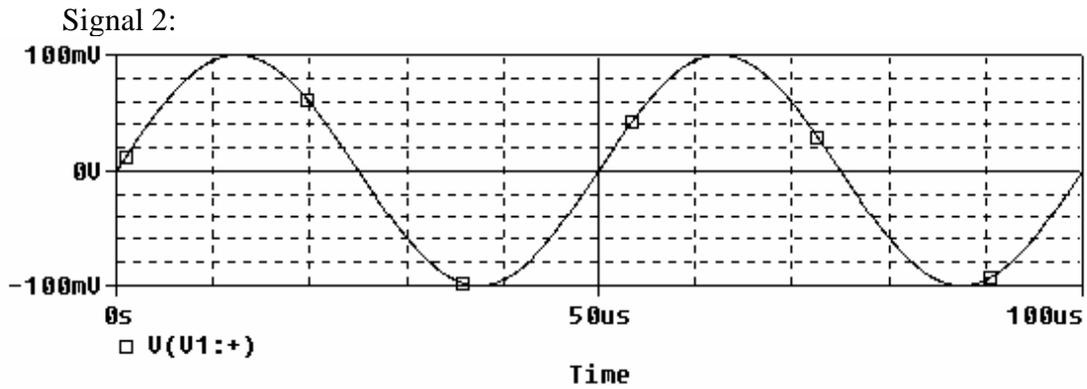
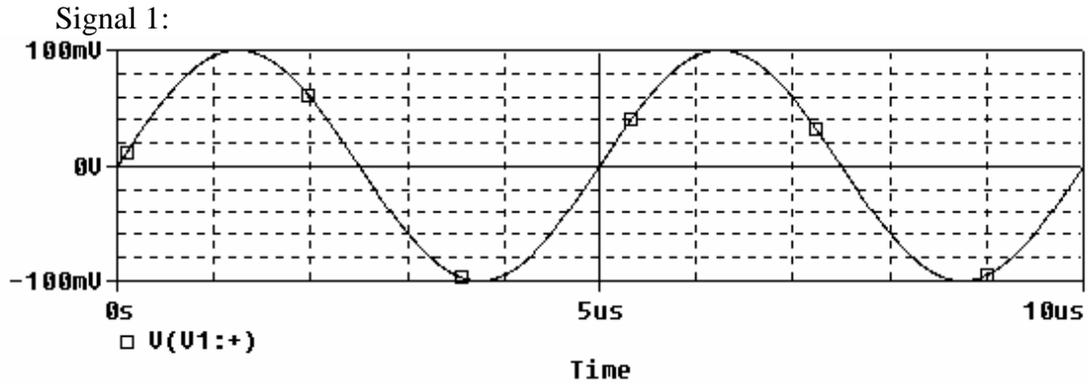
4. Fill out the following chart. Enter “lower” if the amplitude of the output of the given circuit will be substantially lower than the input amplitude. Enter “higher” if the amplitude of the output of the given circuit will be substantially higher than the input amplitude. Enter “same” if the amplitude of the output of the given circuit will be about the same as the input amplitude. Note that the circuits are the ones you analyzed in question I (with the component values we gave you) and the signals are those pictured on the previous page. You can assume the filter transitions are close to ideal. (6 points)

	Signal 1	Signal 2	Signal 3
Circuit A			
Circuit B			

Fall 2004 solution

Question II – Filters (16 points) (See Fall 2004 question I in Transfer Functions for first part)

Consider the following three input signals:



5. What is the frequency of each of the signals above (in Hertz)? (6 points)

Signal 1: $T = 5\mu\text{s}$ $f = 200,000\text{ Hz}$

Signal 2: $T = 50\mu\text{s}$ $f = 20,000\text{ Hz}$

Signal 3: $T = 0.5\text{ms}$ $f = 2000\text{ Hz}$

6. What type of filter is circuit A from question I ? (2 point)

High pass filter

7. What type of filter is circuit B from question I ? (2 point)

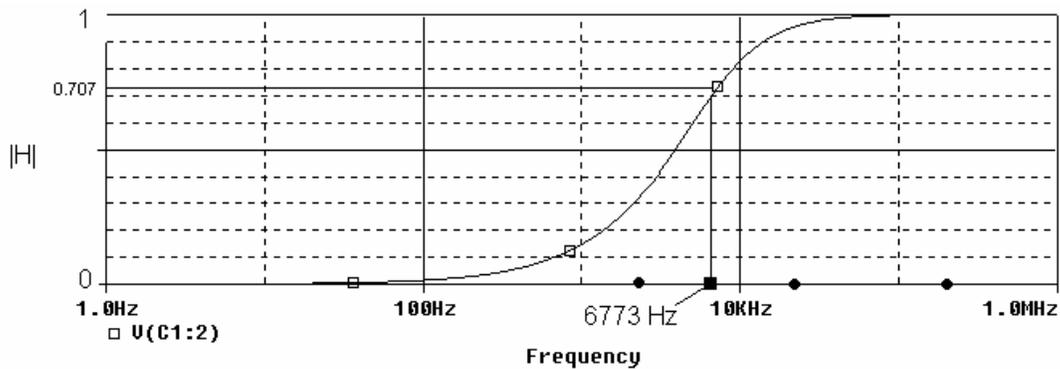
Low pass filter

8. Fill out the following chart. Enter “lower” if the amplitude of the output of the given circuit will be substantially lower than the input amplitude. Enter “higher” if the amplitude of the output of the given circuit will be substantially higher than the input amplitude. Enter “same” if the amplitude of the output of the given circuit will be about the same as the input amplitude. Note that the circuits are the ones you analyzed in question I (with the component values we gave you) and the signals are those pictured on the previous page. You can assume the filter transitions are close to ideal. (6 points)

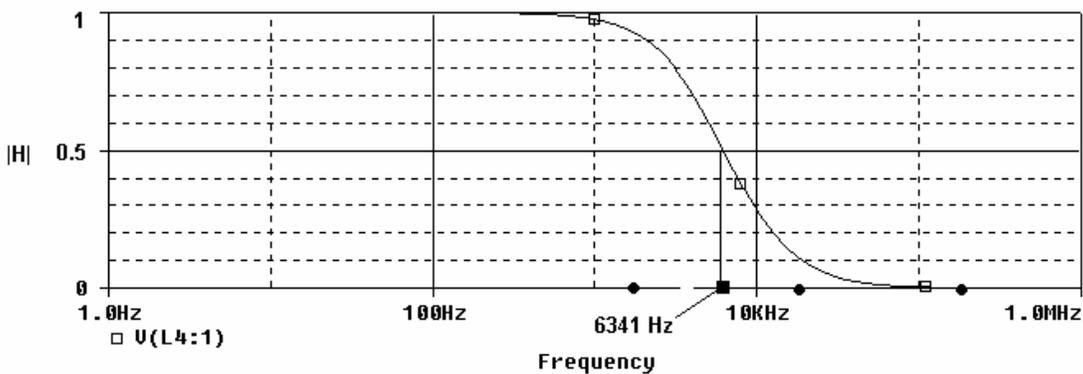
	200K	20K	2K
	Signal 1	Signal 2	Signal 3
Circuit A	same	same	lower
Circuit B	lower	lower	same

(see dots marked on plot below from TransferFunctions.pdf)

Circuit A:

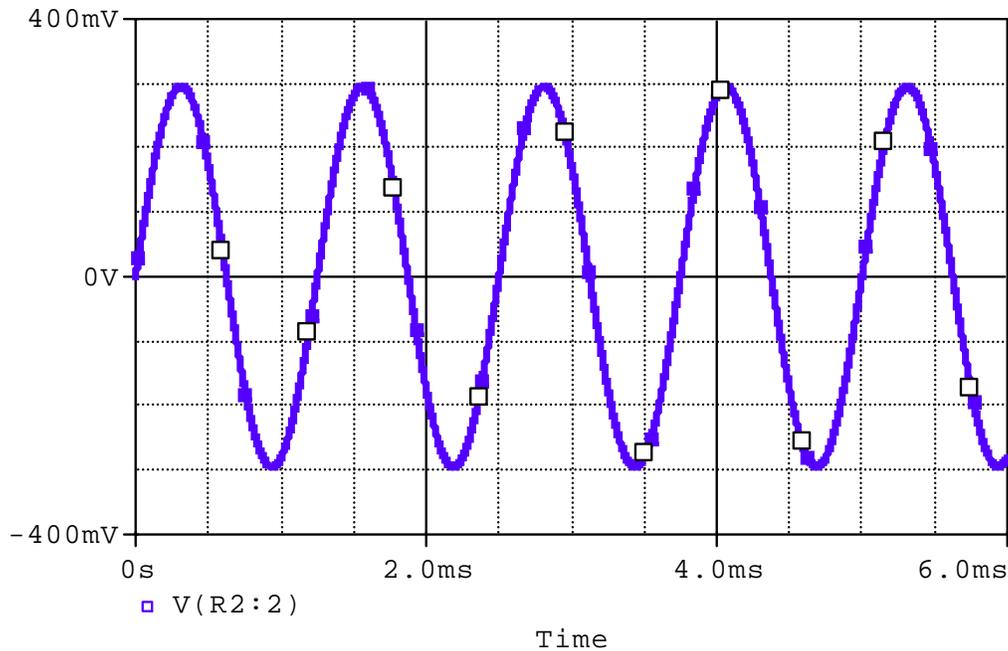


Circuit B:



3. Is this a high pass filter, a band pass filter, a low pass filter or a band reject filter? Explain your answer. (4 points)

4. Given the following source input:



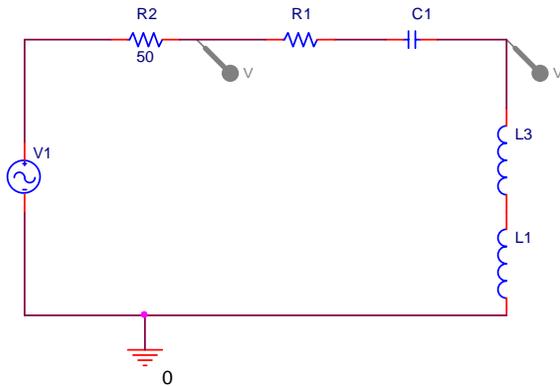
Write the mathematical expression for V_{in} in the form: $V_{in}=A\sin(\omega t)$ (4 points)

5. If $L_1=2\text{ms}$, $L_2=2\text{ms}$, $C_1=0.5\mu\text{s}$ and $R_1=3\text{K}$, what is the resonant frequency of this circuit in Hertz? (4 points)

6. Create a rough sketch of the magnitude of the transfer function of this circuit as a function of frequency. You need only show the general shape. Indicate on the graph where the resonant frequency and input frequency (from part 4) are located. Please give a numerical value. (4 points)

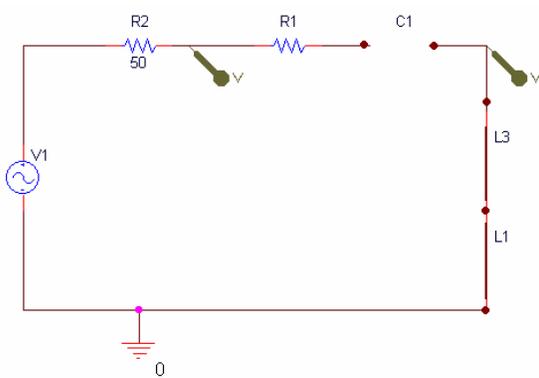
7. Given your knowledge of corner and resonant frequencies, will V_{out} have a greater, equal or lesser value than V_{in} when V_{in} is equal to the input pictured in part 4? Explain your answer. (3 points)

Spring 2004 solution
Question 3 – Filters (25 points)

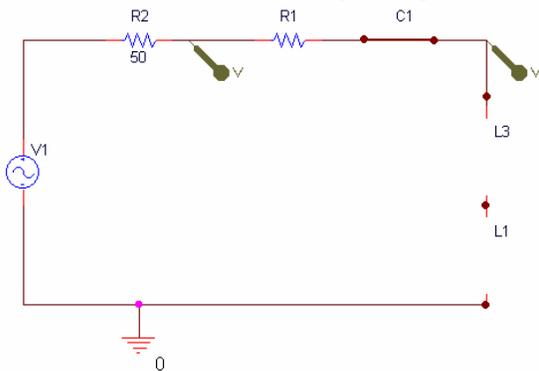


In the circuit above, V1 is a sinusoidal source and R2 is the 50 mohm impedance of the function generator. The remaining four components define some type of filter.

1. Redraw this circuit at low frequencies (3 points)



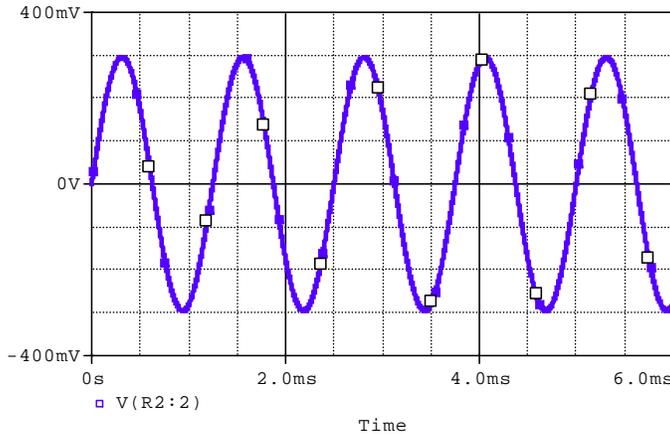
2. Redraw this circuit at high frequencies (3 points)



3. Is this a high pass filter, a band pass filter, a low pass filter or a band reject filter?
 Explain your answer. (4 points)

This is a high pass filter. At low frequencies, the output point is connected to ground (0) [$V_{out}=0$] and at high frequencies the output point is connected to the input voltage and, since there is no current flowing, there is no voltage drop across the resistors [$V_{out}=V_{in}$].

4. Given the following source input:



Write the mathematical expression for V_{in} in the form: $V_{in}=A\sin(\omega t)$ (4 points)

$$A=300mV \quad T=2.5ms/2cycles=1.25ms \quad f=800Hz \quad \omega=2\pi f=1.6K\pi rad/sec=5.03Krad/s$$

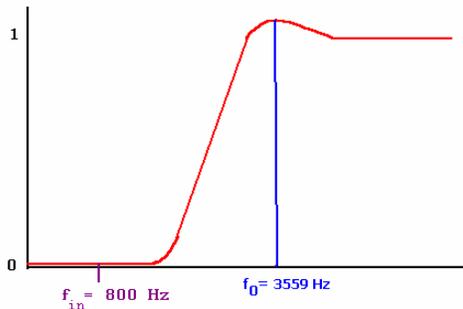
$$v(t) = 300mV \sin(5.03K t)$$

5. If $L_1=2mH$, $L_2=2mH$, $C_1=0.5\mu F$ and $R_1=3K$, what is the resonant frequency of this circuit in Hertz? (4 points)

$$L_{total}=L_1+L_2=2m+2m=4mH \quad \omega_0=1/\sqrt{LC}=1/\sqrt{(4m)(0.5\mu)}=22361rad/s$$

$$f_0=\omega_0/2\pi=3559Hz$$

6. Create a rough sketch of the magnitude of the transfer function of this circuit as a function of frequency. You need only show the general shape. Indicate on the graph where the resonant frequency and input frequency (from part 4) are located. Please give a numerical value. (4 points)

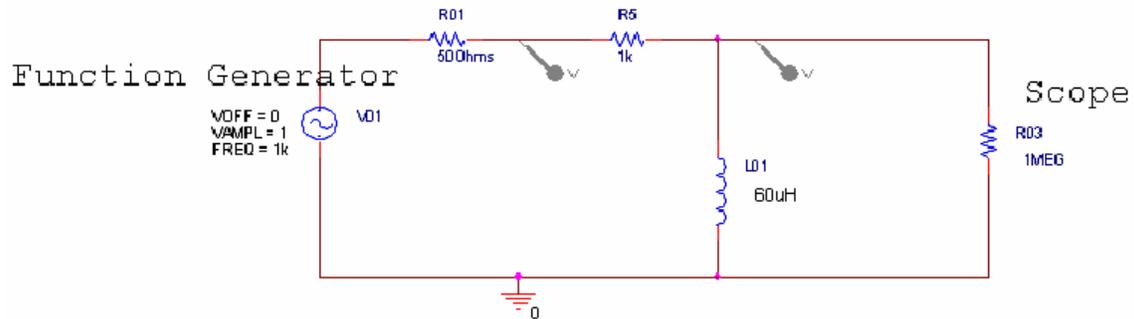


7. Given your knowledge of corner and resonant frequencies, will V_{out} have a greater, equal or lesser value than V_{in} when V_{in} is equal to the input pictured in part 4? Explain your answer. (3 points)

V_{out} will be much less than V_{in} when the input frequency is 800 Hertz. At that value the transfer function is near zero. The output, V_{out} , is equal to the input, V_{in} , times the transfer function. V_{in} times near zero is very small.

Fall 2003

The following circuit consists of a sinusoidal source, an inductor and a resistor.



If V_{in} is the sinusoidal source and V_{out} is the voltage across the inductor, is this configuration a high-pass filter, a low-pass filter, a band-pass filter or a band-reject filter? Explain your answer. (8 points)

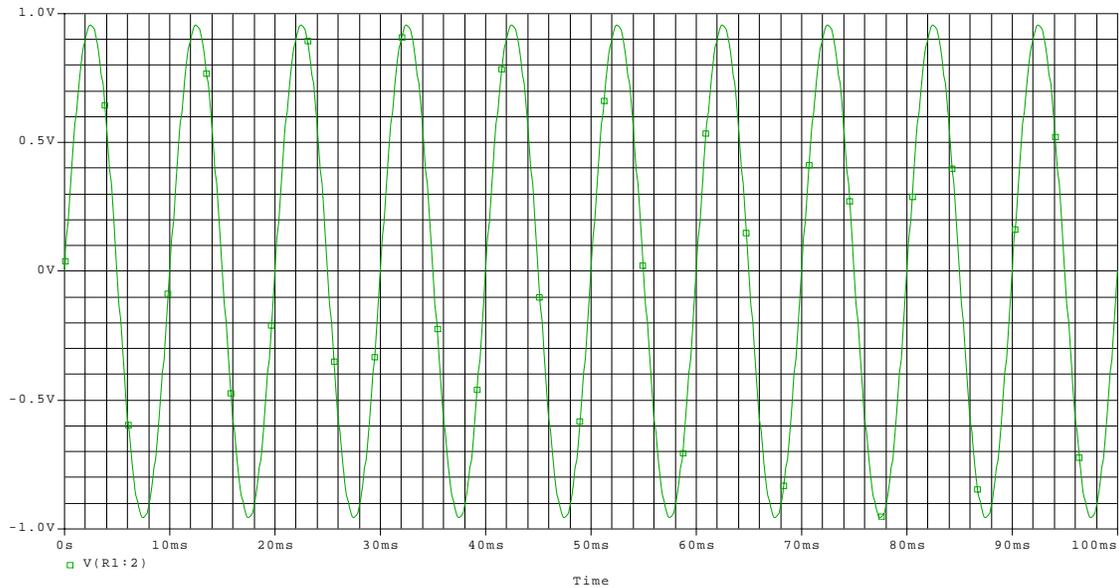
The source is a sinusoidal voltage with some amplitude and frequency. The source voltage, as a function of time, is shown on the next page. Write out the mathematical expression for this voltage function in the form $V_{in} = V_o \sin(\omega t + \phi_o)$. Be sure that you give values for V_o , ω , and ϕ_o . (5 points)

$$V_o =$$

$$\omega =$$

$$\phi_o =$$

$$V_{in} =$$

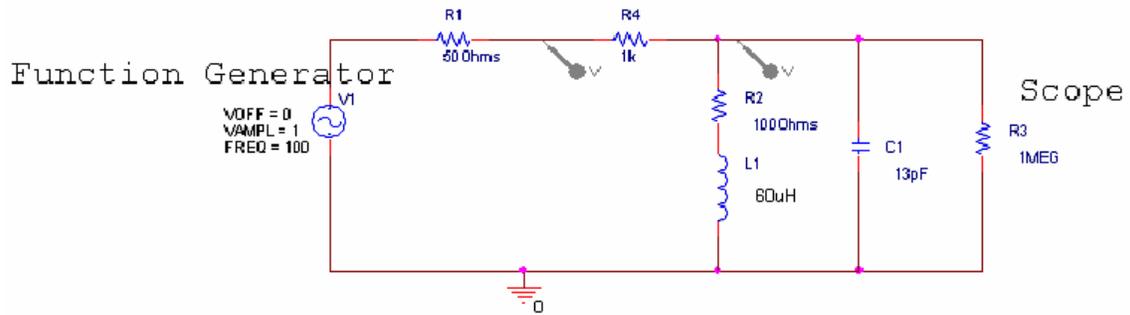


Now that you have determined the magnitude, frequency and phase of the input voltage, you should have some idea of what will happen at the output. From your knowledge of the corner frequency for this circuit, will the output voltage be about the same as the input, substantially smaller or substantially larger than the input? Explain your answer. (5 points)

Would you say that, for this circuit, the frequency of the source is high or low? *Very roughly* sketch the magnitude of the transfer function for this circuit as a function of frequency. You only need to show the general shape of the magnitude, not the phase. Indicate on the graph where the corner frequency is and give the numerical value. Your frequency should cover the entire range that can be measured using the 'scope in the classroom – 1Hz to 60MHz. (6 points)

Extra Credit: (2 points)

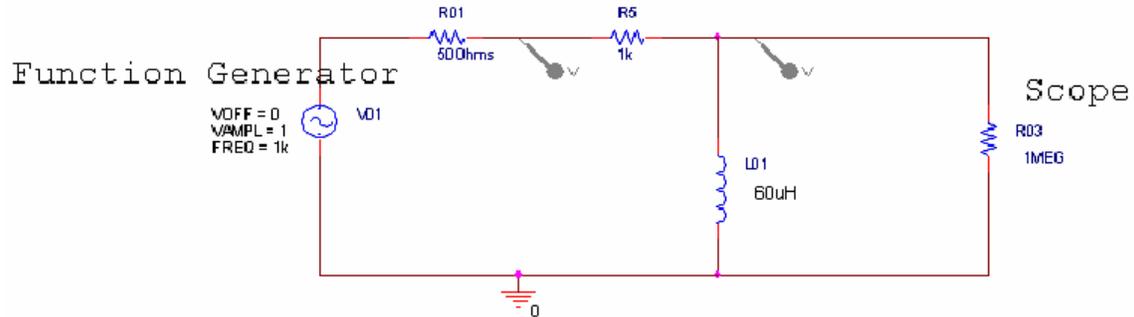
Realistically, the inductor will have finite resistance and the 'scope has an input capacitance. Answer the last question again for the following circuit, where the inductor now has a resistance of 100 Ohms and the 'scope input capacitance is 13 pF.



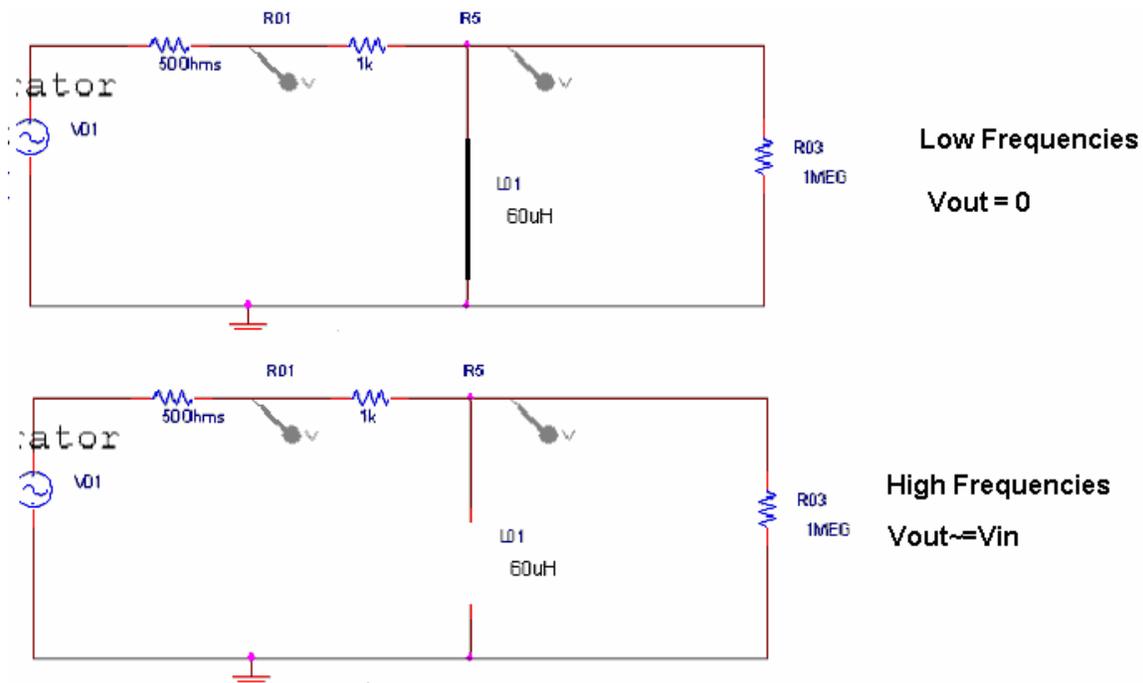
Fall 2003 solution

Question 3 - Filters (25 points)

The following circuit consists of a sinusoidal source, an inductor and a resistor.



If V_{in} is the sinusoidal source and V_{out} is the voltage across the inductor, is this configuration a high-pass filter, a low-pass filter, a band-pass filter or a band-reject filter? Explain your answer. (8 points)



This is a **HIGH PASS FILTER** because it rejects low frequencies (top figure) and passes high frequencies (bottom figure).

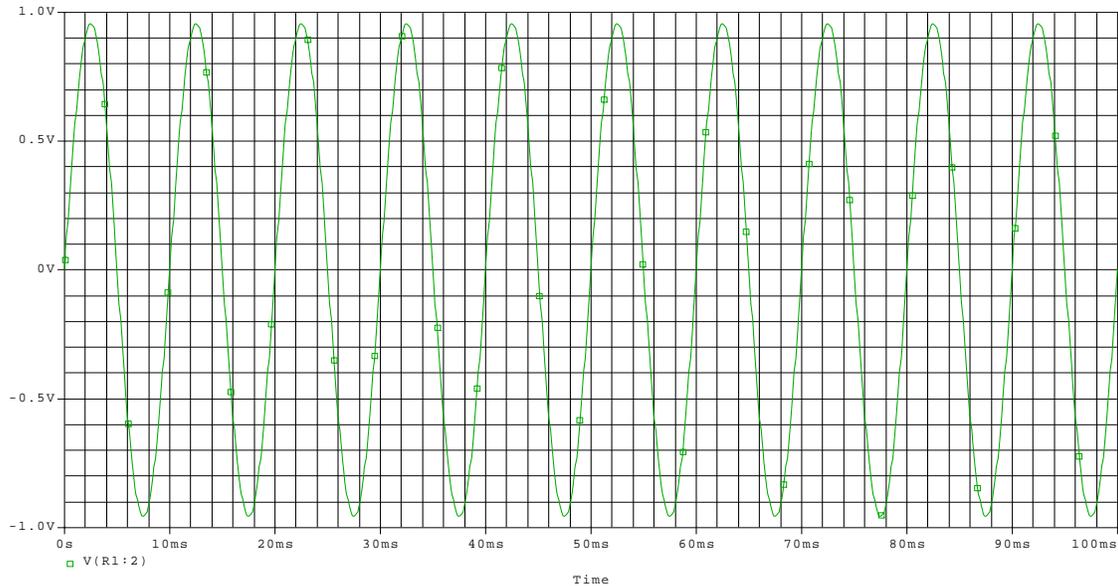
The source is a sinusoidal voltage with some amplitude and frequency. The source voltage, as a function of time, is shown on the next page. Write out the mathematical expression for this voltage function in the form $V_{in} = V_o \sin(\omega t + \phi_o)$. Be sure that you give values for V_o , ω , and ϕ_o . (5 points)

$$V_o = 0.94 \text{ V}$$

$$\omega = 2\pi / T = 2\pi / 10\text{ms} = 2\pi \text{ K} = 628 \text{ rad/sec}$$

$$\phi_o = 0 \text{ rad}$$

$$V_{in} = 0.94 \text{ V} \sin(628 t)$$



Now that you have determined the magnitude, frequency and phase of the input voltage, you should have some idea of what will happen at the output. From your knowledge of the corner frequency for this circuit, will the output voltage be about the same as the input, substantially smaller or substantially larger than the input? Explain your answer. (5 points)

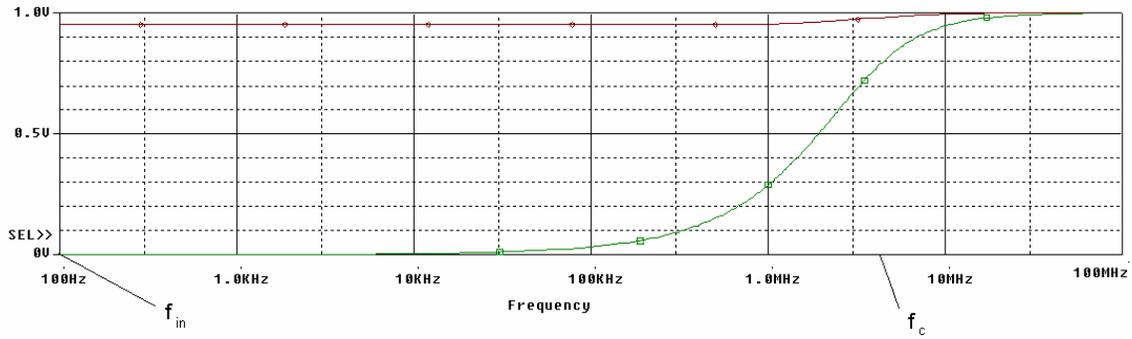
$$\omega_c = R/L = 1\text{K}/60\text{EE}-6 = 1.67\text{EE}+7 \quad \omega_c = 17\text{Meg rad/sec} \quad \omega_{in} = 628\text{rad/sec}$$

Since $\omega_c \gg 628\text{rad/sec}$ and this is a high pass filter, the output at ω_{in} will be substantially smaller than the input.

Would you say that, for this circuit, the frequency of the source is high or low? *Very roughly* sketch the magnitude of the transfer function for this circuit as a function of frequency. You only need to show the general shape of the magnitude, not the phase. Indicate on the graph where the corner frequency is and give the numerical value. Your frequency should cover the entire range that can be measured using the 'scope in the classroom – 1Hz to 60MHz. (6 points)

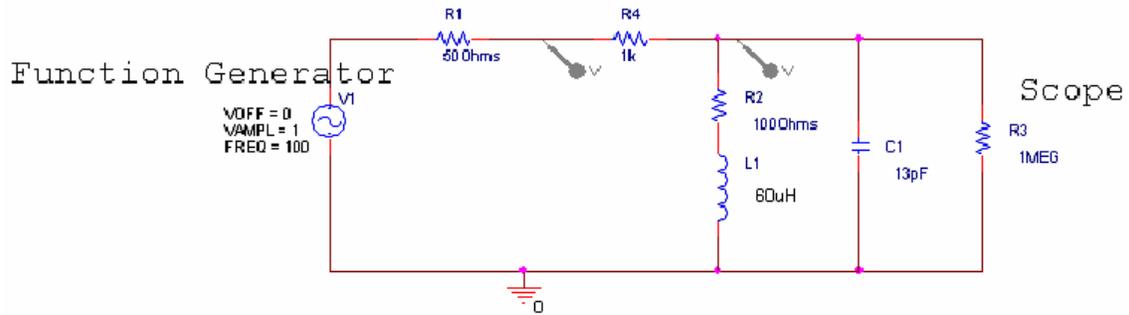
$$f_c = \omega_c / 2\pi = 17\text{Meg} / 2\pi = 2.7\text{MegHz}. \quad f_{in} = \omega_{in} / 2\pi = 628 / 2\pi = 100\text{Hz}$$

The frequency of the source is low.



Extra Credit: (2 points)

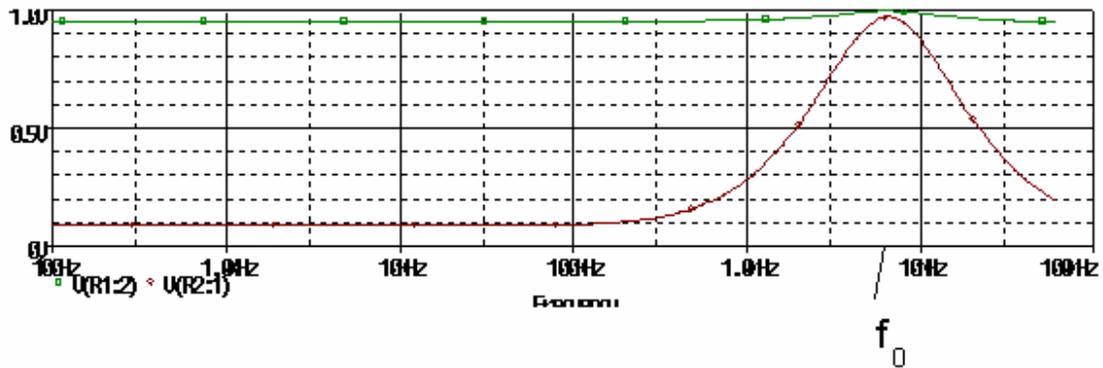
Realistically, the inductor will have finite resistance and the 'scope has an input capacitance. Answer the last question again for the following circuit, where the inductor now has a resistance of 100 Ohms and the 'scope input capacitance is 13 pF.



$$\omega_0 = 1/(LC)^{1/2} = 1/(60 \times 10^{-6} \times 13 \times 10^{-12})^{1/2} = 3.6 \times 10^7 \quad \omega_0 = 36 \text{ Meg rad/sec}$$

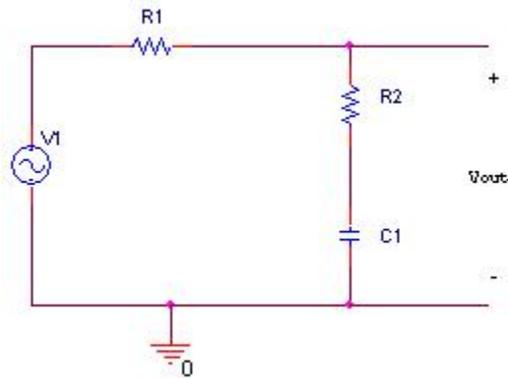
$$f_0 = \omega_0 / 2\pi = 36 \text{ Meg} / 2\pi = 5.7 \text{ MegHz.} \quad f_{in} = \omega_{in} / 2\pi = 628 / 2\pi = 100 \text{ Hz}$$

The input frequency is still low. These additions do not change the transition frequencies very much. (They shouldn't if the devices are well designed.) Below is a picture:



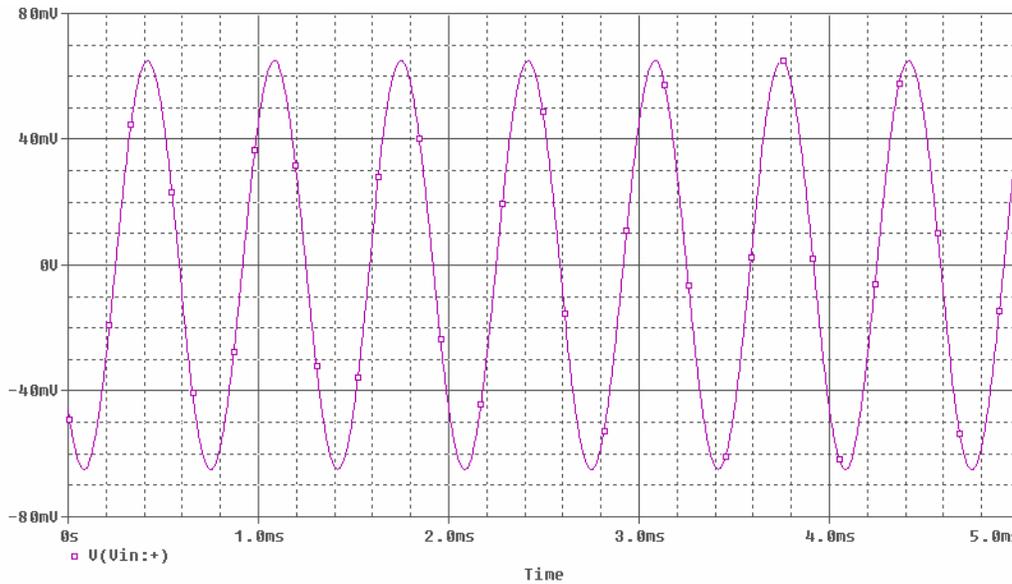
Spring 2003

1. Filters (25 pts)



a) Find the transfer function for the above circuit (6 pts).

b) Assume $R_1=R_2=1K$ and $C_1=1\mu F$, find the magnitude and phase of the transfer function at low and high frequencies (6 pts).



f) Write the mathematical expression for v_{in} in the form $v_{in} = v_0 \sin(\omega t + \phi)$. Provide values for v_0 , ω , and ϕ . (6 pts)

g) What is the phasor \vec{V}_1 for v_{in} ?

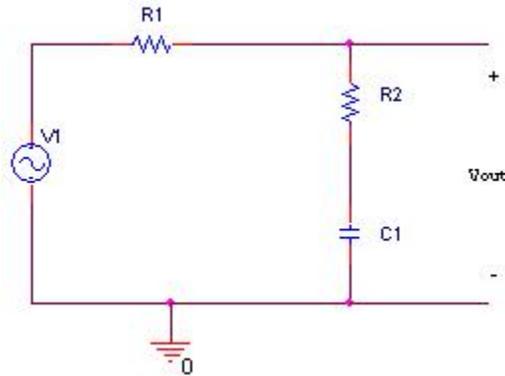
Hint: $\vec{V} = Ae^{j\omega t + \phi}$ or $\vec{V} = A\cos(\omega t + \phi) + jA\sin(\omega t + \phi)$ (4 pts)

h) What type of circuit is this? (3 pts)?

- a) high pass filter
- b) low pass filter
- c) band pass filter
- d) band reject filter
- e) none of the above

Spring 2003 solution

2. Filters (25 pts)



a) Find the transfer function for the above circuit (6 pts).

$$H = \frac{R2 + \frac{1}{j\omega C1}}{R1 + R2 + \frac{1}{j\omega C1}}$$

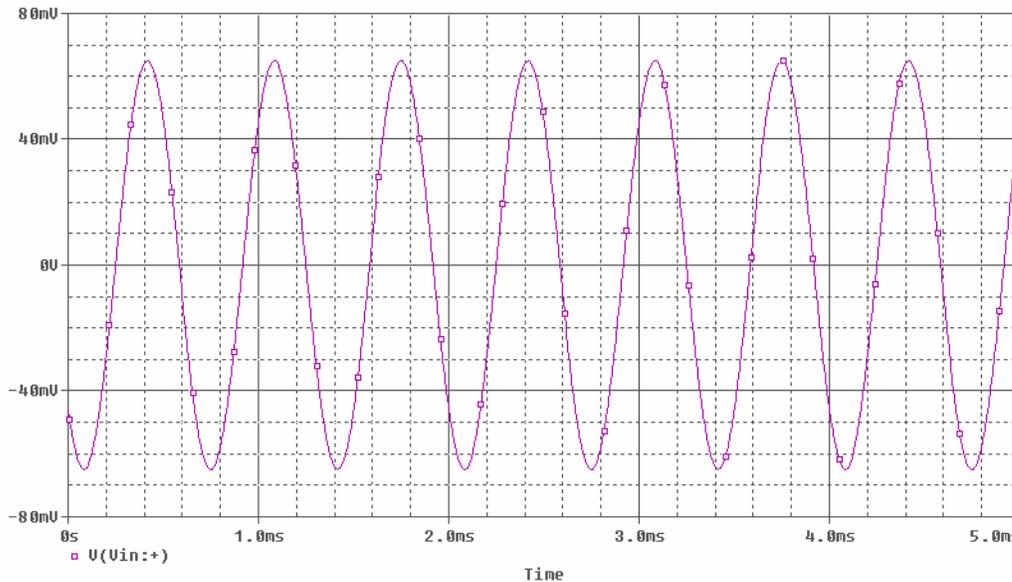
$$H = \frac{j\omega R2 C1 + 1}{j\omega(R1 + R2)C1 + 1}$$

b) Assume $R1=R2=1K$ and $C1=1\mu F$, find the magnitude and phase of the transfer function at low and high frequencies (6 pts).

$$H_{LO}(j\omega) = 1 \quad \underline{|H_{LO}| = 1} \quad \underline{\angle H_{LO} = 0 \text{ radians}}$$

$$H_{HI}(j\omega) = \frac{j\omega R2 C1}{j\omega(R1 + R2)C1} = \frac{R2}{R1 + R2}$$

$$H_{HI}(j\omega) = \frac{1}{2} \quad \underline{|H_{HI}| = \frac{1}{2}} \quad \underline{\angle H_{HI} = 0 \text{ radians}}$$



f) Write the mathematical expression for v_{in} in the form $v_{in} = v_0 \sin(\omega t + \phi)$. Provide values for v_0 , ω , and ϕ . (6 pts)

$$f = 3 \text{ cycles} / 2 \text{ ms} = 1.5 \text{ K Hz} \quad \omega = 3\pi \text{ K} = 9.4 \text{ K rad/sec}$$

$$\phi = -\omega t_0 \quad \phi = -(9.4 \text{ K})(0.25 \text{ ms}) = -2.35 \text{ rad}$$

$$v_{in}(t) = 65 \text{ mV} \cos(9.4 \text{ K} t - 2.35)$$

g) What is the phasor \vec{V}_1 for v_{in} ? (4 pts)

$$\vec{V} = 65 \text{ m} e^{9.4 \text{ K} t - 2.35} \quad \text{or} \quad \vec{V} = 65 \text{ m} e^{-2.35}$$

h) What type of circuit is this? (3 pts)?

a) high pass filter

b) low pass filter

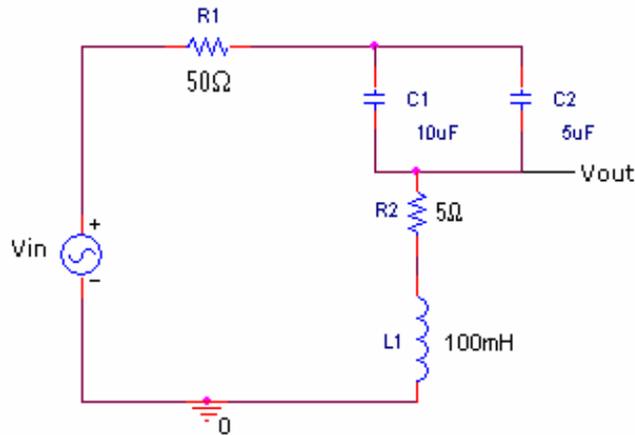
c) band pass filter

d) band reject filter

e) none of the above (It passes all of low and $\frac{1}{2}$ of high. It doesn't really fit any of the classic patterns. You might be tempted to argue that it is some type of band reject filter, but without an inductor, it has no resonance and therefore, cannot have a band.)

Fall 2002

The following circuit analysis consists of a sinusoidal source and a circuit containing resistors, capacitors, and inductors.

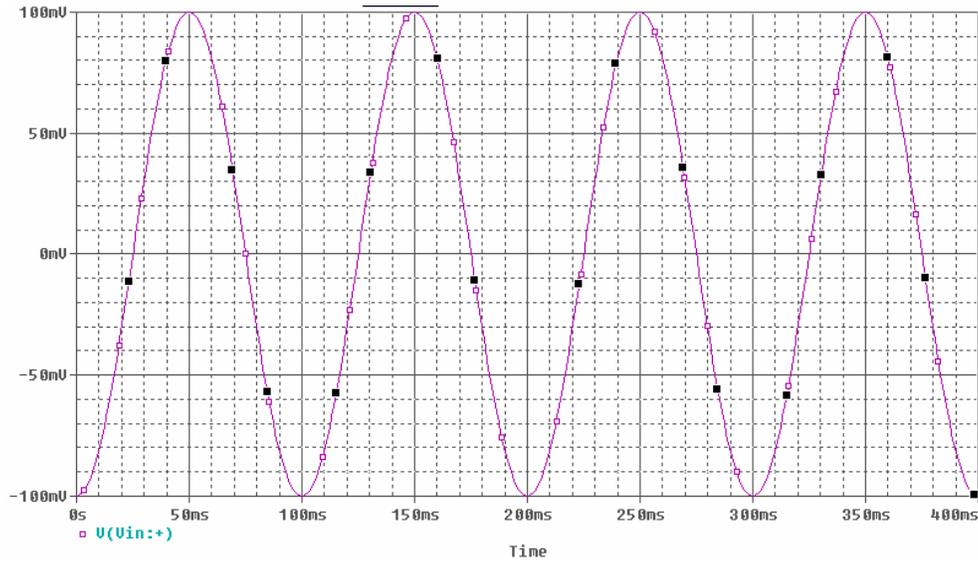


V_{in} is the sinusoidal source and R_1 denotes its internal impedance. Redraw this circuit at high frequencies. (3pt)

Redraw this circuit at low frequencies. (3pt)

Is this a high pass filter, a low pass filter, band pass, or band reject filter? Explain your answer (2pt)

The source voltage V_{in} as a function of time is shown below.



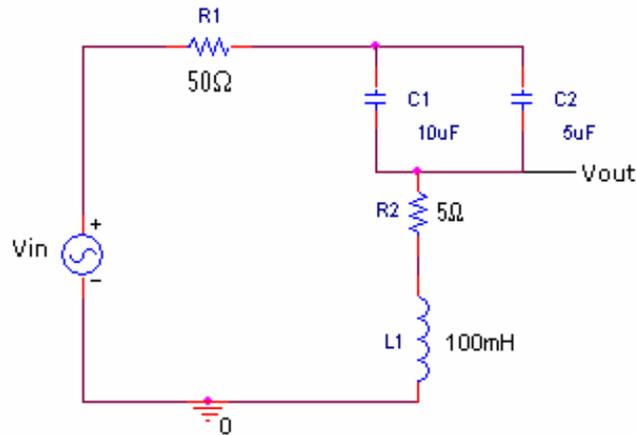
Write the mathematical expression for v_{in} in the form $v_{in} = v_0 \sin(\omega t + \phi)$. Provide values for v_0 , ω , and ϕ . (6pt)

Create a rough sketch of the magnitude of the transfer function for this circuit as a function of frequency. You need only show the general shape, not the phase. Indicate on the graph where the resonant frequency is located. Please give a numerical value. (6pt)

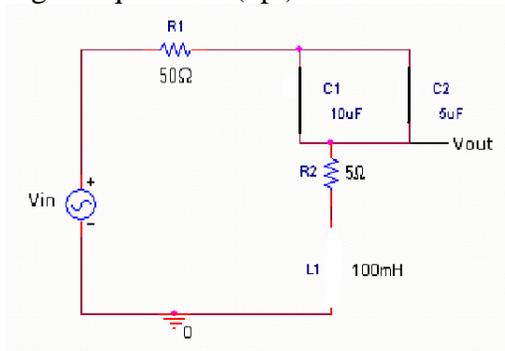
Given your knowledge of corner and resonant frequencies, will v_{out} have greater, equal, or less amplitude than v_{in} ? Explain your answer. (5pt)

Fall 2002 solution

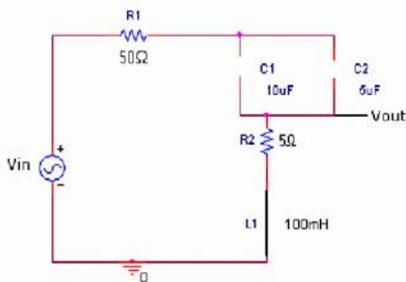
The following circuit analysis consists of a sinusoidal source and a circuit containing resistors, capacitors, and inductors.



V_{in} is the sinusoidal source and R_1 denotes its internal impedance. Redraw this circuit at high frequencies. (3pt)



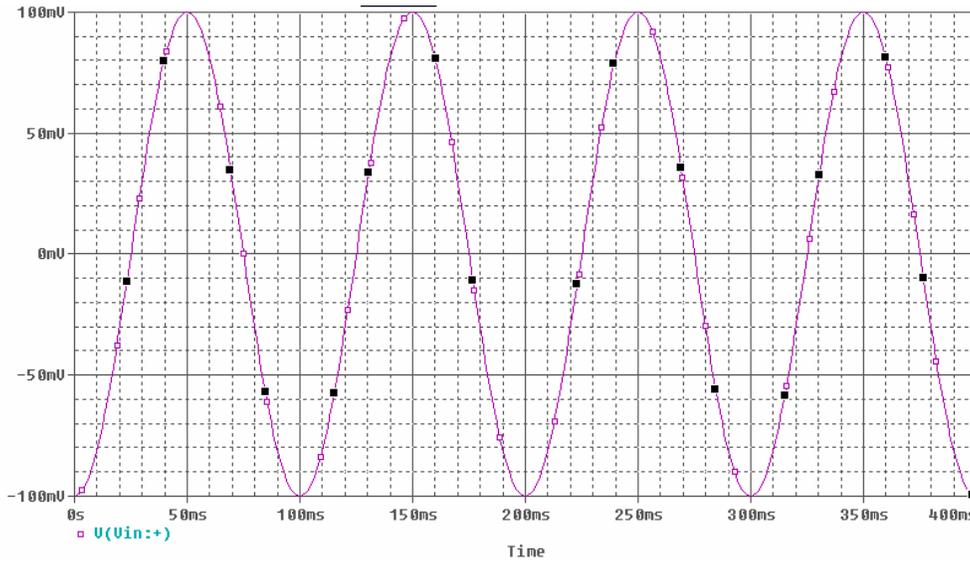
Redraw this circuit at low frequencies. (3pt)



Is this a high pass filter, a low pass filter, band pass, or band reject filter? Explain your answer (2pt)

At low frequencies, the voltage at V_{out} is 0V. At high frequencies, the voltage at V_{out} is V_{in} . This is a high pass filter.

The source voltage V_{in} as a function of time is shown below.



Write the mathematical expression for v_{in} in the form $v_{in} = v_0 \sin(\omega t + \phi)$. Provide values for v_0 , ω , and ϕ . (6pt)

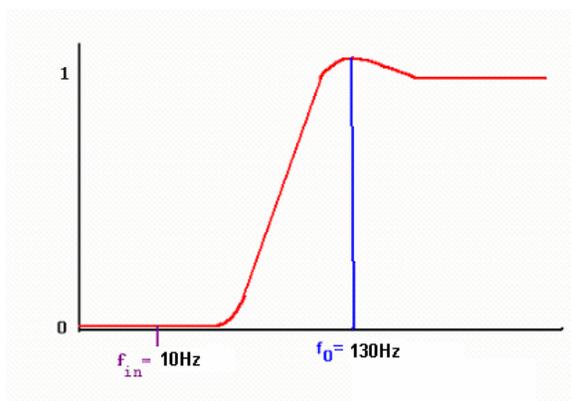
$$f = 1/100m = 10\text{Hz} \quad \omega = 62.8 \text{ rad/sec}$$

$$\phi = -\omega t_0 = -(62.8)(25m) = -1.57 \text{ radians (This is } -\pi/2 \text{ by inspection.)}$$

$$v_{in} = 100\text{mV} \sin(62.8t - 1.57)$$

Create a rough sketch of the magnitude of the transfer function for this circuit as a function of frequency. You need only show the general shape, not the phase. Indicate on the graph where the resonant frequency is located. Please give a numerical value. (6pt)

$$\omega_0 = 1/(\text{sqrt}(L(C_1+C_2))) = 1/(\text{sqrt}((100m)(15\mu))) = 816.5 \text{ rad/sec} \quad f_0 = 130\text{Hz}$$



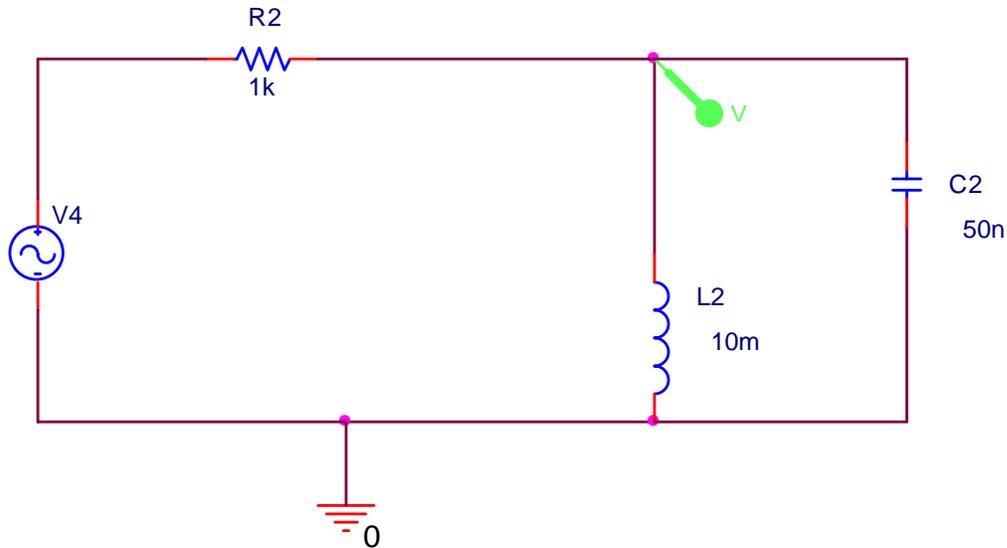
Given your knowledge of corner and resonant frequencies, will v_{out} have greater, equal, or less amplitude than v_{in} ? Explain your answer. (5pt)

v_{out} will have a smaller amplitude than v_{in}

Spring 2002

3. Filters (25 points)

The following circuit consists of a sinusoidal source, an inductor, a capacitor and a resistor.



If v_{in} is the sinusoidal source (including the 50 ohm internal impedance) and v_{out} is the voltage across the inductor, is this configuration a high pass filter, a low pass filter, a band pass filter or a band reject filter? Explain your answer.

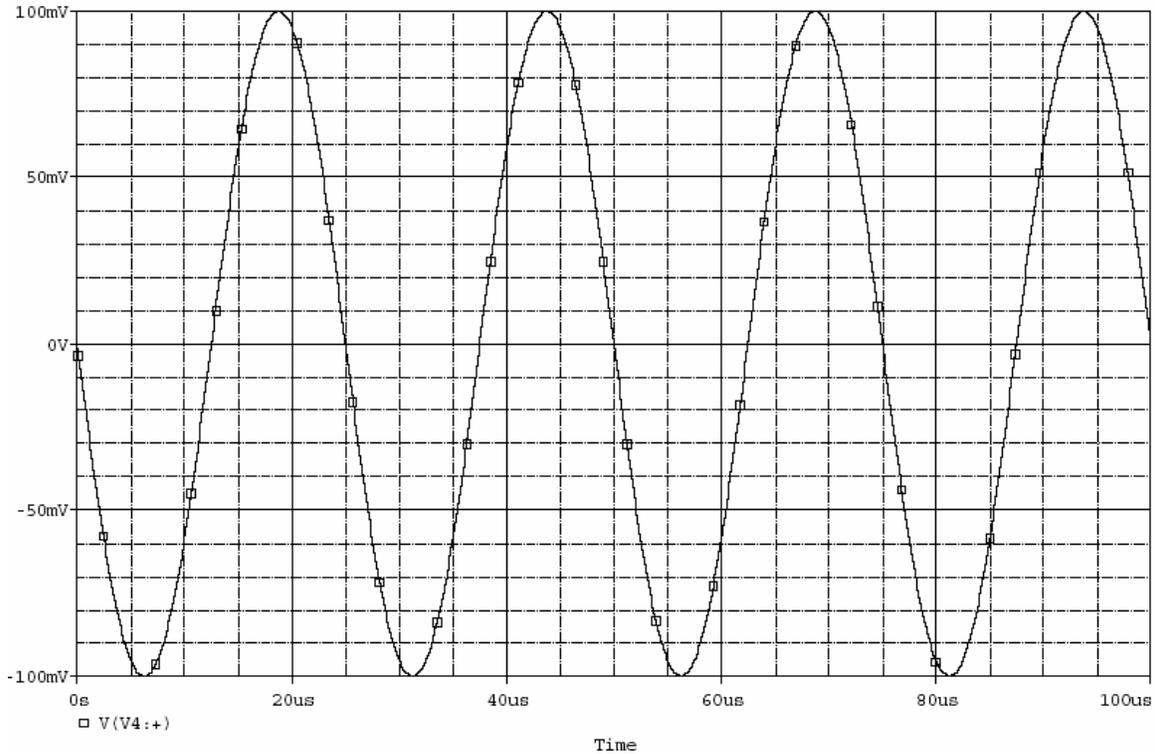
The source is a sinusoidal voltage with some amplitude and frequency. The source voltage, as a function of time, is shown on the next page. Write out the mathematical expression for this voltage function in the form $v_{in} = v_0 \sin(\omega t + \phi_0)$. Be sure that you give values for v_0 , ω , and ϕ_0 .

$v_0 =$

$\omega =$

$\phi_0 =$

$v_{in} =$



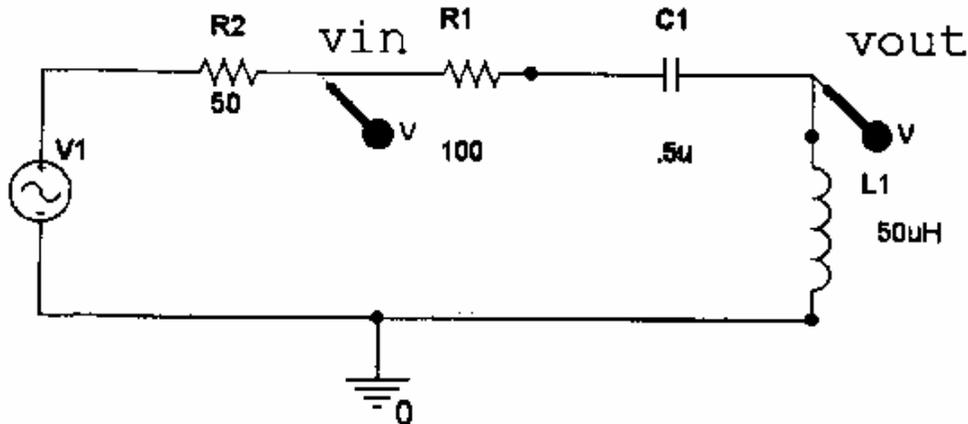
Now that you have determined the magnitude, frequency and phase of the input voltage, you should have some idea of what will happen at the output. From your knowledge of the corner or resonant frequency of this circuit, will the output voltage be about the same as the input, substantially smaller or substantially larger than the input? Explain your answer.

Very roughly sketch the magnitude of the transfer function for this circuit as a function of frequency. You only need to show the general shape of the magnitude, not the phase. Indicate on the graph where the resonance frequency is. Please give a numerical value.

*Spring 2002 solution
(none available)*

3. Filters (25 points)

The following circuit consists of a sinusoidal source, an inductor, a capacitor and a resistor.



If v_{in} is the sinusoidal source (including the 50 ohm internal impedance) and v_{out} is the voltage across the inductor, is this configuration a high pass filter, a low pass filter, a band pass filter or a band reject filter? Explain your answer.

Low f : $C \rightarrow$ open $L \rightarrow$ short $\Rightarrow V_{out} = 0$

High f : $C \rightarrow$ short $L \rightarrow$ open $\Rightarrow V_{out} = V_{in}$

High-pass Filter

The source is a sinusoidal voltage with some amplitude and frequency. The source voltage, as a function of time, is shown on the next page. Write out the mathematical expression for this voltage function in the form $v_{in} = v_0 \sin(\omega t + \phi_0)$. Be sure that you give values for v_0 , ω , and ϕ_0 .

$v_0 = 100 \text{ mV}$

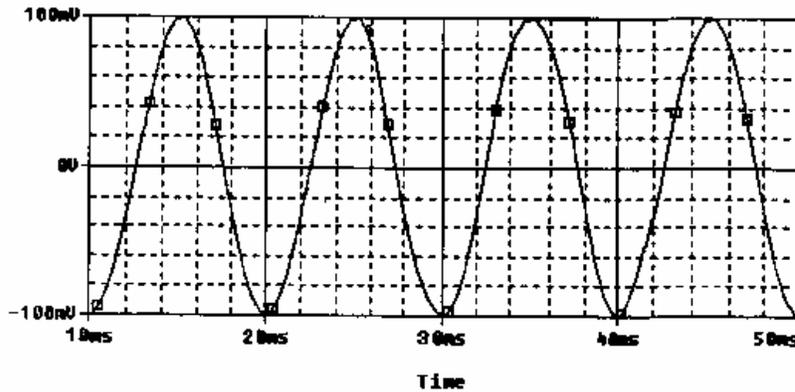
$T = 10 \text{ ms} \Rightarrow f = \frac{1}{T} = 100 \text{ Hz}$

$\omega = 2\pi f = 2\pi \times 100 \text{ rad/sec}$

$\phi_0 = -\frac{\pi}{2}$

$t_0 = 2.5 \text{ ms} \Rightarrow \phi_0 = -\frac{\pi}{2}$

$v_{in} = 100 \text{ mV} \sin(2\pi \times 100 t - \frac{\pi}{2})$



Now that you have determined the magnitude, frequency and phase of the input voltage, you should have some idea of what will happen at the output. From your knowledge of the corner or resonant frequency of this circuit, will the output voltage be about the same as the input, substantially smaller or substantially larger than the input? Explain your answer.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \mu\text{H} \times 0.5 \mu\text{F}}} = \frac{1}{\sqrt{50 \times 10^{-6} \times 0.5 \times 10^{-6}}} = \frac{1}{5 \times 10^{-6}}$$

$$= 200,000 \text{ rad/sec}$$

$\omega \ll \omega_0 \Rightarrow$ Substantially smaller

Would you say that, for this circuit, the frequency of the source is high, low or neither? Very roughly sketch the magnitude of the transfer function for this circuit as a function of frequency. You only need to show the general shape of the magnitude, not the phase.

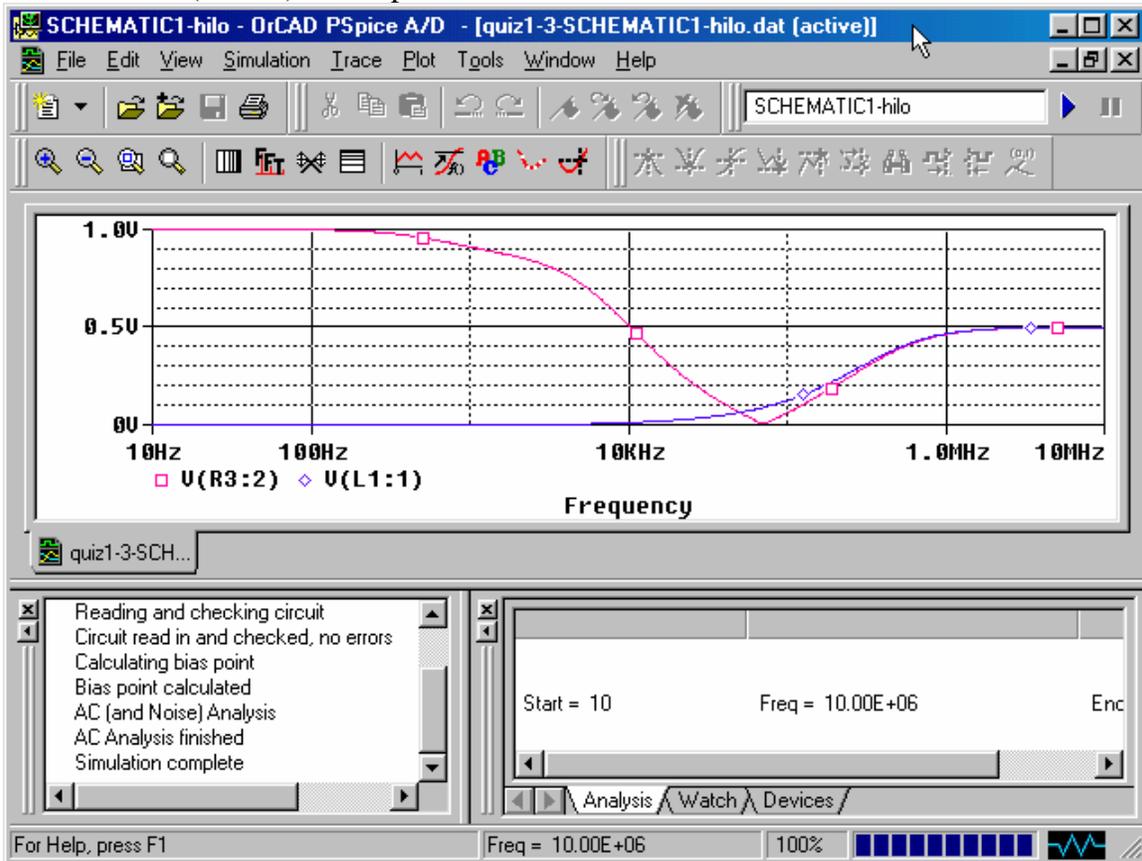
Low: $\omega \ll \omega_0$



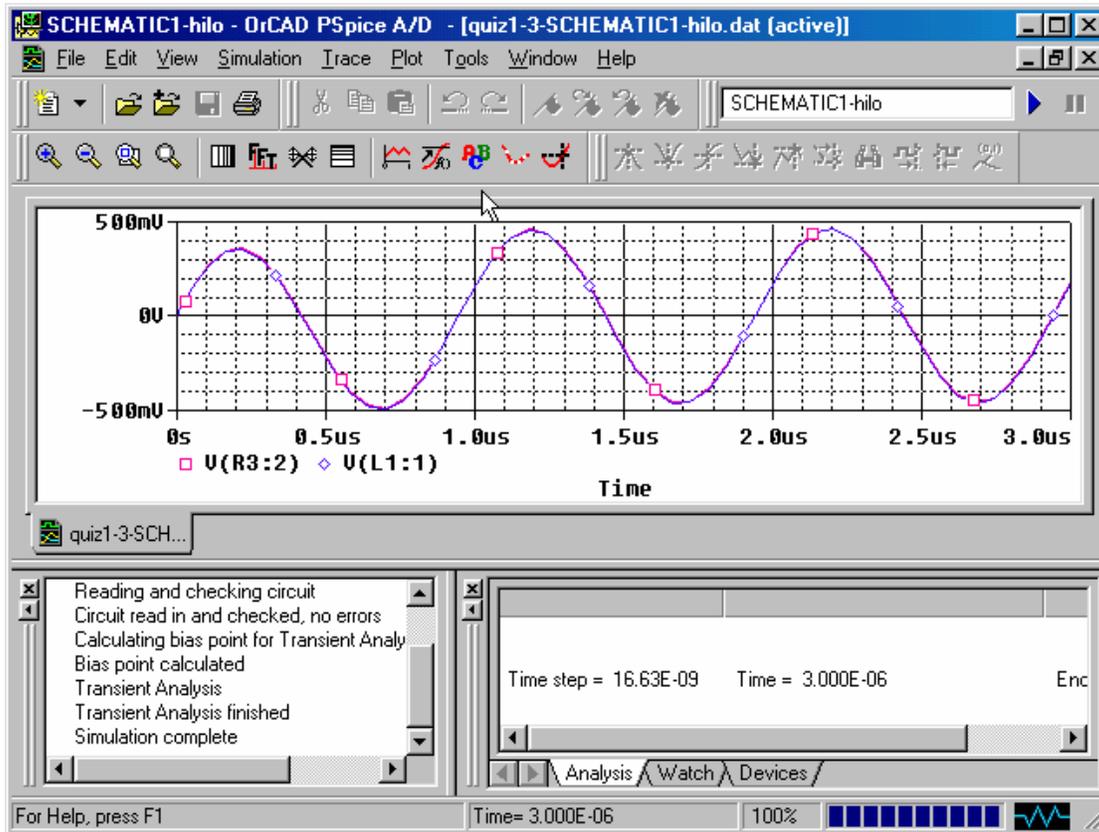
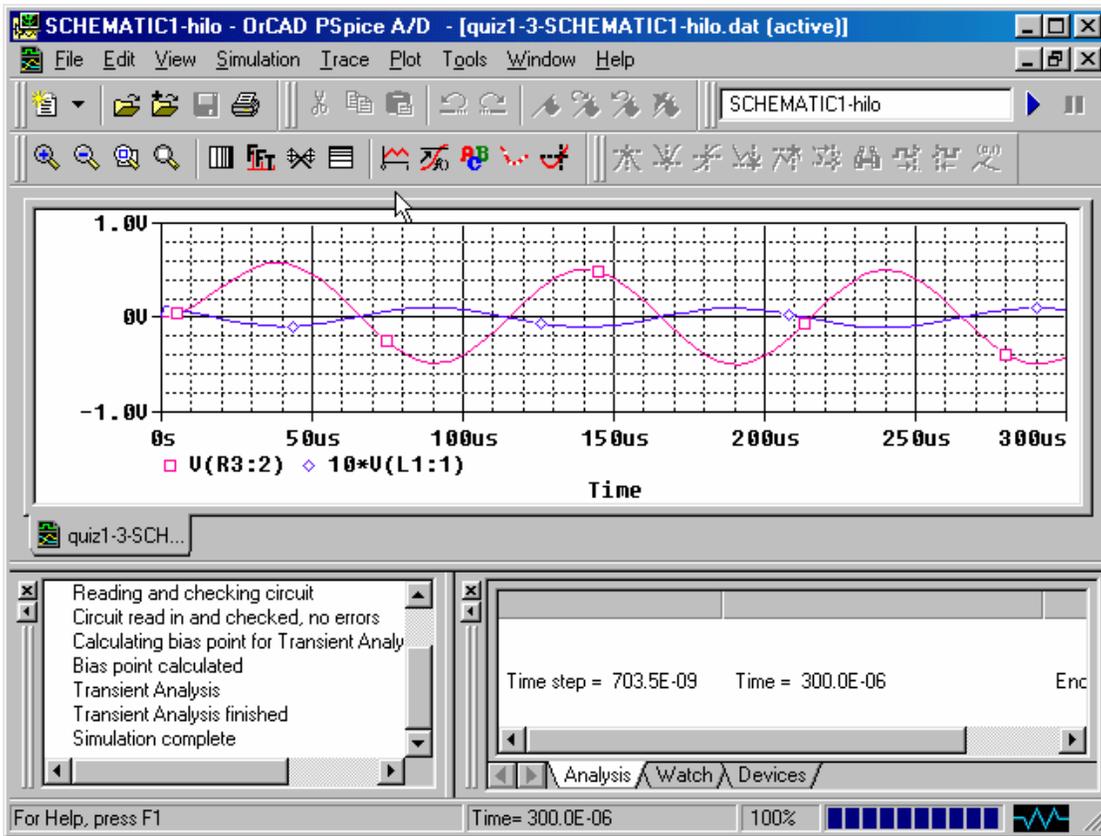
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3. Simulation Results (15 Points)

a. For the circuit of the previous problem, an AC sweep analysis has been performed, with the voltages at points A and B displayed as a function of frequency. Label each of the two traces (A or B) on the plot below.



A transient analysis was also performed at two different frequencies, with the results shown for the same voltages on the next page. Note that on the top plot, one of the voltages displayed is so small that it was multiplied by 10 so that it could be seen on the same scale as the other voltage. First, for each of these plots, determine the frequency of the voltage source that was used and write the frequency on the plots. Second, mark these two frequencies on the voltage vs. frequency plot above.



*Fall 2000 solution
(none available)*