## Electronic

## Instrumentation

## Quiz 2 Solution

Spring 2018

| 1. | $/ 20$ |
| :---: | :---: |
| 2. | $/ 20$ |
| 3. | $/ 20$ |
| 4. | $/ 20$ |
| 5. Online Quiz | $/ 20$ |
| Total | $/ 100$ |

Name $\qquad$

Please write your name at the top of every page (following this page)!

Notes:
DO NOT WRITE ON THE BACK OF PAGES!! EXTRA PAGES ARE PROVIDED IN QUIZ! SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification. Read the entire quiz before answering any questions. Also it may be easier to answer parts of questions out of order. Round answers to 3 significant digits.

## Name

$\qquad$

## Problem 1: Thevenin Equivalent Circuits (20 pts)



The Thevenin Equivalent circuit consists of a voltage source (Vth) in series with a resistor (Rth), which is then in series with a defined load (RL). Note: A load can be more than one component. If there are mulitple, the value of the load resistance should be calculated to create one component.

In the circuits below, you are to find Vth and Rth and then re-draw the thevenin circuit with the defined RL. It will help you and the grader to redraw the circuit as much as needed to find the answers. Extra space is available in the problem and at the end of the exam if needed! (DO NOT WRITE ON THE BACK OF THE EXAM IF YOU WANT IT TO BE GRADED!!!)

In this problem, you are to find the Thevenin voltage and resistance for a series of related circuits. While the circuits and their analysis are similar, treat each circuits as a separate problem.
1.1: Find and sketch the Thevenin Equivalent for for the following circuit. (4 pts)

$\qquad$

## Extra space for 1.1:

## Thevenin Circuit Drawing

$$
\begin{aligned}
& \mathrm{R}_{23 \mathrm{P} 1}:=\frac{\mathrm{R}_{2 \mathrm{P} 1} \cdot \mathrm{R}_{3 \mathrm{P} 1}}{\mathrm{R}_{2 \mathrm{P} 1}+\mathrm{R}_{3 \mathrm{P} 1}}=2 \times 10^{3} \Omega \\
& \mathrm{~V}_{\mathrm{R} 678}:=\mathrm{V}_{1 \mathrm{P} 1} \cdot \frac{\mathrm{R}_{4 \mathrm{P} 1}}{\mathrm{R}_{23 \mathrm{P} 1}+\mathrm{R}_{4 \mathrm{P} 1}+\mathrm{R}_{1 \mathrm{P} 1}}=5.4 \mathrm{~V} \\
& \mathrm{~V}_{\text {th }}:=\mathrm{V}_{\mathrm{R} 678}=5.4 \mathrm{~V}
\end{aligned}
$$

To find Rth

$$
\begin{aligned}
& \mathrm{R}_{\text {th } 1}:=\left[\frac{\left(\mathrm{R}_{23 \mathrm{P} 1}+\mathrm{R}_{1 \mathrm{P} 1}\right) \cdot \mathrm{R}_{4 \mathrm{P} 1}}{\mathrm{R}_{23 \mathrm{P} 1}+\mathrm{R}_{1 \mathrm{P} 1}+\mathrm{R}_{4 \mathrm{P} 1}}\right]+\mathrm{R}_{5 \mathrm{P} 1} \\
& \mathrm{R}_{\text {th } 1}=1.14 \times 10^{4} \Omega
\end{aligned}
$$

Name $\qquad$
1.2: Find and sketch the Thevenin Equivalent for for the following circuit. ( 6 pts )


Find the voltage across R7. Take out the load and do a few voltage dividers.
Finding VR4 which is the same as VR568. Reducing the circuit...

$$
\begin{aligned}
& \mathrm{R}_{68 \mathrm{P} 1}:=\frac{\mathrm{R}_{6 \mathrm{P1}} \cdot \mathrm{R}_{8 \mathrm{P} 1}}{\mathrm{R}_{6 \mathrm{P} 1}+\mathrm{R}_{8 \mathrm{P} 1}}=5.455 \times 10^{3} \Omega \\
& \mathrm{R}_{568 \mathrm{P} 1}:=\mathrm{R}_{68 \mathrm{P} 1}+\mathrm{R}_{5 \mathrm{P} 1}=1.445 \times 10^{4} \Omega \\
& \mathrm{R}_{4568 \mathrm{P} 1}:=\frac{\mathrm{R}_{4 \mathrm{P} 1} \cdot \mathrm{R}_{568 \mathrm{P} 1}}{\mathrm{R}_{4 \mathrm{P} 1}+\mathrm{R}_{568 \mathrm{P} 1}}=4.24 \times 10^{3} \Omega \\
& \mathrm{~V}_{\mathrm{R} 4568 \mathrm{P} 1}:=\mathrm{V}_{1 \mathrm{P} 1} \cdot \frac{\mathrm{R}_{4568 \mathrm{P} 1}}{\mathrm{R}_{1 \mathrm{P} 1}+\mathrm{R}_{23 \mathrm{P} 1}+\mathrm{R}_{4568 \mathrm{P} 1}}=4.631 \mathrm{~V} \\
& \mathrm{~V}_{\mathrm{R} 68 \mathrm{P} 1}:=\mathrm{V}_{\mathrm{R} 4568 \mathrm{P} 1} \cdot \frac{\mathrm{R}_{68 \mathrm{P} 1}}{\mathrm{R}_{5 \mathrm{P} 1}+\mathrm{R}_{68 \mathrm{P} 1}}=1.748 \mathrm{~V} \\
& \mathrm{~V}_{\text {th2 }}:=\mathrm{V}_{\mathrm{R} 68 \mathrm{P} 1} \\
& \text { To find Rth } \\
& \mathrm{R}_{\mathrm{th} 2}:=\frac{\left.\left[\frac{\left(\mathrm{R}_{23 \mathrm{P} 1}+\mathrm{R}_{1 \mathrm{P} 1}\right) \cdot \mathrm{R}_{4 \mathrm{P} 1}}{\mathrm{R}_{23 \mathrm{P} 1}+\mathrm{R}_{1 \mathrm{P} 1}+\mathrm{R}_{4 \mathrm{P} 1}}\right]+\mathrm{R}_{5 \mathrm{P} 1}\right] \cdot \mathrm{R}_{68 \mathrm{P} 1}}{\left[\frac{\left(\mathrm{R}_{23 \mathrm{P} 1}+\mathrm{R}_{1 \mathrm{P} 1}\right) \cdot \mathrm{R}_{4 \mathrm{P} 1}}{\mathrm{R}_{23 \mathrm{P} 1}+\mathrm{R}_{1 \mathrm{P} 1}+\mathrm{R}_{4 \mathrm{P} 1}}\right]+\mathrm{R}_{5 \mathrm{P} 1}+\mathrm{R}_{68 \mathrm{P} 1}}
\end{aligned}
$$

## Name

Extra space for 1.2:
Thevenin Circuit Drawing

$$
\begin{gathered}
\mathrm{R}_{\mathrm{th} 2}=3.689 \times 10^{3} \Omega \\
\mathrm{~V}_{\mathrm{th} 2} \cdot \frac{1 \mathrm{k} \Omega}{\mathrm{R}_{\mathrm{th} 2}+1 \mathrm{k} \Omega}=0.373 \mathrm{~V}
\end{gathered}
$$

Name $\qquad$
1.3: The circuit below is much more complicated. PSpice simulation results for node voltages are shown on the diagram. In the first plot the load resistor is very large ( $100 \mathrm{M} \Omega$ ) while in the second plot the load resistor is much smaller ( $1 \mathrm{k} \Omega$ ). Using the given voltages at each of the nodes for both RLoad (open circuit-like and $1 \mathrm{k} \Omega$ ) determine Vth and Rth. Note: A schematic of the thevenin circuit may be helpful to you but it is not necessary for grading. (8 pts)


| $\mathrm{V}_{\text {th }}$ | ${ }^{\text {[V] }}$ |
| :---: | ---: |
| $\mathrm{R}_{\text {th }}$ | [ohms] |

## Name

Vthevenin is open circuit voltage after RLoad is taken off. Therefore, the voltage a the node above 100M ohm RLoad is Vth.

$$
\mathrm{V}_{\mathrm{th} 3}:=0.7 \mathrm{~V} \quad \text { can round up or not }
$$

To find Rth, you should use the thevenin equivalent circuit where the voltage across the 1 k load is 415.7 mV .

$$
\mathrm{V}_{\text {RLoad }}:=415.7 \mathrm{mV}
$$

Therefore: $\quad \mathrm{V}_{\mathrm{RL} \text { oad }}=\frac{1 \mathrm{k} \Omega}{1 \mathrm{k} \Omega+\mathrm{R}_{\text {th } 3}} \cdot \mathrm{~V}_{\text {th3 }}$

$$
\mathrm{R}_{\mathrm{th} 3}:=\frac{\mathrm{V}_{\mathrm{th} 3} \cdot \mathrm{k} \Omega}{\mathrm{~V}_{\mathrm{RLoad}}}-1 \mathrm{k} \Omega
$$

$$
\mathrm{R}_{\mathrm{th} 3}=683.907 \Omega \quad \mathrm{R}_{\mathrm{th} 3}=684 \Omega
$$

1.4: What is the purpose of a thevenin equivalent circuit? (In other words, if it is an analysis tool, what is it used for?) How does this purpose relate to the procedure to find the thevenin equivalent circuit? (2 pts)

The thevenin equivalent allows you to switch out different loads without doing analysis all over again. It is a design tool to simplify.

You are looking to find the voltage and resistance as the load sees it (from the perspective of the thing you are trying to switch out or design for.)

Note to graders there may be a wide range of answers here. Simplifying the circuit and something about the perspective of the load should be written here.

## Name

$\qquad$

## Problem 2: Harmonic Oscillators (20 pts)

The velocity measured for an oscillating cantilever beam is shown in graphical form as:

2.1: Find the decay constant, $\alpha$, and the angular frequency, $\omega$, for this function. Pay close attention to axes. You must label the graph (arrows) with the points you are using for time and voltage for full credit. (6 pts)
$\omega=2 \pi \mathrm{f}$
period $:=6.5 \mathrm{~s}$

$$
\mathrm{f}:=\frac{1}{\text { period }}=0.154 \frac{1}{\mathrm{~s}}
$$

$$
\omega_{21}:=2 \cdot \pi \cdot f=0.967 \frac{1}{\mathrm{~s}}
$$

$$
\mathrm{V}_{\mathrm{A}}=0.3
$$

$$
\text { freq }=0.15
$$

$$
\mathrm{DC}=0.025
$$

anything from 5.1 to 7 period
anything from 0.27 to 0.3

Can find it using known voltage and known time on the graph: 100 mV at 42 sec

$$
\begin{aligned}
& 0.3 \cdot \mathrm{e}^{-\alpha \mathrm{t}}=100 \mathrm{mV} \quad \frac{100 \mathrm{mV}}{0.3 \mathrm{~V}}=0.333 \\
& -\alpha \cdot 42 s=\ln (0.333) \\
& \alpha:=\frac{-\ln (0.333)}{42}=0.026 \quad \text { close }
\end{aligned}
$$

| $\boldsymbol{\omega}$ |  |
| :---: | :--- |
| $\boldsymbol{\alpha}$ |  |

2.2: Write the mathematical expression for the voltage in the form $\mathrm{Ae}^{-a t} s i n \omega t$. (3 pts)

$$
0.3 \cdot \mathrm{e}^{-0.025 t} \cdot \sin (0.967 \mathrm{t})
$$

## Name

$\qquad$
2.3: Find the acceleraton $a(t)$ of the beam from your answer to part 2.2. Again, use real values for the constant and provide units where appropirate. (3 pts)

$$
a=0.3 \cdot(-0.025) e^{-0.025 t} \cdot \sin (0.967 t)
$$

$$
\begin{aligned}
& 0.3 \cdot-0.025=-7.5 \times 10^{-3} \quad 0.967 \cdot 0.3=0.29 \\
& a=-7.5 \cdot 10^{-3} e^{-0.025 t} \cdot \sin (0.967 t)+0.29 e^{-0.025 t} \cdot \cos (0.967 t)
\end{aligned}
$$

2.4: Find the decay constant, $\alpha$, and the angular frequency, $\omega$, for this function. Pay close attention to axes. You must label the graph (arrows) with the points you are using for time and voltage for full credit. (6 pts)


There is a 3 V offset given the center of the sinusoid at 3 V .

$$
\begin{aligned}
& \text { period2 }:=15 \mathrm{~s} \quad \mathrm{f}_{2}:=\frac{1}{\text { period2 }}=0.067 \frac{1}{\mathrm{~s}} \\
& \omega:=2 \cdot \pi \cdot \mathrm{f}_{2}=0.419 \frac{1}{\mathrm{~s}} \quad 3.9-2.2=1.7 \quad \frac{1.7}{2}=0.85 \\
& \mathrm{~V}_{\mathrm{A} 1}:=3.9-3=0.9 \quad \mathrm{t} 2:=84 \\
& \mathrm{~V}_{\mathrm{A} 2}:=3.1-3=0.1
\end{aligned} \quad \underset{\mathrm{w}}{\mathrm{t} 1:=4} \begin{aligned}
& \mathrm{\alpha}:=\frac{-\ln \left(\frac{\mathrm{V}_{\mathrm{A} 1}}{\mathrm{~V}_{\mathrm{A} 2}}\right)}{\mathrm{t} 1-\mathrm{t} 2}=0.027
\end{aligned}
$$

anything 0.85 to 1

| $\omega$ |  |
| :---: | :---: |
| $\alpha$ |  |
| $\alpha$ |  |

2.5: Write the mathematical expression for the voltage. (2 pts)

$$
3+0.85 \cdot e^{-0.027 t} \cdot \sin (0.419 t)
$$

## Name

$\qquad$

## Problem 3: Op Amp Applications (20 pts)


$\pm 9 \mathrm{~V}$ power supplies have been properly connected to all five op-amps in the circuit above
3.1: (5 pts) The circuit has $5 \mathrm{op}-\mathrm{amps}$ labeled as U1 through U5. State what the op-amp circuit is for each.

U1 Circuit: $\qquad$ U2 Circuit: $\qquad$ U3 Circuit: $\qquad$
Follower
Non-inverting
Differentiator

U4 Circuit: $\qquad$ U5 Circuit: $\qquad$
Integrator
Adder (inverting summer)
3.2 (4 pts): Using values listed below, determine the voltage values at $\mathrm{Va}(\mathrm{t}), \mathrm{Vb}(\mathrm{t}), \mathrm{Vc}(\mathrm{t})$ and $\mathrm{Vd}(\mathrm{t})$ as function fo Vin(t). Please include the symbolic solutions with R1, R2...as preliminary steps. Then you can substituve resistor values. (Vin(t) will be part of your final answer)
$\mathrm{R}_{1}:=10 \mathrm{k} \Omega$
$\mathrm{R}_{2}:=2.5 \mathrm{k} \Omega$
$\mathrm{R}_{3}:=2 \mathrm{k} \Omega$
$\mathrm{R}_{4}:=4 \mathrm{k} \Omega \quad \mathrm{R}_{5}:=20 \mathrm{k} \Omega \quad \mathrm{R}_{6}:=40 \mathrm{k} \Omega$
$\mathrm{R}_{7}:=10 \mathrm{k} \Omega$
$\mathrm{R}_{8}:=10 \mathrm{k} \Omega$
$\mathrm{R}_{9}:=2 \mathrm{k} \Omega$
$C_{1}:=100 \mu \mathrm{~F} \quad \mathrm{C}_{2}:=47 \mu \mathrm{~F}$
a) Voltage at point $\mathrm{Va}(\mathrm{t})$ :

$$
\operatorname{Va}(\mathrm{t})=\mathrm{Vin}(\mathrm{t})
$$

Name $\qquad$
b) Voltage at point $\mathrm{Vb}(\mathrm{t})$ :

$$
\begin{aligned}
& \operatorname{Vb}(\mathrm{t})=\left(1+\frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}\right) \mathrm{V}_{\mathrm{a}}(\mathrm{t})=\left(1+\frac{10 \mathrm{k}}{2.5 \mathrm{k}}\right) \cdot \operatorname{Vin}(\mathrm{t})=5 \cdot \operatorname{Vin}(\mathrm{t}) \\
& \left(1+\frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}\right)=5
\end{aligned}
$$

c) Voltage at point $\mathrm{Vc}(\mathrm{t})$ :

$$
\mathrm{Vc}(\mathrm{t})=-\left(\mathrm{R}_{3} \cdot \mathrm{C}_{1}\right) \cdot \frac{\mathrm{dVa}(\mathrm{t})}{\mathrm{dt}}=-0.2 \cdot \frac{\mathrm{dVin}(\mathrm{t})}{\mathrm{dt}}
$$

$$
-\left(\mathrm{R}_{3} \cdot \mathrm{C}_{1}\right)=-0.2 \mathrm{~s}
$$

d) Voltage at point $\mathrm{Vd}(\mathrm{t})$ :

$$
\begin{aligned}
\operatorname{Vd}(\mathrm{t})= & \frac{-1}{\mathrm{R}_{4} \cdot \mathrm{C}_{2}} \cdot \int \operatorname{Va}(\mathrm{t}) \mathrm{dt}=-5.319 \cdot \int \operatorname{Vin}(\mathrm{t}) \mathrm{dt} \\
& \frac{-1}{\mathrm{R}_{4} \cdot \mathrm{C}_{2}}=-5.319 \frac{1}{\mathrm{~s}}
\end{aligned}
$$

3.3: (3 pts) Determine the output voltage, $\operatorname{Vout}(\mathrm{t})$, as a function of $\mathrm{Vb}(\mathrm{t}), \mathrm{Vc}(\mathrm{t})$ and $\mathrm{Vd}(\mathrm{t})$.

$$
\begin{aligned}
& \mathrm{V}_{\text {out }}(\mathrm{t})=\frac{-\mathrm{R}_{8}}{\mathrm{R}_{5}} \cdot \mathrm{~V}_{\mathrm{b}}(\mathrm{t})-\frac{\mathrm{R}_{8}}{\mathrm{R}_{6}} \cdot \mathrm{~V}_{\mathrm{c}}(\mathrm{t})-\frac{\mathrm{R}_{8}}{\mathrm{R}_{7}} \cdot \mathrm{~V}_{\mathrm{d}}(\mathrm{t}) \\
& \frac{-\mathrm{R}_{8}}{\mathrm{R}_{5}}=-0.5 \quad \frac{-\mathrm{R}_{8}}{\mathrm{R}_{6}}=-0.25 \quad \frac{-\mathrm{R}_{8}}{\mathrm{R}_{7}}=-1 \\
& \mathrm{~V}_{\text {out }}(\mathrm{t})=-0.5 \cdot \mathrm{~V}_{\mathrm{b}}(\mathrm{t})-0.25 \mathrm{~V}_{\mathrm{c}}(\mathrm{t})-\mathrm{V}_{\mathrm{d}}(\mathrm{t})
\end{aligned}
$$

Name $\qquad$
3.4: (3 pts) Find Vout $(\mathrm{t})$ as a function of $\operatorname{Vin}(\mathrm{t})$.

$$
\begin{aligned}
& \mathrm{V}_{\text {out }}(\mathrm{t})=-0.5 \cdot(5 \cdot \operatorname{Vin}(\mathrm{t}))-0.25\left(-0.2 \cdot \frac{\mathrm{dVin}(\mathrm{t})}{\mathrm{dt}}\right)-\left(-5.319 \cdot \int \operatorname{Vin}(\mathrm{t}) \mathrm{dt}\right) \\
& \quad-0.5 \cdot 5=-2.5 \\
& \quad-0.25 \cdot-0.2=0.05 \\
& \mathrm{~V}_{\text {out }}(\mathrm{t})=-2.5 \cdot \operatorname{Vin}(\mathrm{t})+0.05 \cdot \frac{\mathrm{dVin}(\mathrm{t})}{\mathrm{dt}}+5.319 \cdot \int \operatorname{Vin}(\mathrm{t}) \mathrm{dt}
\end{aligned}
$$

3.4 (5 pts) Design and draw an amplifier circuit (multiple stages may be needed) with three inputs (V1, V2, and V3) such that:

Vout $=\mathrm{V} 1-2 \mathrm{~V} 2-4 \mathrm{~V} 3$ (note: all values of of input voltages are positive to start.)

This is one solution. There are a few...


Second stage: Summing amplifier with gains $-1,-2$, and -4

## Name

$\qquad$

## Problem 4: Op Amps (20 pts)

4.1: (5 pts) The input voltage is shown below. Solve for and sketch the output voltage for the following circuit.


$\mathrm{V}_{\text {out }}=\frac{-\mathrm{R}_{2}}{\mathrm{R}_{1}} \cdot 0.2 \mathrm{~V}$
$\mathrm{V}_{\text {out }}:=-4 \cdot 0.2 \mathrm{~V}=-0.8 \mathrm{~V} \quad$ peak at -0.2 goes to +0.8 , peak at +0.2 goes to -0.8
4.2: (5 pts) The input voltage is shown below. Solve for and sketch the output voltage for the following circuit.



$$
\begin{gathered}
\mathrm{V}_{\text {out }}=\left(1+\frac{\mathrm{R}_{2}}{\mathrm{R}_{1}}\right) \cdot \mathrm{V}_{\text {in }} \\
1+\frac{2 \mathrm{k}}{1 \mathrm{k}}=3
\end{gathered}
$$

4.3: (5 pts) The input voltage is shown below. Solve for and sketch the output voltage for the followina circuit.


$\mathrm{V}_{\text {out }}=-\mathrm{R}_{1} \cdot \mathrm{C}_{2} \cdot \frac{\mathrm{dVin}}{\mathrm{dt}}$

$$
\mathrm{V}_{\mathrm{in}}(\mathrm{t})=\frac{0.4 \mathrm{~V}}{1 \mathrm{~ms}} \cdot \mathrm{t} \quad 0.68 \mu \mathrm{~F} \cdot 2 \mathrm{k} \boldsymbol{\Omega} \cdot 400=0.544 \mathrm{~s}
$$

$$
V_{i n}(t)=400 t
$$

From 0 to $1+400$ slope * $-R C=-0.544$ line
From 1 to $2-400$ slope * - RC $=+0.544$ line

## Basic Lab Questions

4.4: (3 pts) What circuits were used to create a velocity output from the strain gauges in the cantilever beam experiment? (Extra credit 2 pts: Draw them as one connected circuit, no partial credit)

Bridge circuit, difference amp, differentiator circuit
4.5: (2 pts) Name at least two names of the TAs from your section of El. (You can write first names only if that's all you can remember.)

| Section 1: | Section 2: |
| :--- | :--- |
| Waleed | Joe |
| Olivia | Michael |
| Ziyi | Kun |

