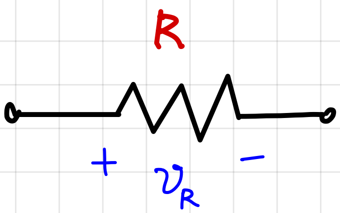
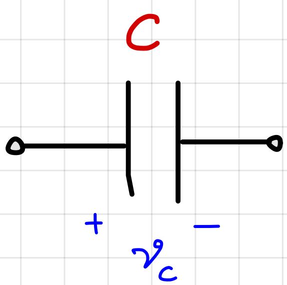
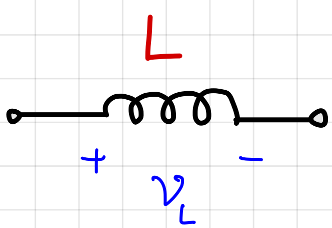
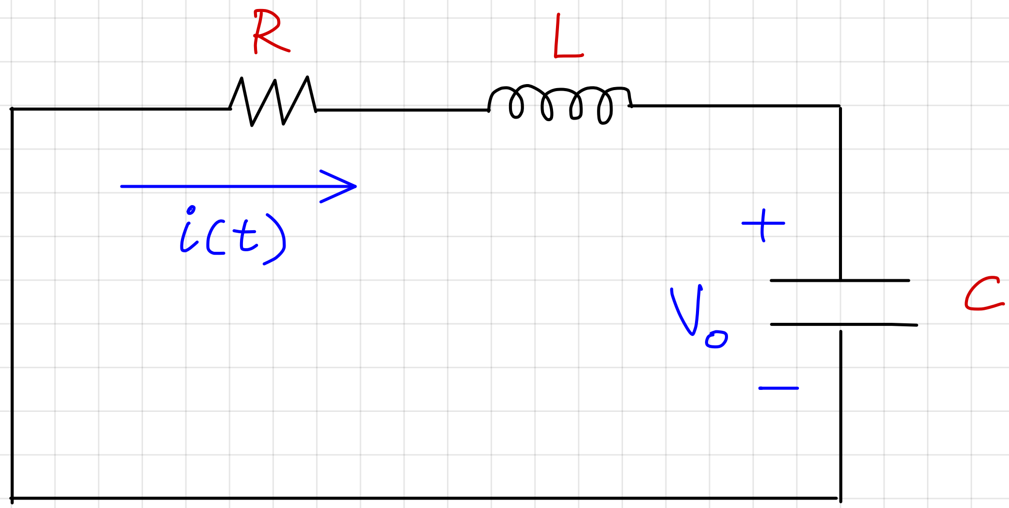
Series RLC circuits

Second order transient response

Current - Voltage Relationships

Component	Symbol	i-v relation
Resistor		$v_R = i_R \cdot R$
Capacitor		$i_C = C \frac{dv_C}{dt}$
Inductor		$v_L = L \frac{di_L}{dt}$



KVL : $$iR + v_L + v_c = 0$$

$$v_L = L\frac{di_L}{dt} = L\frac{di}{dt}$$

$$i_c = C\frac{dv_c}{dt} \Rightarrow \int_0^{t'} i(t)\,dt = C v_c$$

$$\Rightarrow v_c(t) = \frac{1}{C}\int_0^{t'} i\,dt + V_0$$

$$\Rightarrow i \cdot R + L\frac{di}{dt} + \frac{1}{C}\int_0^{t'} i\,dt + V_0 = 0$$

Differentiate with respect to 't'.

$$i.R + L\frac{di}{dt} + \frac{1}{C}\int_0^{t'} i\,dt + V_0 = 0$$

$$R.\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

Divide by 'L'

$$\boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0}$$

SOLVE Differential equation $\rightarrow$ $$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = 0$$

Assume $$i(t) = K e^{st}$$

then $$\frac{d^2 i}{dt^2} = K s^2 e^{st}$$ and $$\frac{di}{dt} = K s e^{st}$$

$$\Rightarrow \quad K s^2 e^{st} + \frac{R}{L} K s e^{st} + \frac{1}{LC} K e^{st} = 0$$

$$\Rightarrow \quad \boxed{s^2 + \frac{R}{L} s + \frac{1}{LC} = 0}$$

Characteristic equation.

Solve quadratic equation: $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$$s = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

next slide: Roots could be real or complex

Define

ω_0 = Resonant frequency (rad/s)

$$= \frac{1}{\sqrt{LC}}$$

$\omega_0 = 2\pi f_0$

ω_0: radians per second; f_0: Hz or sec^{-1}

$$\alpha = \frac{R}{2L} = \text{attenuation constant}$$

$\alpha = \frac{1}{\tau}$

τ = Time constant

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$\alpha > \omega_0 \rightarrow$ Roots are real

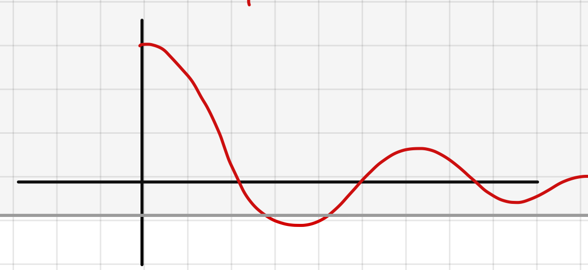
OVERDAMPED SYSTEM

$\alpha = \omega_0 \rightarrow$ Roots are real and equal

CRITICALLY DAMPED

$\alpha < \omega_0 \rightarrow$ Roots are complex conjugates

UNDERDAMPED SYSTEM



Damped Frequency β

β = frequency of damped oscillations

$$= \sqrt{\omega_0^2 - \alpha^2}$$

radians per second = 2π(frequency in Hertz).

ω_0: resonant frequency

α: attenuation constant