Matrix Inverse

size 3x3

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Intro to ECSE

Is the matrix invertible?

- Matrix $\mathbf{A}_{nxn}$ is invertible if there exists $\mathbf{B}_{nxn}$ such that

$$\mathbf{AB = BA = I_n}$$

- Invertible = nonsingular = nondegenerate
- Non-Invertible = singular = degenerate
- Matrix **A** is invertible if and only if $\det(A) \neq 0$

Invert Z_{3x3} to determine Z^{-1}

$$\boldsymbol{Z} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times 3}$$

$$\boldsymbol{Z}^{-1} = \frac{1}{\det(\boldsymbol{Z})} adj(\boldsymbol{Z}) = \frac{1}{\det(\boldsymbol{Z})} \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}^T$$

Transpose

$adj(\boldsymbol{Z})$ = Transpose of matrix of cofactors

$$\boldsymbol{Z} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad adj(\boldsymbol{Z}) = \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}^T$$

$$A = (ei - fh) \qquad B = -(di - fg) \qquad C = (dh - eg)$$
$$D = -(bi - ch) \qquad E = (ai - cg) \qquad F = -(ah - bg)$$
$$G = (bf - ce) \qquad H = -(af - cd) \qquad I = (ae - bd)$$

$$\det(\boldsymbol{Z}) = aA + bB + cC$$

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

Example

$$Z^{-1} = ?$$

$$\boldsymbol{Z} = \begin{bmatrix} -1 & 1 & 0 \\ 2 & 1 & -1 \\ 1 & 2 & -2 \end{bmatrix}_{3\times 3}$$

First find cofactors:

$$\mathrm{adj}(Z) = \begin{bmatrix} 0 & 3 & 3 \\ 2 & 2 & 3 \\ -1 & -1 & -3 \end{bmatrix}^T = \begin{bmatrix} 0 & 2 & -1 \\ 3 & 2 & -1 \\ 3 & 3 & -3 \end{bmatrix}$$

$$\det(Z) = aA + bB + c(C)$$
$$= \cancel{-1\times(0)} + \underline{1\times(3)} + \cancel{0\times(C)}$$
$$= \underline{\underline{3}}$$

$$\boxed{Z^{-1} = \frac{1}{3}\begin{bmatrix} 0 & 2 & -1 \\ 3 & 2 & -1 \\ 3 & 3 & -3 \end{bmatrix}}$$

Check:

$$Z^{-1} \cdot Z = I$$