

Matrix Inverse

size 2×2
rows ↑
→ *columns*
square

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Intro to ECSE



Invert $A_{2 \times 2}$ to determine A^{-1}

$$A = \begin{bmatrix} \underline{a} & \underline{b} \\ \underline{c} & \underline{d} \end{bmatrix}_{2 \times 2}$$

$$\underline{\underline{A^{-1}}} = \frac{1}{\underline{\underline{|A|}}} \text{adj}(A)$$

adjugate.

determinant

$$|A| = \underline{ad - bc}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \begin{bmatrix} \frac{d}{ad - bc} & \frac{-b}{ad - bc} \\ \frac{-c}{ad - bc} & \frac{a}{ad - bc} \end{bmatrix}_{2 \times 2}$$



Example

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$$

$$\begin{aligned} |A| &= \det(A) \\ &= 4 - 6 = -2 \end{aligned}$$

$$\text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}_{2 \times 2}$$