



Class #23: Second order (RLC) circuits

Purpose: In this experiment we will investigate how circuits with more than one capacitor or inductor responds to changing sources.

Background: Before doing this experiment, students should be able to

- Review online background materials.
- Build and operate simple circuits on a Protoboard.
- Make transient measurements using the Oscilloscope function on the Analog Discovery Board.
- Analyze simple circuits consisting of combinations of resistors, especially voltage dividers.
- Do a transient (time dependent) simulation of circuits using LTspice
- Review the background for the previous experiments.

Learning Outcomes: Students will be able to

- Determine when a second order circuit is overdamped.
- Determine when a second order circuit is underdamped.
- Find the attenuation constant and oscillation frequency of an

Resources Required:

- M1K board with Part Kit
- Alice tools
- Matlab
- LTspice

Helpful links for this experiment can be found on the course website under Class #23.

Part A – LR Circuits

In the last experiment, we investigated the exponential function and how it related to the RC (resistor-capacitor) circuit. A similar investigation can be applied to LR (inductor-resistor) circuits. If we once again were to connect and disconnect a power source, we would again see an exponential response when measuring voltage (and current) as a function of time. However, unlike capacitors, inductors don't behave very well when you disconnect the source, making an open circuit. The physics of inductors is that they need a constant current path and if that path is broken (like when you disconnect a source), the inductor can cause arcing. The inductors in our parts kits are too small to see this effect, but it is still a bad idea to interrupt their current path. Rather than repeat the connect-disconnect experiment from the last class, we will use the square wave pulse from the end of the last experiment.

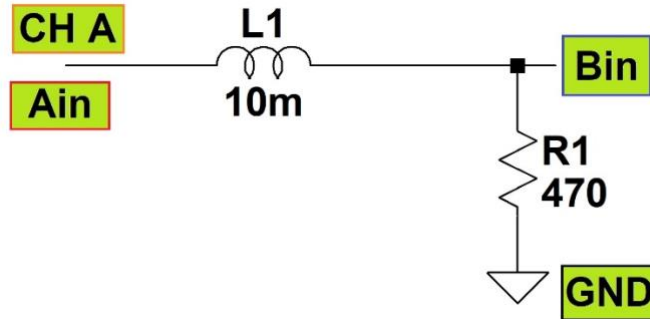


Figure A-1: LR Circuit

- Implement the LR circuit shown in Figure A-1. Configure CH A as a function Generator to source a square wave, with a maximum value of 2V and a minimum value of 0V, and a frequency of 2500 Hz. Measure the voltage across the resistor using Bin. You'd need to put both channels in Split I/O mode to make measurements on Ain and Bin pins. Use the math channel to determine the current through the resistor (Ohm's Law), which is also equal to the current through the inductor (series circuits). More details about math channel on next page. The recommended source frequency has been adjusted such that the current reaches the DC steady state value during both the growing and decaying cycles. The current has reached DC steady state when the plot is approximately flat. An **example** is shown in Figure A-2. The DC steady state regions are shown between the two dashed lines.
- **Include a plot of your input voltage and inductor current in your report. Adjust vertical and horizontal axis on oscilloscope to show two to three times periods.**

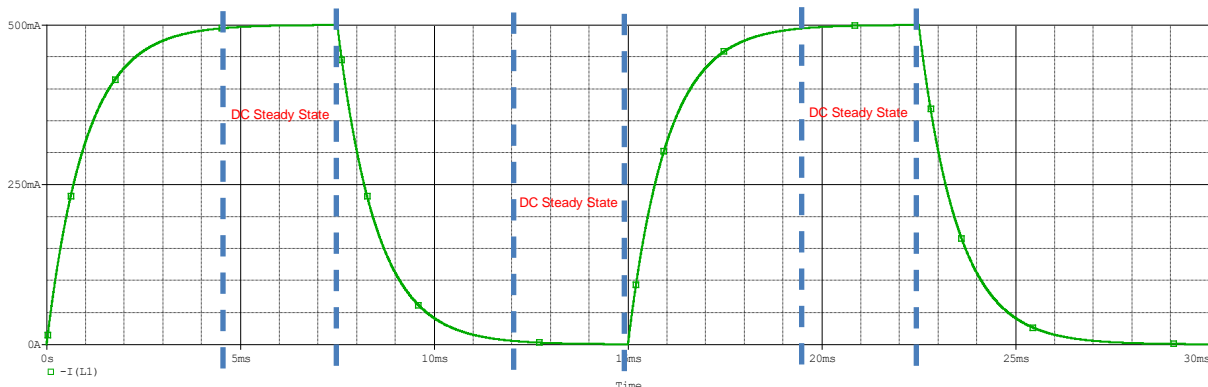


Figure A-2: Circuit with Current Reaching DC Steady State

- In the previous experiment, we discussed time constants, which are a measure of how long the signal takes to reach DC steady state. For LR circuits, the time constant is $\frac{L}{R}$. Based on the components, determine the time constant associated with this circuit.

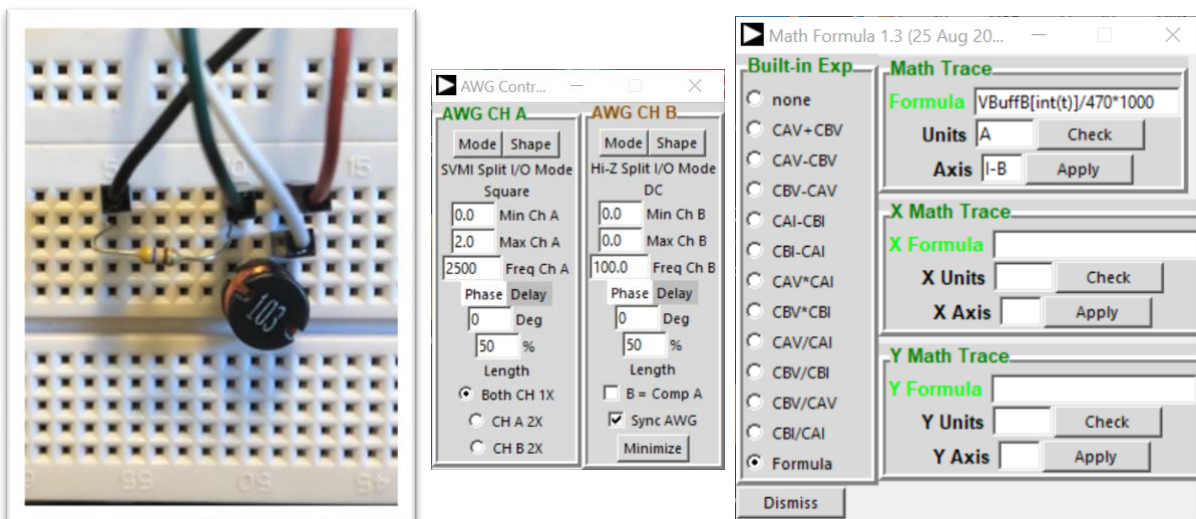


Figure A-3: Images to help identify inductor, AWG controls, and Math channel setup

- We can also find the time constant experimentally (cursors in Alice, or import into Matlab). When looking at the decaying part of the curve, the time constant can be found by determining how long it takes the signal to decay to 36.7% of its starting value. In Figure A-2, the starting value of a decay cycle is 500mA. It takes 1ms to decay to $(0.367 * 0.5) = 183.5\text{mA}$, giving the time constant of 1ms.
- How does your measured time constant compare to the calculated time constant?
- Replace the 470Ω resistor with a 10Ω resistor. Adjust the frequency until you see the current reach DC steady state for each cycle. Include a plot of your current in the report.
- Calculate the time constant and determine the time constant experimentally. Look at the Inductor properties pdf in the Class 20 section of the course web page. Based on the data sheet, why do you think the calculated and measured time constants differed? Using the data sheet, determine the effective resistance of the circuit and recalculate the time constant. Is this value more consistent with the measured value?

Add Math Channel.

For Built-in Exp, select Formula, doing this will plot Math Trace on the oscilloscope. Then Math Trace you will enter: $V_{\text{BuffB}}[\text{int}(t)]/470 * 1000$

Where, V_{BuffB} is voltage B samples, 470 is the resistor value, you multiply 1000 such that you can scale up the plot so it can actually be observed on the software (the software does not allow current measurement/division go below 0.1). To obtain the current value needed to be used in calculation, use cursors and divide by 1000 for the accurate value, or, use Matlab to plot the current plot.

Part B – RLC Circuits

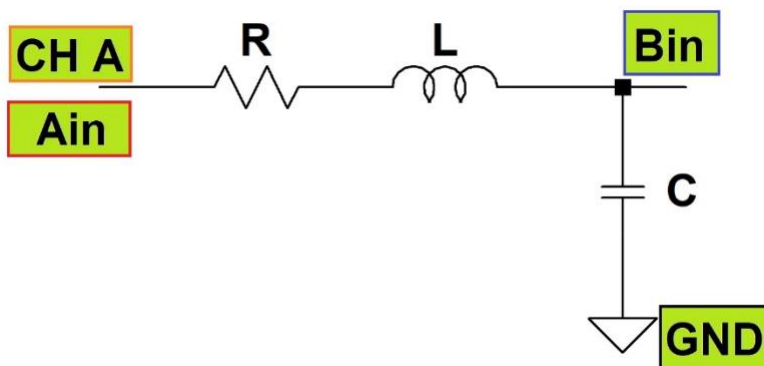


Figure B-1: RLC Series Circuit

The circuit shown in Figure B-1 is an RLC series circuit. This circuit is a second order system. The RL and RC circuits we have studied previously are first order systems. The transient response (response due to a changing source) of a first order system is exponential, as we saw in our plots. The response due to a second order system also includes an exponential component. However, under certain conditions it can also contain a sinusoidal component. There are two parameters of interest when we consider the circuit in Figure B-1, they are the attenuation constant, α , and the resonant frequency (also called the natural frequency), ω_o . For the series RLC circuit, those values are given as

$$\begin{aligned} \text{Attenuation constant, } \alpha &= \frac{1}{2} \frac{R}{L} \\ \text{Resonant frequency, } \omega_o &= \frac{1}{\sqrt{LC}} \end{aligned}$$

Those values are specific to the series RLC circuit and are derived using differential equations, which you will see when you take the class in the math department. For now, we will consider the physical interpretations of those values with regard to the voltage and current measurements in our circuit. The type of functions we will see depends on the relationship between the attenuation constant and the resonant frequency.

If $\alpha > \omega_o$, then the circuit is considered overdamped. All measurements we would take would have similar curves to those seen when we studied RL and RC circuits. The curves would have the exponential functions seen previous, $Ae^{-\frac{t}{\tau}}$ and $A\left(1 - e^{-\frac{t}{\tau}}\right)$, with time constant τ .

For now, we will not consider the overdamped system, but rather study the underdamped system where the relationship between the attenuation constant and the resonant frequency will be $\alpha < \omega_o$. In this case, the response will be a combination of the sinusoid we saw introduced in the last laboratory and the exponential decay we have seen in our first order circuits. The type of response is called a damped sinusoid (damped sine wave). The type of plot is shown in Figure B-2.

We can see that sinusoidal oscillation occurs, however, as time goes on, the amplitude of the sinusoid decreases. We trace an exponentially decaying curve such that it ‘touches’ the top of each sinusoidal peak. This curve the ‘envelope’ of the signal and is seen as the expression, $Ae^{-\alpha t}$. The α in the expression is the attenuation constant we have seen previously, $\alpha = \frac{1}{2} \frac{R}{L}$. This relates to our time constant, τ , with the inverse relationship, $\alpha = \frac{1}{\tau}$.

The (radial) frequency of oscillation for this waveform is defined as β . Note, this value is in radians/s and to convert to Hz we would need to divide by 2π . Experimentally, we can find the oscillation frequency by using the expression,

$\beta = \frac{2\pi}{T}$, where T is the period of one cycle. Using circuit analysis, we can find the oscillation frequency as $\beta = \sqrt{\omega_o^2 - \alpha^2}$.

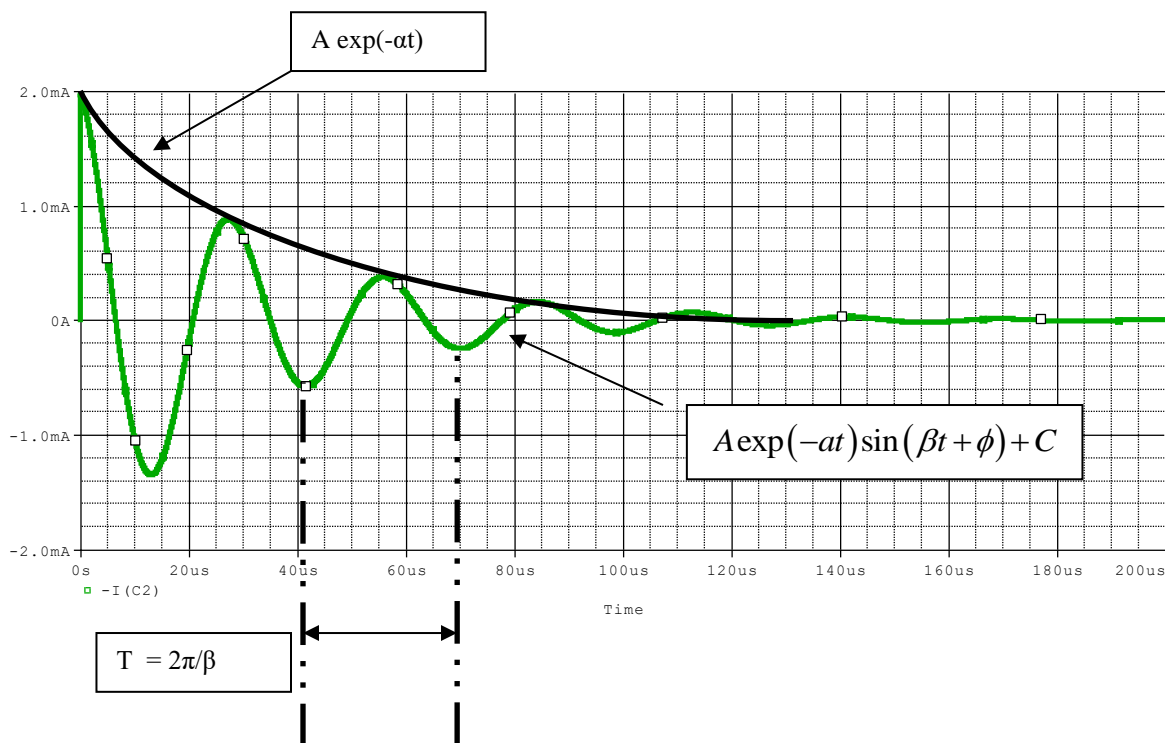


Figure B-2: Damped Sinusoid

- Build the circuit, using the following component values

$$R = 100\Omega$$

$$C = 0.01\mu\text{F} (\mu \rightarrow 10^{-6})$$

$$L = 10\text{mH}$$

Set the function generator to a square wave oscillating between 0V and 2V. Measure the voltage across the capacitor. Set the source frequency (Ch A) to be 400 Hz so that over each cycle, the amplitude of the sinusoid becomes vanishingly small. On the oscilloscope, you will need to adjust the Volts/div and time/div so that you can clearly see the voltage waveform. Include a plot of the capacitor waveform in your report.

- Measure the period of the sinusoid and calculate the oscillation frequency using the expression $\beta = \frac{2\pi}{T}$
- Calculate the oscillation frequency using the circuit expression $\beta = \sqrt{\omega_o^2 - \alpha^2}$
- Pick two peaks on the damped sinusoid curve. You need to pick peaks on the function whose DC steady state value decays to zero. Record the voltage at the peaks, V1 and V2, the times those peaks occur, t1 and t2.

- Use the expression $\alpha = -\frac{\ln\left(\frac{V2}{V1}\right)}{t2 - t1}$ to find the attenuation constant experimentally.

- Calculate the attenuation constant using the circuit expression $\alpha = \frac{1}{2} \frac{R}{L}$

- Repeat all of the above for R = 10Ω. You may need to adjust the source frequency so that over each cycle, the amplitude of the sinusoid becomes vanishingly small.



- For both resistor values, the calculated values and the measured values were likely different. The discrepancy should be worse for the 10Ω measurements. Is that true for your observations? What observation made in Part A would explain these differences?
- Replace the resistor with some other random choices and include the waveforms in your report. Make sure you adjust the oscilloscope so that you can clearly see the waveform. You don't have to do any calculations, just make observations.