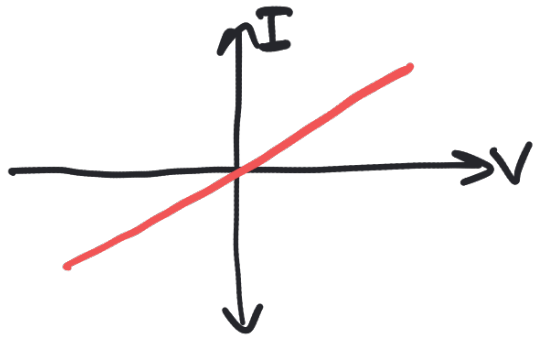
Intro to ECSE Class 16 : Circuit Analysis Method #4: Superposition

I| Properties of Linearity: Elements → Circuits → Systems

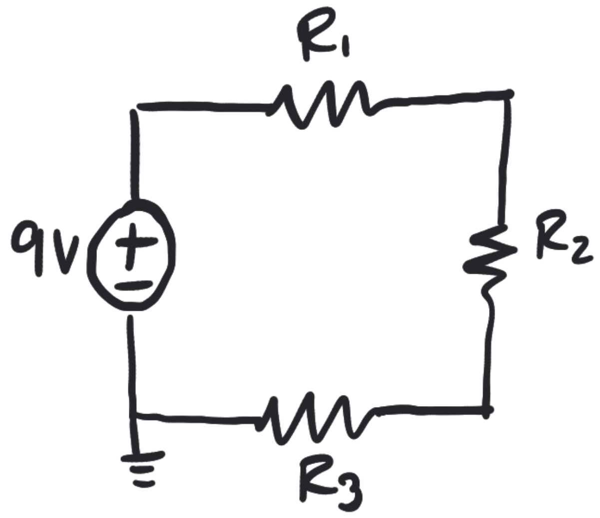
· What have we said linearity means so far?

+ Circuit elements: linear IV characteristic

Resistor

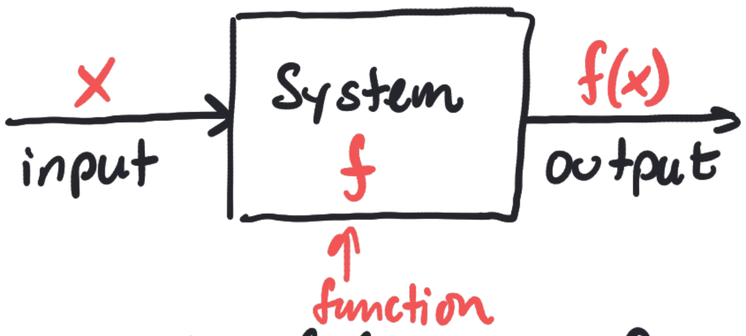


+ Circuits: composed of linear circuit elements



+ What sorts of non-linear elements have we encountered so far?
    - diodes, LEDs
    - op-amps (comparators)
    - transistors

- But what does linearity mean for systems in general?
  - What is a system? something that takes an input, acts on it with a function, then outputs



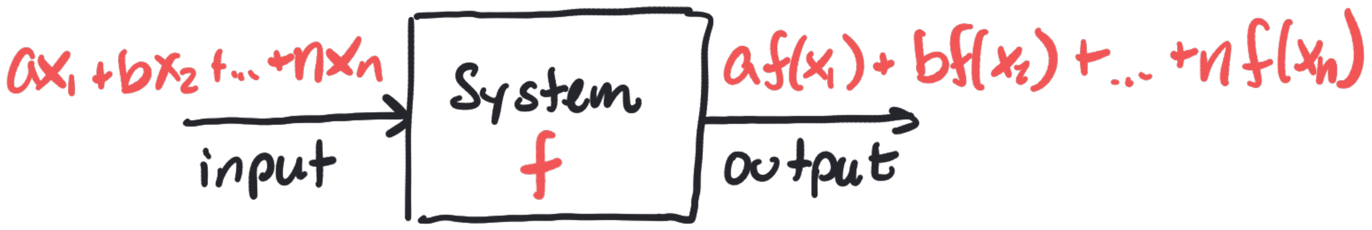
- And in order to determine if our system is linear, we must determine if the function f is linear

a function is linear is linear if it satisfies the Superposition Principle

- A function satisfies the <u>Superposition Principle</u> if it satisfies the following two properties:

  1. Additivity: $f(x_1 + x_2) = f(x_1) + f(x_2)$
     - This means: if instead of inputting a single input x, we input a sum of inputs $x_1 + x_2$, the output is the sum of the outputs for the individual in
  2. Homogeneity: $f(ax) = a f(x)$
     - This means: the output of an input scaled by a constant "a" is "a" times the output for the unscaled input

→ Putting this all together, we have for linear systems:



- And in other words: if we send a $\underbrace{\text{sum of scaled inputs}}_{\text{linear combination}}$ to our system, the output is the scaled output for each of the individual inputs

- Some examples: are these functions linear?
  - 1. Multiplication: amplification, Ohm's law...

$f(x) = -5x$

output ↑ input ↗

✓ Additivity: $f(x_1 + x_2) = f(x_1) + f(x_2) \rightarrow$ inputs $x_1 = 1, x_2 = 2$

linear

- $f(x_1 + x_2) = f(1+2=3) = -5(\underset{\downarrow x_1+x_2}{3}) = -15$ =
- $f(x_1) + f(x_2) = -5(1) + -5(2) = -5 - 10 = -15$

✓ Homogeneity: $f(ax) = af(x)$, $a = 2$, $x = 3$

- $f(ax) = f(2x) = -5(2 \cdot 3) = -5 \cdot 6 = \underline{-30}$ =
- $a f(x) = 2 \cdot f(3) = 2 \cdot (-5 \cdot 3) = 2 \cdot (-15) = \underline{-30}$

2. Addition: $f(x) = x + 1$

✗ Additivity: $x_1 = 1, x_2 = 2$

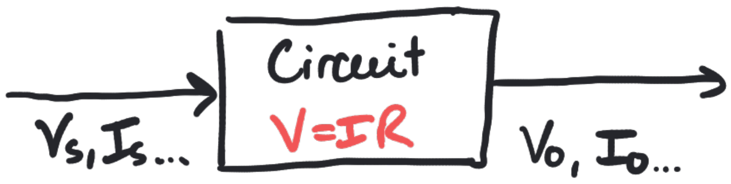
not linear!

- $f(x_1 + x_2) = f(1+2) = f(3) = 3 + 1 = \underline{4}$ ≠
- $f(x_1) + f(x_2) = f(1) + f(2) = \{1+1\} + \{2+1\} = 2 + 3 = \underline{5}$

- differentiation and integration are linear

II] Circuit Analysis Method #4: Superposition

- What function defines our circuit as a system?
  $V = IR$ Ohm's law



- Our system is governed by Ohm's law → multiplication by R or 1/R → linear!

- What does it look like if we apply the Superposition Principle to our circuit?

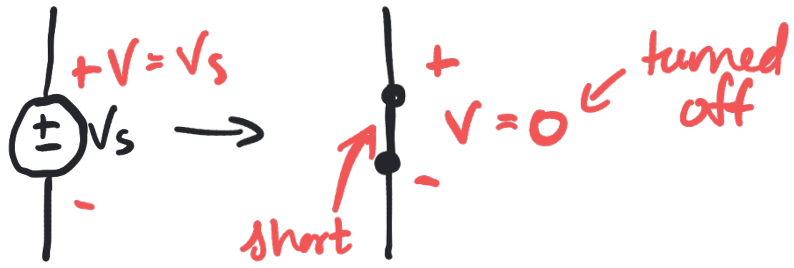
$$V_o(V_{in,1} + V_{in,2} + \dots + V_{in,N}) = V_o(V_{in,1}) + V_o(V_{in,2}) + \dots + V_o(V_{in,N})$$

↖ output voltage

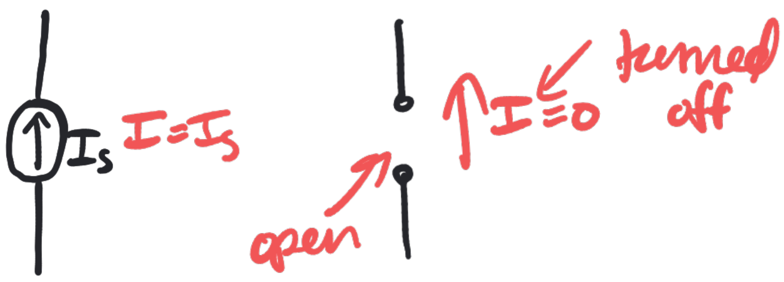
- In words: If we have a circuit with multiple sources, we can calculate the output by summing the outputs from individual sources

- But how do we get the output from an individual source?
  - We need to turn off all other sources

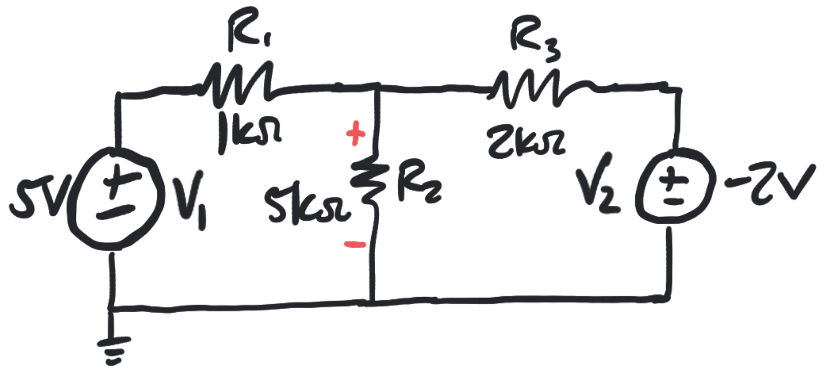
- Voltage sources:
  on : $V = V_s$
  off : $V = 0$
- replace w/ short circuit



- Current sources:
  on : $I = I_s$
  off : $I = 0$
- replace w/ open circuit

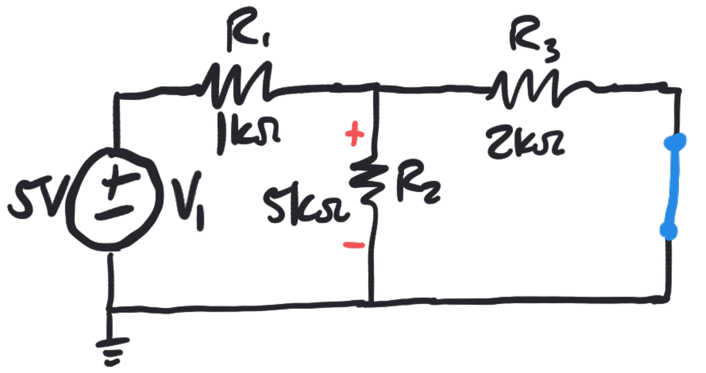


- Let's see how this works via an example



- Solve for V_{R2} using superposition

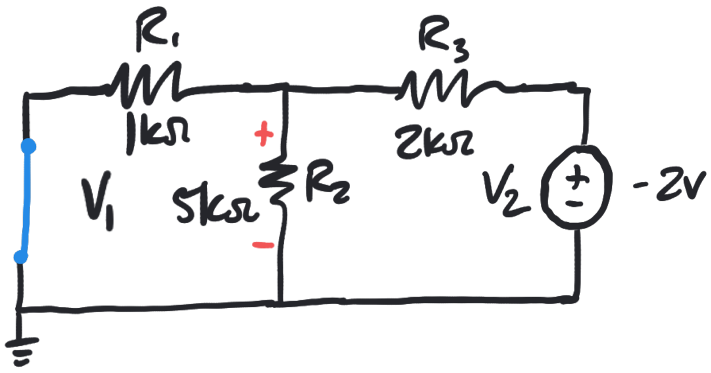
1. Turn off V_2 and solve for V_{R2,V_1}



Using voltage divider eq.:

$$V_{R2,V1} = 5V \cdot \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} = 5V \frac{1.429k\Omega}{1k\Omega + 1.429k\Omega} = \underline{2.942V}$$

2. Turn off V_2 and solve for $V_{R2,V2}$

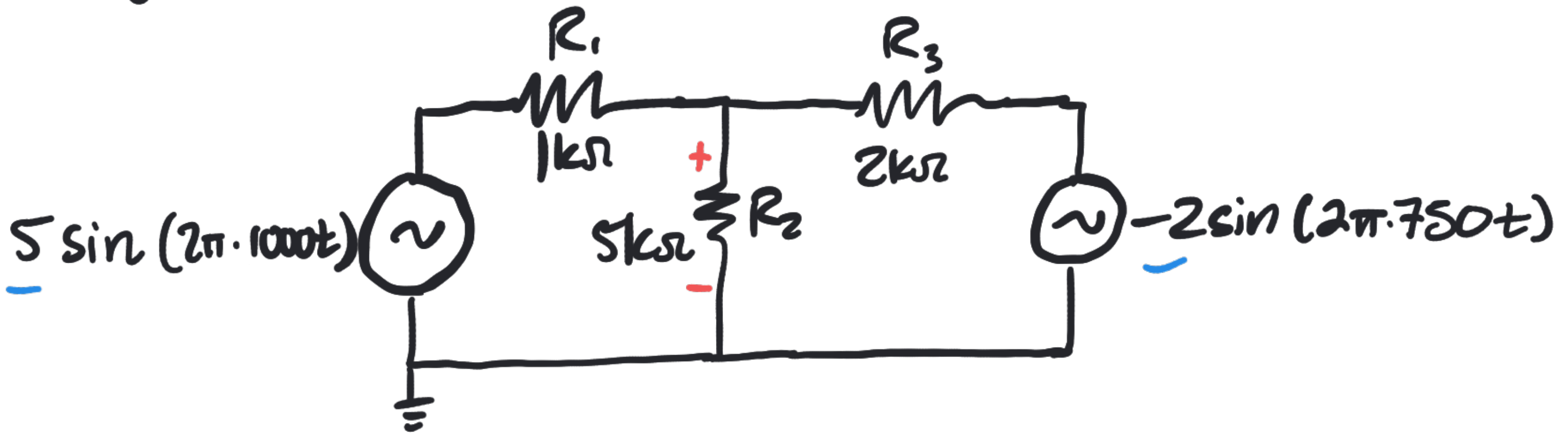


$$V_{R2,V2} = (-2V) \frac{R_1 \| R_2}{R_3 + R_1 \| R_2} = (-2V) \frac{0.833k\Omega}{2k\Omega + 0.833k\Omega} = \underline{-0.588V}$$

3. Add the two results together

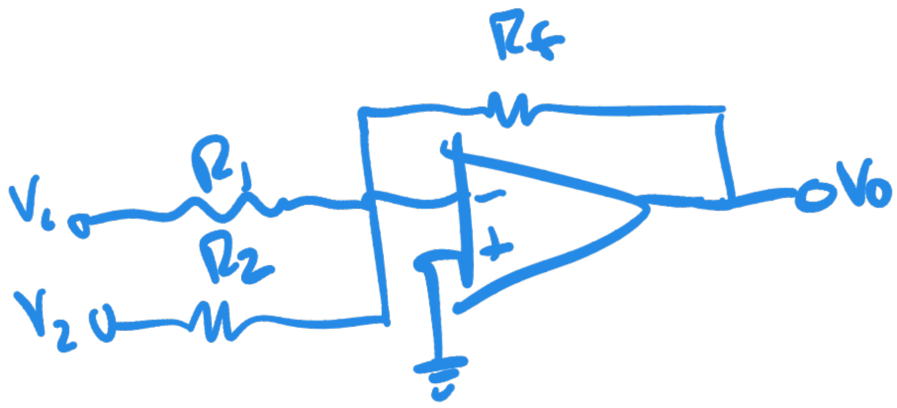
$$V_{R2} = V_{R2,V1} + V_{R2,V2} = 2.942V - 0.588V = \underline{2.354V}$$

- But why bother with this at all? When is this useful? b



- Solve for V_{R2} using superposition

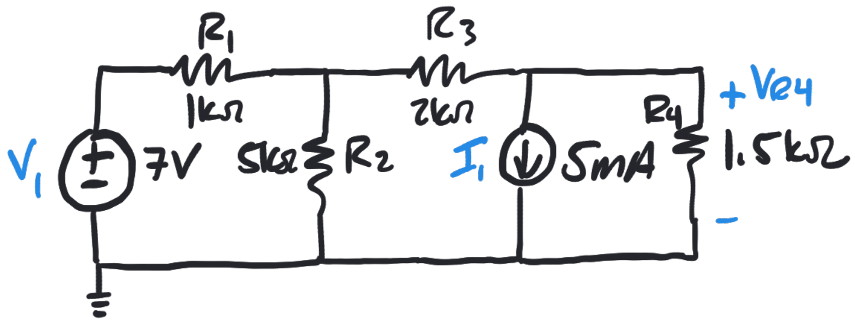
- $V_{R2} = V_{R2,V_1} + V_{R2,V_2} = 2.942\cdot\sin(2\pi\cdot 1000t)$
$-0.568\sin(2\pi 750t)$



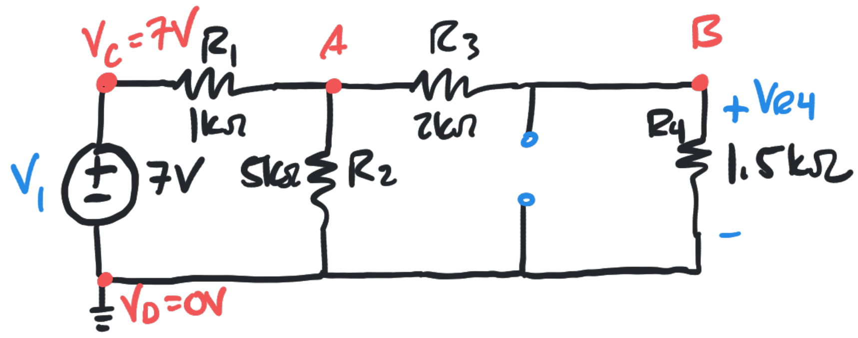
$$V_o = -\frac{R_f}{R_1}V_1 - \frac{R_f}{R_2}V_2$$

Extra Practice Problem

- Find V_{R4} using superposition



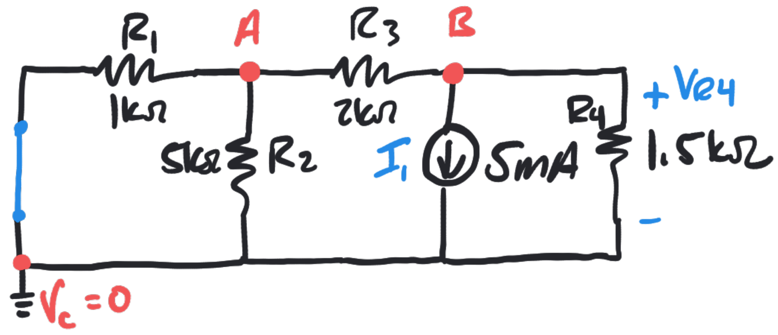
1. V_{R4} due to V_1:



- using nodal analysis
- node A: $\frac{V_A - 7V}{1k\Omega} + \frac{V_A}{5k\Omega} + \frac{V_A - V_B}{2k\Omega} = 0 \rightarrow (1/1k\Omega + 1/5k\Omega + 1/2k\Omega)V_A + (-1/2k\Omega)V_B = 7V/1k\Omega$
- node B: $\frac{V_B - V_A}{2k\Omega} + \frac{V_B}{1.5k\Omega} = 0 \rightarrow (-1/2k\Omega)V_A + (1/2k\Omega + 1/1.5k\Omega)V_B = 0$

$$\begin{bmatrix} (1/1k + 1/5k + 1/2k) & -1/2k \\ -1/2k & (1/2k + 1/1.5k) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 7/1k \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 4.712V \\ \underline{2.019V} \end{bmatrix}$$

2. V_{R4} due to I_1:



- using nodal analysis

node A: $\frac{V_A}{1k\Omega} + \frac{V_A}{5k\Omega} + \frac{V_A - V_B}{2k\Omega} = 0 \rightarrow (1/1k\Omega + 1/5k\Omega + 1/2k\Omega)V_A + (-1/2k\Omega)V_B = 0$

node B: $\frac{V_B - V_A}{2k\Omega} + 5\times10^{-3}A + \frac{V_B}{1.5k\Omega} = 0 \rightarrow (-1/2k\Omega)V_A + (1/2k\Omega + 1/1.5k\Omega)V_B = -5\times10^{-3}A$

$$\begin{bmatrix} (1/1k + 1/5k + 1/2k) & (-1/2k) \\ (-1/2k) & (1/2k + 1/1.5k) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 0 \\ -5\times10^{-3}A \end{bmatrix} \rightarrow \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -1.442V \\ \underline{-4.904V} \end{bmatrix}$$

3. V_{R4} due to V_1 + I_1:

$$V_{R4} = 2.019V - 4.904V = \underline{-2.885V}$$

Upcoming Assignments & Due Dates

1. Lab 01 Optimization   Due Date: 10/26