

## LABORATORY 2: Linear Systems and Beyond...

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### Getting Started!

#### Lab Submission Instructions and Partners:

- 1) Keep your partners! If a student dropped the course, let us know!
- 2) Decide if you'd like to do an Alpha Experiment or Omega Exploration.
- 3) You will go through each lab section to learn some basic steps. ***Then you will prove specified concepts listed at the end of each section.*** This simply means that you will demonstrate how a specific concept works through clear and concise comparisons of mathematical analysis, simulation, and experimental measurements
- 4) The template for the **Proof of Concepts** document that you will submit can be found here:  
[https://sites.ecse.rpi.edu/courses/F24/ECSE-1010/resources/labs/templates/Proof\\_of\\_Concepts\\_Template\\_F24.docx](https://sites.ecse.rpi.edu/courses/F24/ECSE-1010/resources/labs/templates/Proof_of_Concepts_Template_F24.docx)

Please answer any questions related to those concepts and provide mathematical calculation, simulation, and experimental data to support the proof of concept!

## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

**Purpose:** The objective of this experiment is to introduce you to linearity, some mathematical tools that help you understand complicated systems (linear algebra for example) and provide examples of non-linearity to compare to. To start, you'll determine whether a circuit component is linear by finding its I-V characteristic. Then, we'll introduce principles of linearity to systems and control. We'll define a complicated system as a black box with a transfer function and investigate a literal black box to explore operational amplifiers. *In ECSE, the idea of linearity affects multiple sub-disciplines.*

**Student Preparation BEFORE doing this experiment:** (Students should be able to)

- Apply Ohm's Law to determine the current through a resistor
- Determine the values of series and parallel combinations of resistors
- Download and install software on a Windows machine or Mac machine

**Learning Outcomes AFTER doing this experiment:** (Students will be able to)

- Find the current-voltage characteristic of a device using the voltmeter and/or oscilloscope.
- Define a linear component (resistor) and a non-linear component (light emitting diode).
- Use nodal analysis to solve for voltage and current in a linear resistive circuit.
- Use matrix math to solve a set of linearly independent equations.
- Identify different operational amplifier (op-amp) configurations to do math or make a decision.
- Discuss the idea of a transfer function at a high level.
- Articulate a series of questions posed about simple circuits and answer the questions using data obtained from physical experiments.
- Demonstrate to themselves and others the difference between tinkering and engineering by answering the question "Is this right?" through documenting the comparison of mathematical analysis, simulation and experiment to prove a concept or an idea!

**Software and Equipment Required:**

- LTSpice
- MATLAB Simulink
- ADALM2000 (M2K) board with Scopy or Analog Discovery with Digilent Waveforms
- Several different Resistors and wires from the ADALP2000 Parts Kit
- Light Emitting Diodes (LEDs)
- Operational Amplifiers (OP484 or OP482) - LTSpice use "universal opamp2"
- Omega Exploration Optional: resistive sensors, Arduino, push buttons or switches, op-amps
- Protoboard

**Learning from Proof of Skills applied to this lab:**

**Professional Accountability:** I **can clearly document and compare a calculated, simulated, and experimental result** to answer the question "Is this right?" for myself

**Circuit Simulation (LTSpice or equivalent):** Nodal Voltages

I **can label and identify Nodal Voltages** in a circuit. (creating a well labeled schematic!!)

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I can use **transient analysis with a sinusoidal source** to measure voltage across ONE resistor in a resistive circuit (**Total resistor count in the circuit must be two or more**)

I can step through parameters with **parametric analysis** to repeatedly measure voltages as I vary my resistance over a range of values

**Experimental Measurement and Personal Instrumentation:** I can use my instrumentation board's function generator to **create a DC, sinusoid, and pulsed signal** and measure with its oscilloscope directly (*hint: no circuit necessary but need external wires!*)

I can **build a resistive circuit** and **measure dc voltage across ONE resistor using a dc input source** and vary dc voltage at least 3 times (-5,+5 and any voltage in between) (**Must be two or more resistors**, *hint: to do something useful to you, try to simulate a homework or class problem!*)

I can **build a resistive circuit** and **measure the dc current through ONE resistor using a dc source** (OR find another way if needed depending on board!) (**Must be two or more resistors**, *hint: to do something useful to you, try to simulate a homework or class problem!*)

I **can use my cursor function** to show specific voltage and time points.

**MATLAB and Simulink Basics:** I **can analytically determine the amplitude, frequency, period and phase shift of a sinusoid** (*hint: for phase shift you will need a reference point which could be two different sinusoids plotted together!*)

I **can find the solutions for linearly independent equations using the matrix function** on my personal calculator (TI-XX) and compare it to the calculation in MATLAB

I **can import simulation data** (from LTSpice or equivalent) **to MATLAB** and plot the function

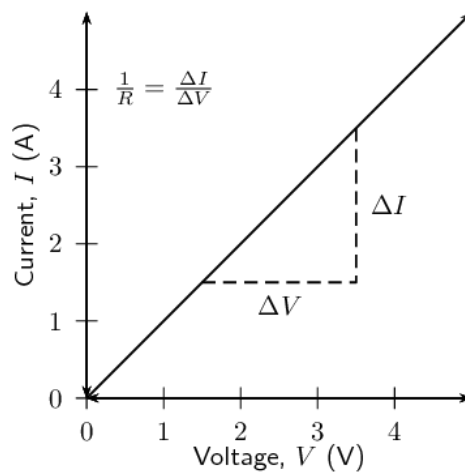
I **can use a regression** in MATLAB to help define my function

## PART A [Core] – Defining Linear and Non-linear Devices with Current-Voltage Characteristics

Material covered: *I-V characteristics of Resistors and LEDs*

I-V Characteristics of Resistors and Light Emitting Diodes Background:

*I-V Characteristic of Resistors:* The relationship between current and voltage in a resistor is defined by Ohm's Law,  $V = IR$  (Voltage = Current\*Resistance). This equation is a linear relationship, following the classic expression,  $y = mx+b$ , where  $m$  is the slope of the line and  $b$  is the y-intercept. When considering Ohm's Law, we can say the y-variable is the voltage,  $V$ , and the x-variable is the current,  $I$ . The slope of the line is then the resistance, with the y-intercept being zero.



**Note: In your experiments and simulation below, the x axis is current (I) and the y axis is voltage (V). This makes the slope R instead of 1/R!**

Figure 1: I-V characteristic of a resistor. Ohm's Law. <https://www.siyavula.com/read/science/grade-11/electric-circuits/11-electric-circuits-02>

*I-V characteristic of a Light Emitting Diode:* In order to understand how a diode functions, it is useful to look at a plot of the voltage across the diode vs. the current through the diode. We call this type of curve an I-V characteristic curve. If we were to create an I-V curve of a resistor, where the current is directly proportional to the voltage ( $V=IR$ ), we would see a straight line with a constant slope of  $R^{-1}$ . When we plot the characteristic curve of an ideal diode (that switches on when the voltage across it goes above zero), we see zero current when  $v_D$  is negative and infinite current as soon as  $v_D$  tries to go positive. This is shown in Figure 2. Note that, when an ideal diode turns on, it is a short circuit and, therefore, the voltage across the ideal diode when it is on is always zero.

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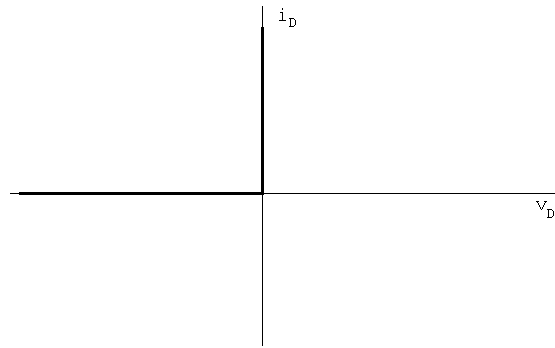


Figure 2: I-V characteristic for an ideal diode

*I-V curve of a diode:* In comparison, Figure 3 shows typical characteristics of a real diode.

Ideally, a diode is a device that allows current to flow in one direction only. In practice, diodes allow large amounts of forward current to flow when the positive voltage across them reaches a small threshold. They also have a small “saturation” current, which flows in the opposite direction when the voltage across the diode is negative, and a “breakdown” region in which a large amount of current will flow in the opposite direction when a large negative voltage is applied. In small signal diodes, the forward current will typically be up to a few tens of mA at a forward voltage of about 1V. The reverse-breakdown voltage might be about 100V, and the saturation current  $I_s$  may be on the order of 1nA. Power diodes may allow forward currents up to many amperes at forward voltage drops of 0.6 to 1.5V or so, depending on the type of diode. The reverse-breakdown voltage of power diodes may range from as low as 50V up to 1000V or even much more.

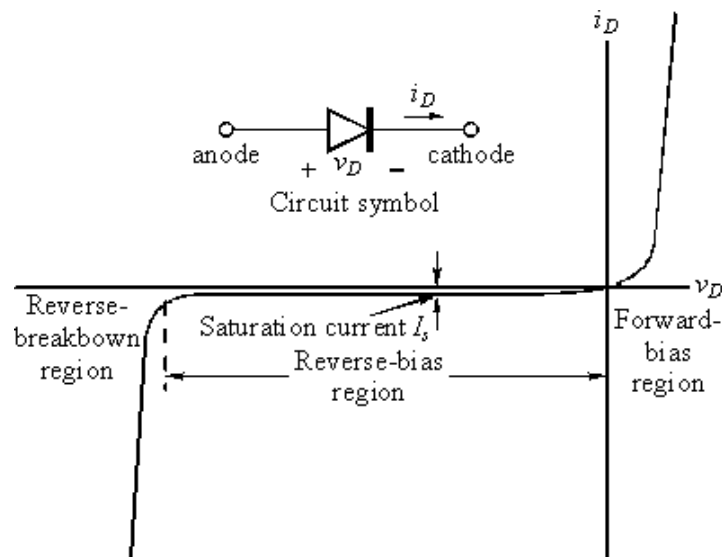


Figure 3: I-V characteristic of a real diode

## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

*The ideal diode equation:* The equation below gives a reasonably good representation of the  $I$ - $V$  characteristics of a diode.

$$i_D = I_S \left( e^{v_D/nV_T} - 1 \right)$$

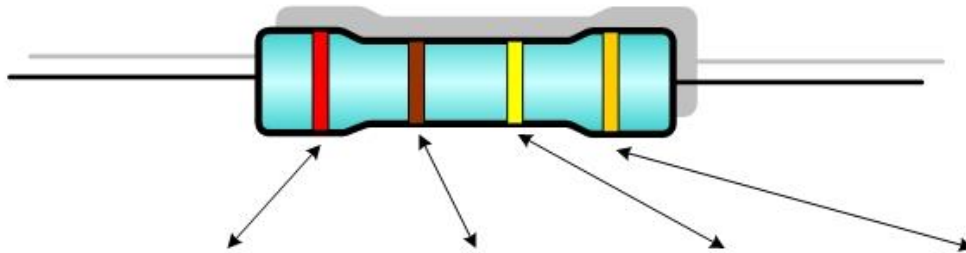
$I_S$  is the saturation current usually measured in microamps, or nanoamps.  $V_T$  is the Thermal Voltage where  $V_T = k_B T/q = 0.0259V$  at 300K and  $n$  is an empirical parameter that depends on how the diode is constructed and usually lies between 1 and 2. Note that this equation characterizes the basic features of the diode  $I$ - $V$  curve, but leaves out some details like reverse breakdown, junction capacitance, etc.

### *Determining Resistor Values*

Select any resistor from your parts box. (It is a good idea to pick a resistor smaller than  $1M\Omega$ ,  $10^6\Omega$ . If you choose a large resistance it may severely affect the accuracy of your voltage measurements.)

- 1) Write down the four-color code for any resistor you choose in your Proof of Concept document.
- 2) Using the following table, determine the resistance of the resistor you selected.

### 4-band Resistor



Color	1 <sup>st</sup> band value	2 <sup>nd</sup> band value	Multiplier	Tolerances
Black	0	0	× 1	
Brown	1	1	× 10	± 1%
Red	2	2	× 100	± 2%
Orange	3	3	× 1000	± 3%
Yellow	4	4	× 10,000	± 4%
Green	5	5	× 100,000	± 0.5%
Blue	6	6	× 1,000,000	± 0.25%
Violet	7	7	× 10,000,000	± 0.10%
Grey	8	8	× 100,000,000	± 0.05%
White	9	9	× 1,000,000,000	
Gold			× 0.1	± 5%
Silver			× 0.01	± 10%
No band				± 20%

You can use an online resistor calculator to verify your calculation, a nice reference can be found at All About Circuits using the following link.

<https://www.allaboutcircuits.com/tools/resistor-color-code-calculator/>



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### I-V Characteristics of Resistors and Light Emitting Diodes Experiment:

#### I-V Characteristic of Resistors

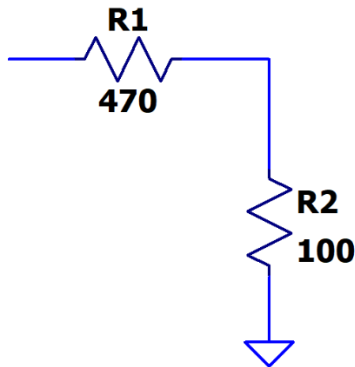


Figure 4: Set of two resistors for I-V measurements.

#### Analysis

1. Draw the I-V characteristics for each of the resistors above by hand!
2. Rock out to this ~1 minute video to learn how... <https://youtu.be/ZDAltCd1M-U>
3. Make the chart for R1 and R2 separately, then plot them together on the same x-y axis (current-voltage axis respectively) ...you may use excel if you'd like or draw by hand. Either one is fine. You should have a line on your IV characteristic for each resistor.

#### Simulation (LTSpice)

1. Simulate the I-V characteristic of each of your resistors using parametric analysis
2. See <https://iexploresiliconvalley.com/2019/02/03/ltspice-lesson-1-generating-iv-curves/>
3. To plot current on the y-axis, click directly on the resistor (an icon with an arrow going through a circle in the direction of how LTSpice has defined current flow should appear). To plot voltage on the x-axis, right click on the x-axis and in the text field "Quantity Plotted", replace "time" with whatever value you want to plot. For example, the voltage across resistor R1 would be V(R1).

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Measurement (M2K or Analog Discovery Board and Multimeter)

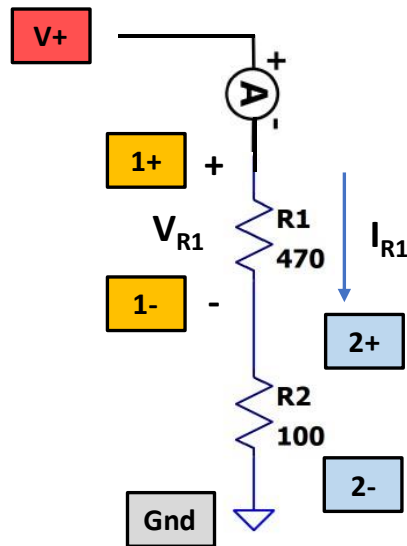


Figure 5: Circuit setup for measuring the IV characteristic of a resistor. The symbol with the “A” in a circle is that of an ammeter, which measures the current.

Using the circuit in Figure 5 you will measure the IV characteristic of a resistor. You will change  $V+$  to obtain different voltages across resistors  $R1$  and  $R2$ , and use an ammeter (a function of your multimeter) to measure the current through the circuit for each different DC voltage.

1. Build the circuit in Figure 5, including connecting the signal wires from your instrumentation board as shown. Be sure that you have both  $V+$  (voltage source) and ground connected.
2. To measure current, you will need to use the ammeter function on your multimeter. In the circuit in Figure 5, the ammeter is represented by the symbol of an “A” in a circle, which is in series with  $R1$ ,  $R2$ , and the DC voltage source  $V+$ . *Note:* although you measure voltage by placing the measurement leads in parallel with the element whose voltage you want to measure, *you must place the ammeter in series with the element whose current you want to measure.* That means you’ll need to leave a gap between the voltage source and  $R1$  on your breadboard, then bridge that gap using the ammeter’s measurement leads. See this [short video on how to measure current using a multimeter](#).
3. Once your circuit is constructed, set the DC voltage source to 3V. Use channel 1 of your instrumentation board to measure the voltage across  $R1$  and channel 2 to measure the voltage across  $R2$  via your instrumentation board software’s voltmeter. Record the values in the table under “ $R1$  Voltage” and “ $R2$  Voltage”.
4. Use the multimeter to measure the current through  $R1$  and  $R2$ . Take note of the units your multimeter is using to report current. Enter the current value in the table under “Current”.
5. Repeat voltage and current measurements for  $R1$  and  $R2$  (separately) for values of  $V+$  ranging from 3V down to 0V and fill out the table.

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<b>V+ Voltage (V)</b>	<b>R1 Voltage (measured)</b>	<b>R2 Voltage (measured)</b>	<b>Current (measured)</b>
0			
0.5			
1			
1.5			
2			
2.5			
3			

6. Enter this data into MATLAB and plot the resistor current  $I$  (x-axis) vs the resistor voltage  $V$  (y-axis) for each of the resistors.
7. Fit a linear regression curve to the data for each of the resistors (if you've forgotten how, see Proof of Skills). From the slope of your fitted curves, determine the resistance of each of your resistors.
8. Using the ohmmeter function of your multimeter, measure the actual resistance of each of your resistors (measure them "in parallel" like you would measure the voltage across them). How close are their values to the resistances you determined from your IV measurement data and the linear regression?

## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

### I-V Characteristic of a Diode

#### Analysis

1. Choose a diode from your kit. It doesn't matter which kind but write down which one you chose.
2. Draw the I-V characteristics of the diode from your kit by hand. Include and label the turn-on voltage, reverse breakdown voltage, and saturation current. You will need to find these values in the LED's data sheet.

#### Simulation (LTSpice)

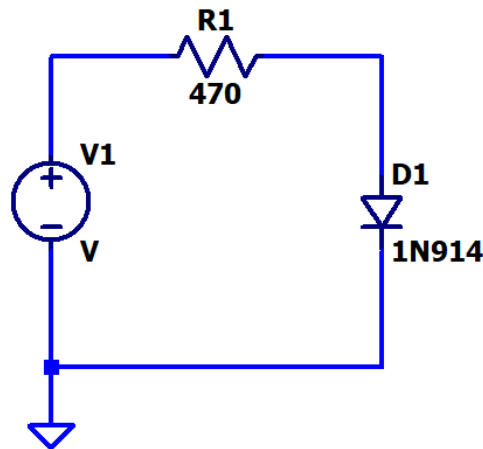


Figure 6: Diode in series with a 470 $\Omega$  current-limiting resistor

1. Simulate the forward I-V characteristic of a diode using a transient simulation and a pulsed voltage source. The pulsed voltage source will supply a ramp signal to your diode, which will sweep through a range of voltages over time.
  - a. To create a pulsed voltage source, add the usual voltage source to your circuit, then right click on it to open the source settings. Click on "Advanced" to open show options.
  - b. On the left-hand side, click on "PULSE" to create a pulsed source.
  - c. You will need to create a voltage ramp, which starts at -5V and ends at 5V.
    - i. The setting "Vinitial[V]" sets the initial voltage of the pulse, which will be -5V.
    - ii. "Von[V]" is the maximum voltage for the pulse, which we want to be 5V.
    - iii. "Trise[V]" is the amount of time it takes the voltage to rise from Vinitial to Von. To be consistent with our experiment later, set this to 0.1s.
    - iv. "Tfall[V]" is the amount of time it takes the voltage to fall from Von to Vinitial; set this to 1e-6s.
    - v. "Ncycles" is the number of times the pulse will repeat. Set this to 3.
2. Run a transient simulation. Set the stop time to be the same as Trise and make sure you have at least 100 points in your simulation (adjust the time step parameter).
3. Once you measure the voltage across the diode, you should see a curve that shows the diode voltage on the y-axis and time on the x-axis. We need to change the x-axis to current.

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- a. Right click on the x-axis and a window should appear with a field for you to enter the quantity to be plotted on the horizontal axis.
  - b. In the field, enter "I(D1)", where "D1" is the name of the diode in your LTspice schematic, and click "ok". This will plot the current flowing through diode D1 on the x-axis and keep the voltage across diode D1 on the y-axis.
4. Your plot should show the exponential I-V characteristic of your diode. What is the turn-on voltage of your diode? Don't worry if it's different from the one in your kit: the diode in LTspice will be different.

### Measurement (M2K or Analog Discovery Board and Multimeter)

We can experimentally find the exponential I-V curve using the Figure 7 circuit (drawn below). You will supply the circuit with a triangle wave, which will sweep through a range of voltages over time, then measure the diode voltage  $V_D$  on channel 1, while also measuring the resistor R1 voltage on channel 2. You'll use a math channel to estimate the current through R1, which is the same as the diode current.

1. Construct and connect the circuit as shown below. You will be supplying a time-dependent voltage to your circuit, so you'll need to use the signal generator (W1 or W2).

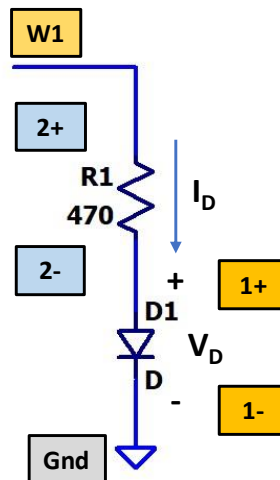


Figure 7: LED with current limiting resistor in series for I-V characteristic measurements.

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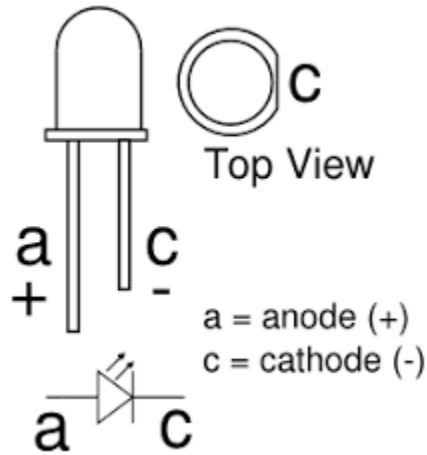


Figure 8: LED configuration for anode and cathode. You must align this correctly for the LED to turn on!

2. You will need to set up your instrumentation board to output a “triangle” wave via the signal or waveform generator.
  - a. Once you’ve opened the waveform generator, choose “triangle”.
  - b. Set the amplitude to 5V (10 volts peak to peak), frequency to 200 Hz, and phase to 90 degrees. This should generate a triangle wave that starts at -5V, takes 10ms to rise to 5V, then ramps back down to -5V for 10ms. This cycle repeats every 5ms.
3. Create a math channel from channel 2 that will plot the current through the resistor (if you don’t know how to do this, see Proof of Skills!). *Note:* Since the current divided by the resistance may be a very small number in amperes, your oscilloscope may not plot it smoothly and it could look digitized. One possible solution is to multiply your math channel result by 1000 (i.e.  $VR1/R1*1000$ ) and report your current in milliamperes instead. This will generate a smoother curve, but remember you’ll have to convert back to amperes when you plot your data in Matlab!
4. Adjust your oscilloscope to show at least two time periods of the diode voltage and the resistor current (math channel) and follow these steps to sketch the I-V characteristic.

### *For the M2K*

- a. In the oscilloscope, click on the settings (the gear wheel in the top right). Turn on the setting called “X-Y”.
- b. Set the diode current (the math channel) to be plotted on the x-axis and the diode voltage (channel 1) to be plotted on the y-axis.

### *For the Analog Discovery 2*

- a. In the oscilloscope, click on “View” and then “Add XY”. A new plot should appear.
- b. You may need to switch which channel is shown on the x-axis and which channel is shown on the y-axis. Plot the diode current (math channel) on the x-axis and diode voltage

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- Export the data from the software so that you can plot it in Matlab. Additionally, save this plot for submission with your proof of concepts.
- You will also need to show that the differential resistance varies at different points along the IV curve. You can approximate the differential resistance around a point by choosing two adjacent current-voltage pairs ( $I_1, V_1$ ) and ( $I_2, V_2$ ) and finding the slope of the line between the two of them:

$$R_{diff} \approx \frac{\Delta V}{\Delta I} = \frac{V_2 - V_1}{I_2 - I_1}$$

You will need to do this for at least 2 locations on the curve to show that the differential resistance changes along the I-V characteristic, as demonstrated in Figure 9.

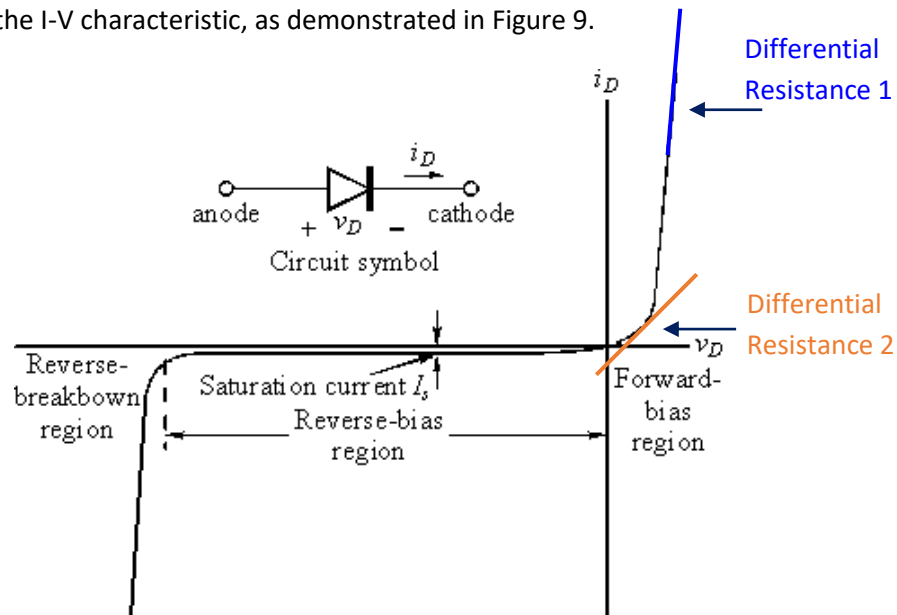


Figure 9: Differential resistance in a real light emitting diode I-V characteristic

### Part A: Proof of Concepts List

1. Prove that the slope of an I-V curve corresponds with ohm's law for two different resistor values.
2. Prove the non-linear I-V curve for a light emitting diode.
3. Show/demonstrate that the differential resistance changes in different regions in the diode I-V curve.

## Part B [Core] – Solving a Linear Resistive Network using Nodal Analysis and Linear Algebra (Matrix Math)

*Material covered: Nodal Analysis, Linear Independent Equations and Matrices*

### Nodal Analysis and Linear Algebra Background:

#### Nodal Analysis Mechanics with Voltage Differences and Ohm's Law

In previous experiments we have discussed the relationship between voltage and current in a resistor (Ohm's Law). In the last experiment we used nodal analysis to determine the voltage across a resistor (or combination of resistors). The voltage we use in Ohm's Law is always a voltage difference, i.e. the voltage on one side of the resistor minus the voltage on the other side of the resistor.

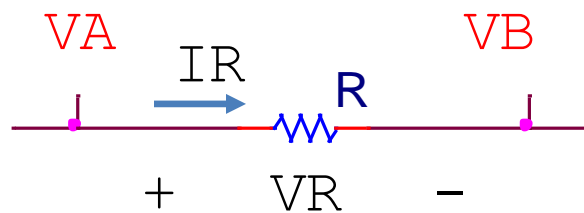


Figure 10: Voltage of resistor between nodes VA and VB

In Figure 10, the voltage across the resistor is  $V_R = V_A - V_B$ , as we saw in the previous laboratory. Likewise, the current through the resistor is  $I_R = V_R/R$ . We can then use the voltage difference expression and rewrite the current through the resistor as,  $I_R = \frac{V_A - V_B}{R}$ . This form of the equation is used to set up a system of equations to find the voltage at every node in a circuit.

Important note: Current in a resistor has direction and will always flow from a higher voltage node to a lower voltage node. In Figure 9, the '+' side of the resistor is assumed to be a higher voltage, indicating that current will flow to the 'right'. These designations are called the **polarity** of the voltage and current. It is possible that VB is a higher voltage than VA, which would result in VR being negative (using the expression  $V_R = V_A - V_B$ ). If the voltage across a resistor is negative, then the current will also be negative since we always use the same polarity designations.

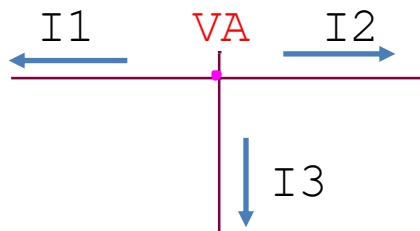


Figure 11: Currents leaving node VA



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*Kirchoff's Current Law (KCL):* In words, KCL indicates that the total amount of current entering a node must be equal to the total amount of current leaving a node. In other words, there is no accumulated charge (electrons) at a node. Visually, we can represent that concept in Figure 11. Node VA has three paths (connections to various components. If we draw currents leaving the node, as shown in the figure, we can express KCL mathematically as,

$$I_1 + I_2 + I_3 = 0$$

In this expression, we notice that at least one of the currents must be negative. In other words, at least one of the currents must be entering the node. As mentioned above, a negative current just means we 'guessed' the wrong direction when we drew the arrow.

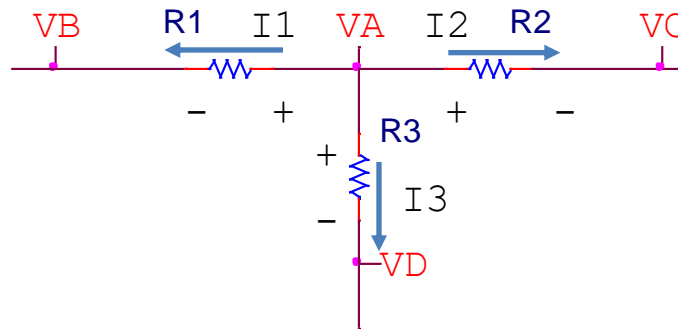


Figure 12: KCL applied at node VA with current leaving node

We can apply KCL to a node connected to resistors, as shown in Figure 12. Again, we assume currents are leaving the node. Based on that current polarity, we must assign a consistent voltage polarity, with the + side on the VA side of each resistor and the – side on the other node for each resistor (VB for R1, etc.). Applying KCL, we get the same expression as seen on the previous page,

$$I_1 + I_2 + I_3 = 0$$

We can now apply Ohm's Law to each of those currents and use voltage difference concepts to include the nodal voltages,

$$I_1 = \frac{V_A - V_B}{R_1}$$

$$I_2 = \frac{V_A - V_C}{R_2}$$

$$I_3 = \frac{V_A - V_D}{R_3}$$

We can substitute these Ohm's Law expressions back into the KCL expression, to obtain an equation

$$\frac{V_A - V_B}{R_1} + \frac{V_A - V_C}{R_2} + \frac{V_A - V_D}{R_3} = 0$$

We now have one equation with four unknowns (VA, VB, VC, and VD). In order to find the values of each of those nodal voltages, we would need three more **linearly independent** equations.

### Summarizing Nodal Analysis:

To solve for the voltages at the nodes of a circuit, a summary of the process is as follows.

- 1) Label all the nodes in the circuit.

## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

- 2) Pick a node to be common ground and set the voltage at the node to be zero.
- 3) For each voltage source, use voltage differences to find the voltage at another node (note, the voltage source needs to be connected to ground).
- 4) At each of the remaining unknown nodal voltage, apply KCL to get an equation
- 5) Implement the system of equations as a matrix expression,  $Ax = b$ .
- 6) Solve the matrix expression

### Nodal analysis design problem

In many design problems, we can find there is tremendous flexibility. Often there will be more variables than constraints, leaving many choices to the designer. For example, in the circuit shown in Figure 13, if the only constraint is that the voltage at A must 2V, then an infinite number of circuits are possible. There are three design choices to make, the voltage of  $V_1$  and the resistance values of the two resistors  $R_1$  and  $R_2$ . We do recognize some additional design restrictions, for example, we know that the voltage source must be greater than 2V since the voltage divider always results in a 'step down' in voltage. Additional, once we pick a source voltage, we know the voltage divider expression determines a relationship between  $R_1$  and  $R_2$ ,  $V_A = \frac{R_2}{R_2 + R_1} V_1$ . For example, we could pick  $V_1 = 4V$ , leading to the relationship  $R_1 = R_2$ .

There are still an infinite number of possibilities, even though the source voltage has been chosen. Some practical considerations now emerge. Picking resistors that are too small can be problematic since we have verified that voltage sources have an internal resistance that we tend to neglect. If we chose resistance values,  $R_1 = R_2 = 1\Omega$ , simulation results would give us  $V_A = 2V$ , however, experimental results would likely produce a very different result. Another practical consideration with resistors is choosing a resistor value that is easy to locate (i.e. in our parts kit). Choosing  $R_1 = R_2 = 6k\Omega$  would require using other resistors in series and parallel to obtain that value.

In our design process, we may wind up with a resistor value that is not a standard or readily available. That is fine, but let that be a single resistor value where we need to design a small resistor network (parallel and series) to implement that single value.

As you approach the design problems on the following page, you might

- 1) Lay out a basic circuit that meets the provided constraints
- 2) Assign voltage values that you feel will be sufficient to meet the goals
- 3) Assign values to some (not all) of the resistor values based on what is available in your parts kit.
- 4) Apply circuit analysis to any remaining resistors, determining a value that meets the requirements (Note you can approach from another direction by assigning resistors values first and then choosing voltage source values that meet the requirements.)

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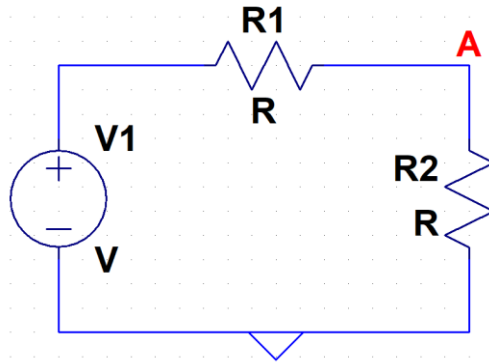


Figure 13: Voltage Divider with Node A

### Nodal Analysis and Linear Algebra Experiment

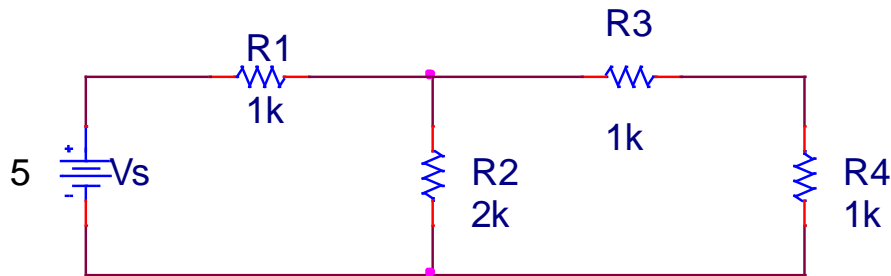


Figure 14: Four node circuit

- 1) In your Proof of Concepts, (Building Block) include the circuit schematic with each node labelled. You may choose any resistor values you want. You can also change the input voltage if you'd like.
- 2) Identify a ground node (it is best to pick a node connected to the source)
- 3) Use the source to identify the voltage at one of the labelled nodes. Which node and what voltage?
- 4) How many unknown voltages are left?
- 5) Apply KCL to each of those unknowns. Include the expressions in your report.
- 6) Combine your simultaneous equations into matrix form, filling in values for the matrix  $A$  and vector  $b$ .
- 7) Solve the matrix 'by hand'.
- 8) Verify your solution using Matlab for the matrix mathematics (take screen shots of your process and include them in your report).
- 9) Simulate the circuit in LTspice and check your answers. (Comparing calculations and simulations is a frequent part of engineering.)
- 10) Build the circuit and confirm your findings in your Proof of Concepts.

### Design Challenges with Nodal Analysis

Choose one of the design challenges and include in your Proof of Concepts document:

#### Design Challenge 1:

Written by S. Sawyer and A. Patterson  
Rensselaer Polytechnic Institute

Fall 2024: 11/13/2024  
Troy, New York, USA

## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

Design a circuit with a node that has 2V (not the source node). Your circuit should consist of one voltage source and three equivalent resistors\*. The resistors cannot be in series. Two of the resistors may be in parallel.

Design Challenge 2:

Design a circuit with a node that has 2V (not the source node). Your circuit should consist of two voltage sources and three equivalent resistors\*. None of the resistors should be in parallel or series. The voltage sources should have a difference of at least 1V.

Design Challenge 3:

Design a circuit with one node at 2V and one node at 3V (neither are source nodes). Your circuit should consist of two voltage sources and resistors as needed. None of the resistors should be in parallel or series. The voltage sources should have a difference of at least 1V.

\*equivalent resistors – resistor values that can be (but don't have to be) constructed by using resistors in parallel and/or series configurations available in the parts kit.

### Part B: Proof of Concepts List

**4. Prove that nodal analysis solves unknown nodal voltages in a circuit.**

**5. Prove/demonstrate your approach to designing a circuit using nodal analysis.**

## Part C [Core] – Operational Amplifiers

Material covered: *Operational Amplifiers and MATLAB Simulink*

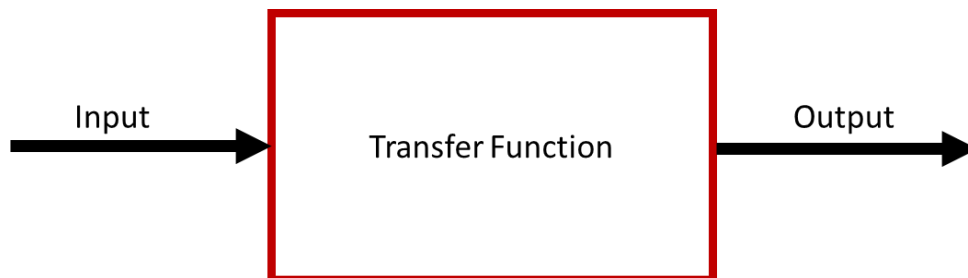
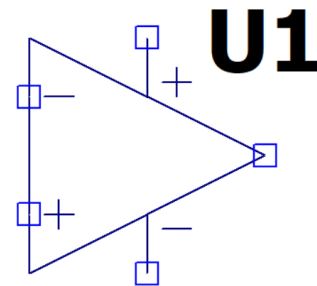
### General Operational Amplifier Basics

The op amp is another non-linear device that we treat as a black box. It is an amplifier with properties that are important for circuit isolation, amplification, mathematical operations, and binary decision making. There are hundreds, if not thousands of different types of op amps. All have datasheets and different specifications to meet needs.

The circuit schematic symbol of the connections for op amps are below. You can find this component in LTSpice “UniversalOpamp2”:

- input, - (left), inverting input
- input, + (left), non-inverting input
- -(bottom), V-: Negative power, -9 V
- +(top), V+, Positive power, 9V
- Right node: Vout, output voltage

In this lab, we introduce the fundamental idea and give you a chance to build and test an op amp circuit and/or represent its **transfer function** in Simulink. A transfer function is how a circuit or component affects an input to create an output. In fact, Transfer Function =  $H = V_{out}/V_{in}$ . It is the ratio of  $V_{out}$  to  $V_{in}$ . The idea of a **transfer function is a universal engineering concept.**



## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

### Operational Amplifier Applications

Content style modified from <http://site.iugaza.edu.ps/yyazji/files/lab-51.pdf>. All example output signals are referenced to this document.

#### Decision-Making Op Amps

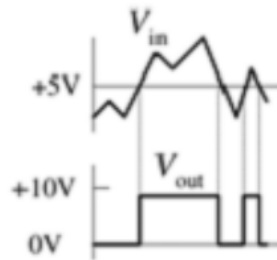
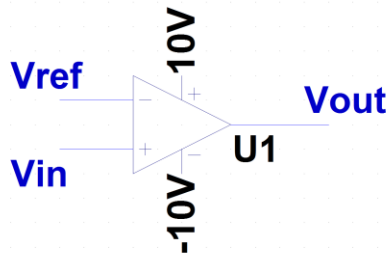
##### 1. Non-inverting Comparator

*Vref is a voltage you set, usually by a voltage divider with your source or just your source*

*Vin is the input that you want to compare to Vref.*

*If input is above Vref saturates positive.*

*If input is below Vref saturates negative.*



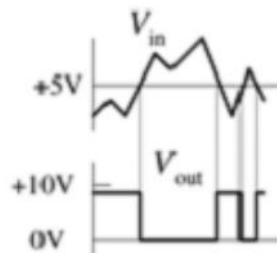
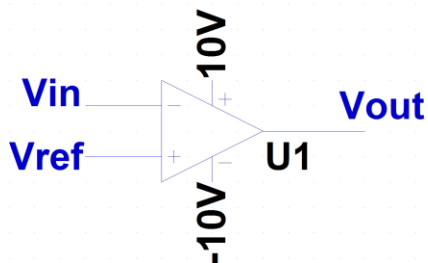
##### 2. Inverting Comparator

*Vref is a voltage you set, usually by a voltage divider with your source or just your source*

*Vin is the input that you want to compare to Vref.*

*If input is below Vref saturates positive.*

*If input is above Vref saturates negative.*



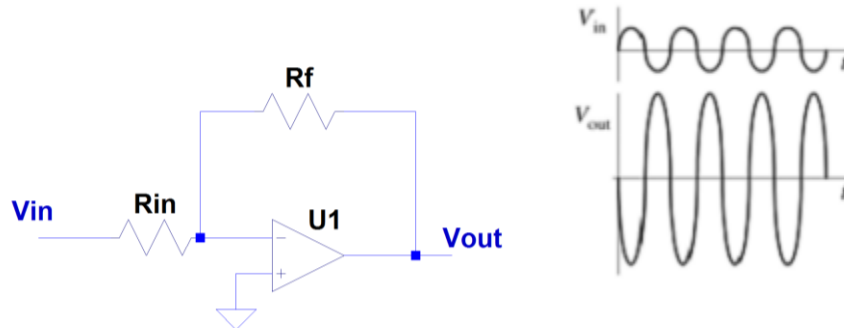
## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

### Mathematical Op Amps

1. **The inverting amplifier:** *Inverts a signal and amplifies it*

$$V_{out} = \frac{-R_f}{R_{in}} * V_{in}$$

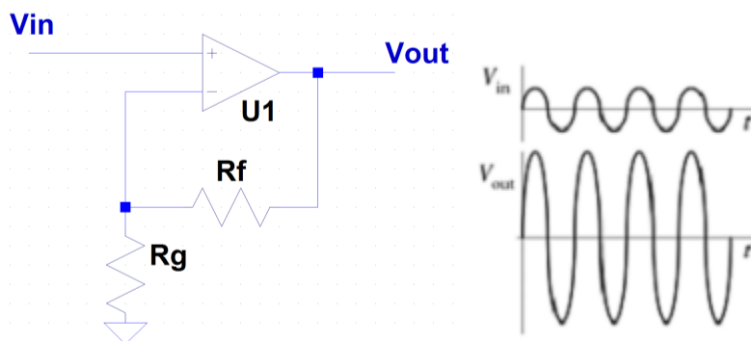
Inverting amplifier transfer function (also known as closed loop gain) =  $\frac{-R_f}{R_{in}}$



2. **The non-inverting amplifier:** *Amplifies a signal*

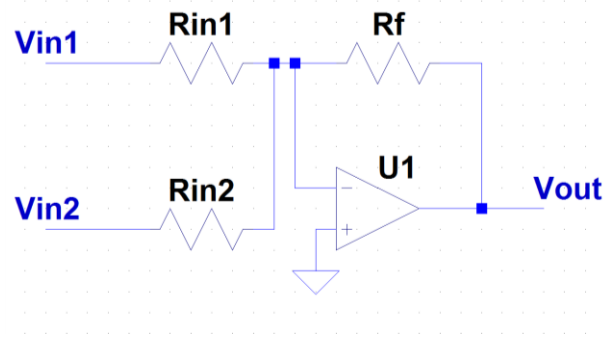
$$V_{out} = \left(1 + \frac{R_f}{R_g}\right) * V_{in}$$

Non-inverting amplifier transfer function (also known as closed loop gain) =  $\left(1 + \frac{R_f}{R_g}\right)$



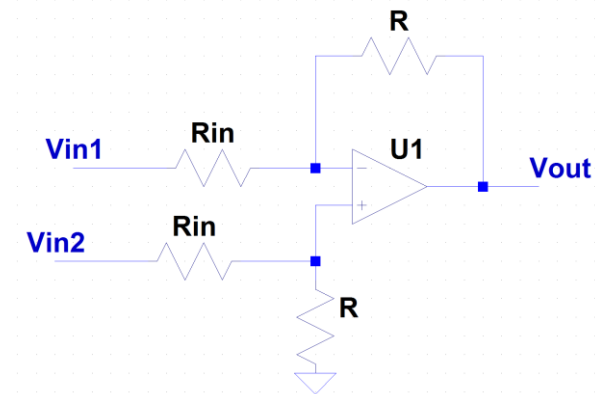
3. **The Inverting Summer:** *Adds signals together and amplifies as a weighted ratio and inverts the signal*

## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations



$$V_{out} = -\left(\frac{R_f}{R_{in1}} * V_{in1} + \frac{R_f}{R_{in2}} * V_{in2}\right)$$

4. The difference amplifier: *Subtracts signals*



$$V_{out} = \frac{R}{R_{in}} (V_{in2} - V_{in1})$$

*Note: R is the same value in both places, Rin is the same value in both places*



## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

### Operational Amplifier and Transfer Function Experiment

#### Decision Making Op Amp Circuit Experiment

- 1) In your Proof of Concepts, (Building Block) choose ONE Decision Making Op Amp circuit (either inverting or non-inverting comparator) and pick a reference that is NOT 5V, for example 1.3V or -2.7V or 3.6V. Make your schematic in LTSpice.
- 2) Simulate the function of your comparator using a changing input... triangle wave, sinusoid, ramp signal... any input that shows your output switching will work!

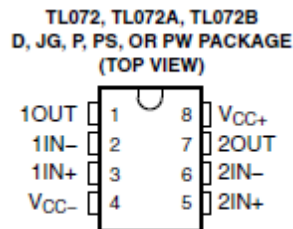
**For best results in LTSpice, use the "UniversalOpamp2" component.**

Build your experiment on your protoboard. Use your M2K or Analog Discovery Board function generator to provide an input that will show the function of your comparator. For your op-amp, **use the TL072CP chip** (dual op-amp) which will be available from the instructor.

The data sheet for the chip can be found online from any number of sites. One is provided below (it is long and contains several chips)

<http://www.ti.com/lit/ds/symlink/tl071.pdf>

A copy of the pin connections is shown below



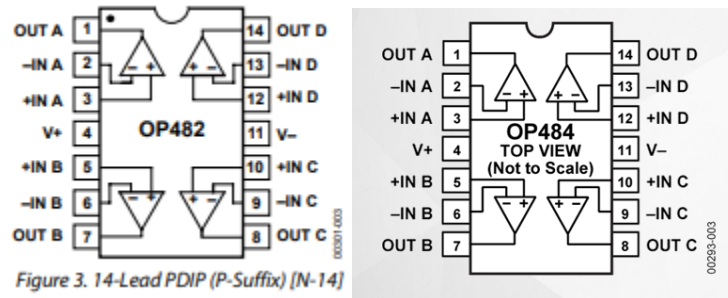
- 3) There are two op-amps on the chip, indicated by the '1' and the '2' pin labels. For example, 1IN+ is the V+ and 1IN- is the V- of the first op-amp, with 1OUT being the Vout. Power connections are +Vcc at pin 8 and -Vcc at pin 4.

## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

### Math Op Amp Experiment

- 1) In your Proof of Concepts, (Building Block) choose ONE Mathematical Op Amp circuit. **Note: You can also try a differentiator or integrator** not shown in this document, there are more, absolute value op amp, non-inverting summer... Make your schematic in LTSpice.  
**For best results in LTSpice, use the "UniversalOpamp2" component.**
- 2) Simulate the function of your mathematical op-amp using a changing input... triangle wave, sinusoid, ramp signal... any input that shows the mathematical function of your op amp.
- 4) Build your experiment on your protoboard. Use your M2K or Analog Discovery Board function generator to provide an input that will show the function of your comparator.

For mathematical op amp circuits, use the OP482 ([datasheet here](#)) or OP484 ([data sheet here](#)) in your parts kit. The OP482 needs a positive and negative supply voltage (+9V and -9V power) while the OP484 is a single supply op amp that needs (+5V and ground). Both have four op amp chips in the same package!



### Part C: Proof of Concepts List

6. Prove the function of an op amp comparator
7. Prove the function of a mathematical op amp.

**Part D [Alpha/Omega] – Applications: Transfer Functions, Linear Algebra, and Control**  
*Material covered:*

1. Choose an application! (water quality monitor, sun tracker for solar panel, flex sensor for VR glove or body suit in gaming etc.)
2. Make high level building blocks with input and output that define your sensor system
3. Find Op Amps or other components that are functions in each building block

**Choose Your Adventure! Alpha Experiments or Omega Exploration**

### Alpha Experiments

#### Sensors and Decisions

Choose an application of interest like a moisture sensor for an automated aquaponic system or a motion sensor for a smart traffic light.

1. Draw a high-level block diagram.
2. Simulate your application (must include at least one sensor and one op amp): You may simulate a sensor using a potentiometer or [find one in your kit!](#)
3. Calculate the relationship from input to output for each part of your circuit. For example, a non-inverting amplifier would amplify an input voltage by  $(1+R_f/R_g)$  to get an output voltage. This is a transfer function!
4. Build your application (using hardware) and demonstrate its function. Compare.

### Omega Exploration

Explore applications of control theory, sensing, or audio electronics in **one** of the following:

- **PI and PD Controllers** – design a circuit that automatically adapts its output to track a reference voltage
- **Sensor + Alarm System** – design a circuit that amplifies a sensor reading and triggers an alarm
- **Two-Channel Audio Mixer** – design a circuit that can add two signals together and adjust the volume of each channel and the overall output

**Technical and functional requirements for your application are listed [below](#).**

Part D: Proof of Concepts List

8. **Alpha: Prove your sensor decision circuit. You may also amplify your signal if needed.**

**Omega: Prove the concept of transfer functions in your exploration.**

**OR**

**Omega: Semiconductor Devices Exploration – tour RPI’s clean room and do a short research project on how diodes function. There is a special Proof of Concepts template, set of Presentation Guidelines and Exploration Map for this option on the course website.**

Omega Exploration Application Requirements

A short description and the technical specifications for each Omega Exploration option are listed below.

**Proportional-Integral (PI) and Proportional-Derivative (PD) Controllers**

A [proportional-integral-derivative \(PID\) controller](#) calculates the error between the system’s output and a desired reference, then applies a correction that consists of proportional, integral, and derivative terms to bring the output closer to the reference. In this Omega Exploration, you will design a proportional-integral (PI) or proportional-derivative (PD) controller.

*Requirements:*

- ❑ *Circuit architecture:*
  - Choose one: PI controller **OR** PD controller. In your presentation, you must discuss the advantages and disadvantages of both approaches.
  - Input stage must be an adjustable reference voltage (potentiometer, signal generator, etc.)
- ❑ *Operating conditions:*
  - Input voltage range:  $|V_{in}| \leq 5V$
- ❑ *Output voltage:*
  - $V_{out}$  should settle to the reference voltage from the input
  - Error between  $V_{out}$  and setpoint should be less than 5% of the reference voltage value when the voltage is a DC input
- ❑ *Experimental Verification / Test Cases:*
  - In real time, demonstrate that  $V_{out}$  settles to within 5% of the chosen reference when  $V_{in}$  is varied

## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

### **Sensor + Alarm System**

In this project, you will design a circuit that sets off an alarm if the sensor input has exceeded a particular threshold. In this Omega Exploration, you will design, build, and test two different designs for achieving this same goal, then compare them.

#### *Requirements:*

- ❑ *Circuit architecture:*
  - Input stage: [Wheatstone bridge](#) with resistive sensor of your choice
  - Must choose **two** different options for the decision-making stage: comparator **OR** Schmitt trigger **OR** transistor **OR** other of your choice. In your presentation, you must discuss the advantages and disadvantages of both chosen approaches.
  - Output stage: indicator (LED, speaker, etc.) for alarm
- ❑ *Operating conditions:*
  - Define your own input ranges based on the sensor you've chosen.
  - You must set a reference value for your sensor input.
- ❑ *Output voltage:*
  - $V_{out}$  must indicate when the alarm threshold has been crossed
- ❑ *Experimental Verification / Test Cases:*
  - Demonstrate that both circuits' outputs indicate that the threshold has been crossed at the designed input parameter value (light, temperature, etc.) by varying the input parameter

## Intro to ECSE Laboratories: Alpha Experiments and Omega Explorations

### **Two-Channel Audio Mixer**

At the most fundamental level, [audio mixers](#) allow music producers to add multiple different signals, or tracks, together and adjust their relative volume levels. In this Omega Exploration, you will design a circuit that sums two waveforms with individually controllable amplitudes, provides the ability to adjust the overall volume, and plays them from a speaker.

*Requirements:*

- *Circuit architecture:*
  - Input stage: two signal generator channels from your instrumentation board.
  - The amplitude of each of the input signals must be adjustable via amplifiers with variable gain. In your presentation, you must justify why you chose the amplifier types you selected for your design. Explain at least one other way you could have implemented this design using different amplifiers.
  - The amplitude of the sum of the two signals must also be adjustable.
  - Output stage: speaker
- *Operating conditions:*
  - Input voltage range of each signal  $-5V \leq V_{in} \leq 5V$
- *Output voltage:*
  - $-5V \leq V_{out} \leq 5V$
  - Gain of last amplifier stage should not permit its operation in the saturation regime
- *Experimental Verification / Test Cases:*
  - Demonstrate that the amplitude of each of the input signals can be adjusted individually, as designed.
  - Demonstrate that the amplitude of the summed signals can be adjusted, as designed.
  - Demonstrate that the maximum gain of the final amplification stage does not drive the op-amp into saturation.

## SUMMARY of Concepts

*Concept List that must be accounted for in your Proof of Concepts*

### PART A:

1. Prove that the slope of an I-V curve corresponds with ohm's law for two different resistor values.
2. Prove the non-linear I-V curve for a light emitting diode.
3. Show/demonstrate that the differential resistance changes in different regions in the diode I-V curve.

### PART B:

4. Prove that nodal analysis solves unknown nodal voltages in a circuit.
5. Prove/demonstrate your approach to designing a circuit using nodal analysis.

### PART C:

6. Prove the function of an op amp comparator
7. Prove the function of a mathematical op amp.

### PART D:

8. *Alpha:* Prove your sensor decision circuit. You may also amplify your signal if needed.  
*Omega:* Prove the concept of transfer functions in your exploration.

OR

*Omega:* Semiconductor Devices Exploration – tour RPI's clean room and do a short research project on how diodes function. There is a special Proof of Concepts template, set of Presentation Guidelines and Exploration Map for this option on the course website.

## How You'll Be Graded (with Standards Based Assessment):

You will be graded on the following Standards. Please ensure to achieve each standard. If you do not, you can resubmit to the missing standard to the end of the semester. CLEARLY mark the changes you make in you Proof of Concept submission by either Tracking Changes in Word or highlighting changes by writing comments in a different color and/or changing the color of the updated work.

# Lab 02 Standards

1. I can create current voltage characteristics for a linear component.
2. I can create current voltage characteristics for a non-linear component.
3. I can demonstrate that differential resistance varies along the IV curve of a non-linear component.
4. I can solve a circuit's unknown voltages using nodal analysis.
5. I can design a circuit using nodal analysis.
6. I can demonstrate the function of a decision-making op amp.
7. I can demonstrate the function of a mathematical op amp.
8. **Alpha:** I can design and demonstrate a series of components with transfer functions (input, component, output) to make a larger circuit that makes a decision about something.

**Omega:** I can demonstrate the use of transfer functions in my application.

OR

**Omega:** Semiconductor Devices Exploration – tour RPI's clean room and do a short research project on how diodes function. There is a special Proof of Concepts template, set of Presentation Guidelines and Exploration Map for this option on the course website.

Additionally, the following standards apply to proving each of the above concepts:

- I can demonstrate "good failure" whenever applicable by providing accurate results in my experience and speculating what went wrong.
- I can identify non-idealities or unexpected results and attempt to explain why they may exist.
- I can answer for myself "Is this right?" by comparing mathematical calculations to simulation and experimental results.
- I can show plots and diagrams that are easy to read, scaled correctly and clearly labeled.
- I can use consistent variable labels and component values in mathematical calculation, simulation and experimental results for easy comparison.