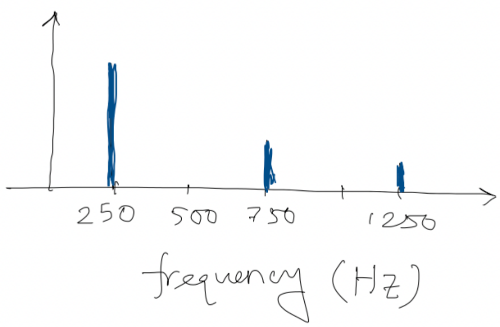
Problem 1

$$V(t) = \sin(2\pi \cdot \overbrace{250}^{f_1} \cdot t) + 0.3\sin(2\pi \cdot \overbrace{750}^{f_2} \cdot t) + 0.15\sin(2\pi \cdot \overbrace{1250}^{f_3} \cdot t)$$

1.1

a)



b) The period of $V(t) = 4\,ms = 0.004\ s$

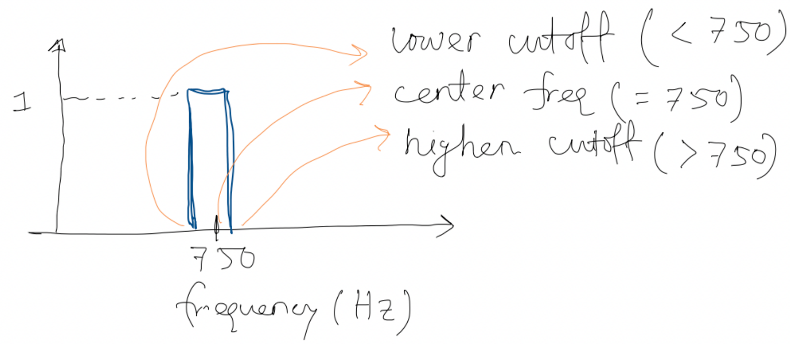
(smallest time after which the signal repeats itself)

$\therefore$ fundamental frequency $= \frac{1}{0.004} = 250\ Hz$

c) BPF (band pass filter)

because only a band (a frequency for the current case, 750 Hz) is being allowed to pass

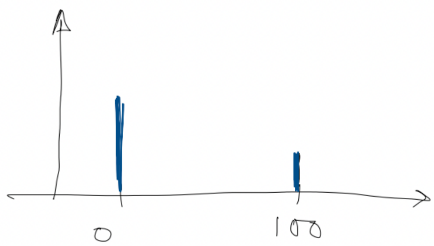
d)



ideal BPF
= rectangle filter

1.2

Given freq domain representation



$$v(t) = A_1 \sin(2\pi \cdot 0 \cdot t + \theta_1) + A_2 \sin(2\pi \cdot 100 \cdot t + \theta_2)$$ ✓

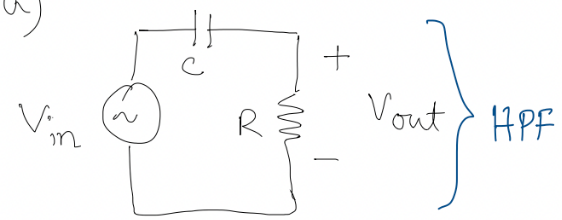
$$= A_1 \sin(\theta_1) + A_2 \sin(200\pi t + \theta_2)$$ ✓

$$= V_{DC} + A_2 \sin(200\pi t + \underbrace{\theta_2}_{\text{optional}})$$ ✓

Problem 2

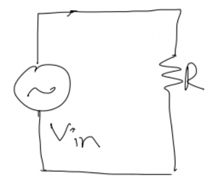
2.1

a)



$\equiv$

at high freq



b) $f_c = \dfrac{1}{2\pi RC}$

$\Rightarrow 6400\pi = \dfrac{1}{RC}$

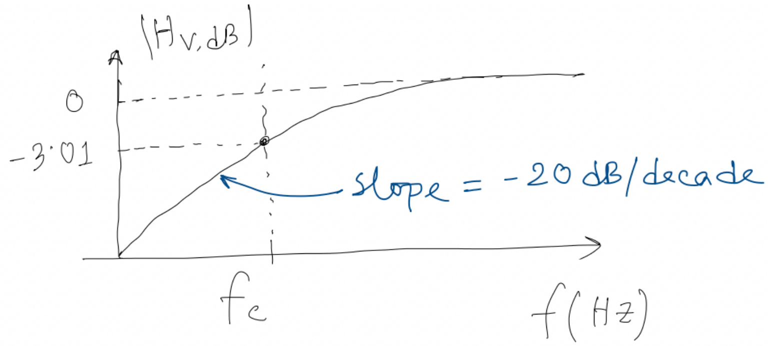
when; $C = 10^{-7}$; $R = 497.359\ \Omega$

$C = 5\times10^{-7}$; $R = 99.47\ \Omega$

$C = 10^{-6}$; $R = 49.73\ \Omega$

so, the closest combination possible is $\boxed{R = 100\,\Omega;\ C = 5\times10^{-7}}$

c)



d) $V_{out} = V_{in}\left(\frac{R}{R + \frac{1}{j\omega C}}\right)$ $j\omega = s$

$$\Rightarrow H(s) = \frac{R}{R + \frac{1}{sC}} = \frac{RS}{RS + 1/C}$$

$$\Rightarrow H(s) = \frac{S}{S + \frac{1}{RC}} \quad \& \quad H(j\omega) = \frac{j\omega}{j\omega + \frac{1}{RC}}$$

2.2

a)

Given; $H(j\omega) = \dfrac{j\omega}{j\omega + 2\times10^3}$

$$|H(j\omega)| = \mathrm{sqrt}\left(\frac{j\omega}{j\omega + 2\times10^3} \cdot \frac{-j\omega}{-j\omega + 2\times10^3}\right)$$

$$= \sqrt{\frac{\omega^2}{(2\times10^3)^2 - (j\omega)^2}}$$

$$= \sqrt{\frac{\omega^2}{4\times10^6 + \omega^2}}$$

b) $V_{in} = 2\sin(\underbrace{1200}_{\omega} t)$

$$|H(1200)| = \frac{(1200)^2}{\sqrt{4\times10^6 + (1200)^2}}$$

$$= 0.514$$

$\therefore$ in decibels $= 20\log(0.514)$

$$= -5.772 \text{ dB}$$

c) $|V_{out}| = |H(1200)| \cdot |V_{in}|$

$\Rightarrow |V_{out}| = 0.514 \times 2$

$$= 1.029 V$$

2.3

a) $V_{out} = V_{in} \cdot \left(\dfrac{\frac{1}{sC} + sL}{\frac{1}{sC} + sL + R} \right)$

$$H(s) = \frac{\frac{1}{C} + s^2 L}{\frac{1}{C} + s^2 L + Rs}$$

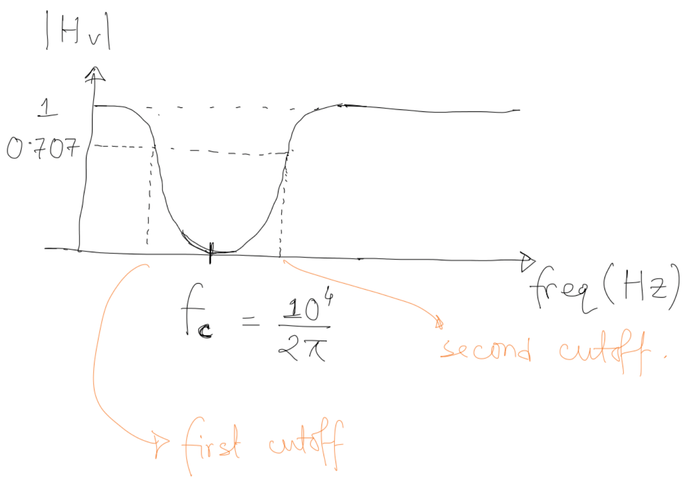
$$= \frac{\frac{1}{LC} + s^2}{\frac{1}{LC} + s^2 + \frac{R}{L} s}$$

$$= \frac{s^2 + \frac{1}{LC}}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}$$

b) $H(s=0) = 1;$ $H(s=\infty) = 1$

$H(s) = 0 \Rightarrow s^2 + \frac{1}{LC} = 0 \Rightarrow \omega^2 = \frac{1}{LC}$

$\therefore f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi} \cdot 10^4$



c) Band stop filter (Notch filter)

Problem 3

3.1

True $\longrightarrow$ from fourier series we know this $\longrightarrow$ so theoritically True

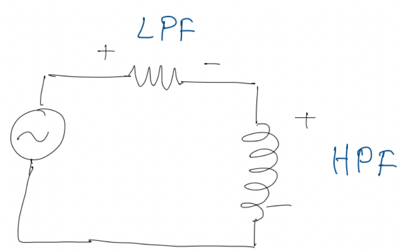
False $\longrightarrow$ generating sinusoidal waves from $-\infty$ time to 0 time is not practical & impossible.

3.2

Square wave consists of infinite amount of sinusoidal waves (from Fourier) having frequencies multiples of the fundamental freq.

Hence; the frequency spectrum of the square wave (signal 1) has more peaks at higher frequencies.

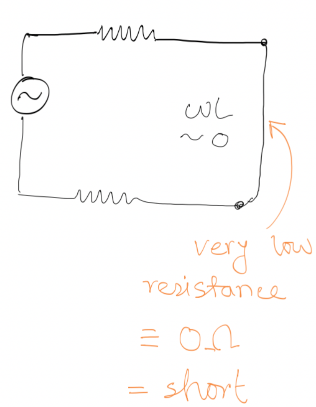
3.3



depending on where the Vout is (ACROSS R or L) the circuit can be LPF or HPF

3.4

Low Frequency



High frequency