

Intro to ECSE

Quiz 3

Spring 2023

1.	/18
2.	/18
3.	/12
Total	/48

Name _____

RIN _____

Notes:

SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification. Use the backs of pages if there is not enough room on the front.

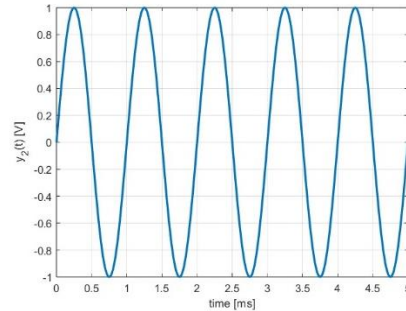
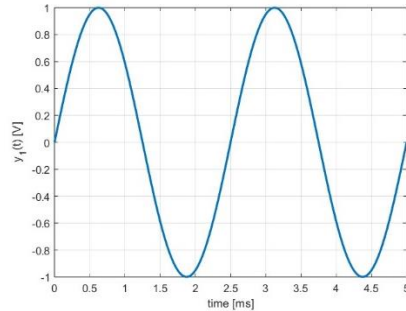
For partial credit on some questions, you may want to re-draw circuit diagrams as you simplify the circuits.

Many problems can be solved using more than one method. check your answers by using a second method.

At least skim through the entire quiz before you begin and then start with the problems you know best. The proctor will only answer clarification questions where wording is unclear or where there may be errors/typos. No other questions will be responded to.

Problem 1: Concepts: Time Domain, Frequency Domain, and Filters
[18 Points]

a) Shown below are two sinusoidal signals: $y_1(t)$ and $y_2(t)$. In this part (a) of Problem 1, you will be asked questions about these signals.

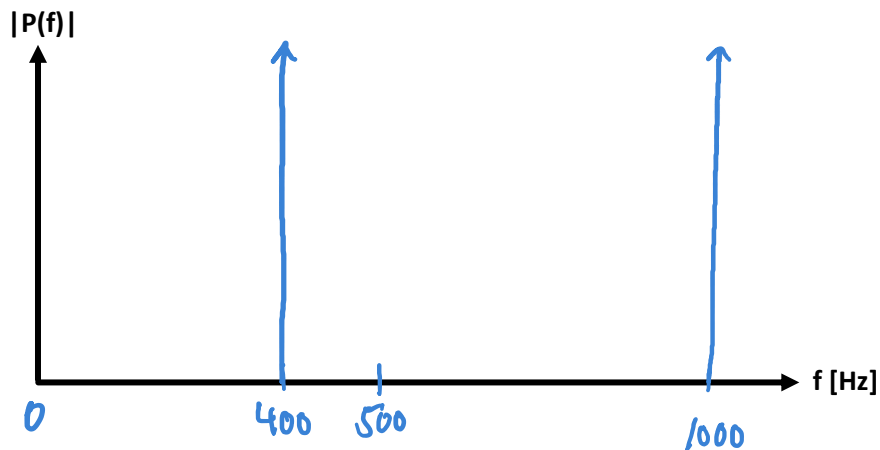


i) [4 pts] Write the time-domain expression for the sum of the two waves: $y(t) = y_1(t) + y_2(t)$.

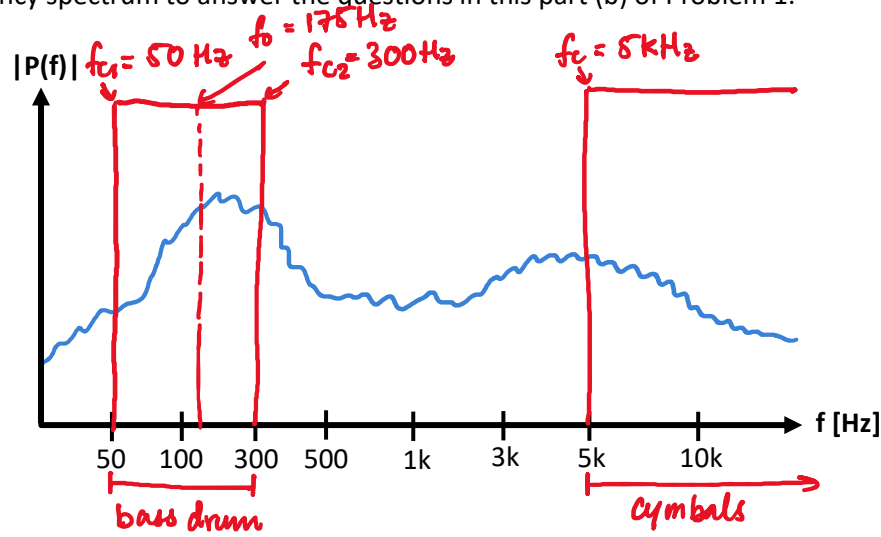
$$f_1 = \frac{1}{T_1} = \frac{1}{2.5 \times 10^{-3} \text{ s}} = 400 \text{ Hz} ; f_2 = \frac{1}{T_2} = \frac{1}{1 \times 10^{-3} \text{ s}} = 1000 \text{ Hz}$$

$$y(t) = \sin(2\pi \cdot 400t) + \sin(2\pi \cdot 1000t)$$

ii) [2 pts] Sketch the frequency spectrum for $y(t) = y_1(t) + y_2(t)$ from part i) on the axes below. You do not have to label the absolute amplitude of the frequency spectrum, but the relative amplitude of the signals that make up $y(t)$ and their locations in frequency should be correct.



- b) The frequency spectrum of an audio file that contains a drum part for a song is shown below. Use this frequency spectrum to answer the questions in this part (b) of Problem 1.



- i) [2 pts] Bass drum frequencies typically fall in the frequency range $50 \text{ Hz} < f < 300 \text{ Hz}$ and cymbals fall in the range $f > 5000 \text{ Hz}$. Indicate where these frequencies are **on the frequency spectrum above**.

- ii) [2 pts] What type of filter would you use to isolate **only** the bass drum frequencies?

bandpass filter

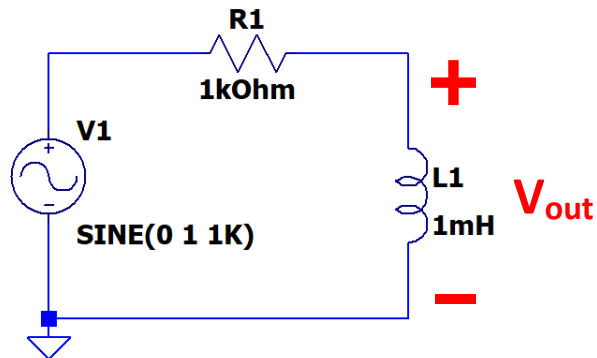
- iii) [2 pts] What type of filter would you use to isolate **only** the cymbal frequencies?

high pass filter

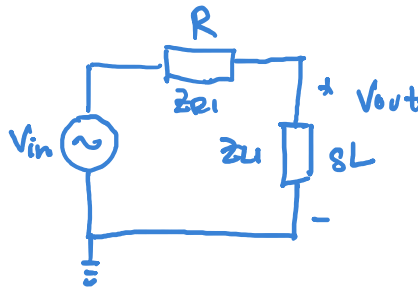
- iv) [3 pts] Draw the transfer function of the ideal filter you would use to isolate the bass drum frequencies and label the appropriate corner frequency (if it's a 1st order filter) and center frequency and corner frequencies (if it's a 2nd order filter) **on the frequency spectrum above**.

- v) [3 pts] Draw the transfer function of the ideal filter you would use to isolate the cymbal frequencies and label the appropriate corner frequency (if it's a 1st order filter) and center frequency and corner frequencies (if it's a 2nd order filter) **on the frequency spectrum above**.

Problem 2: Concepts: 1st Order Filters, Transfer Functions, and Bode Plots
[18 Points]



- a) [2 pts] Redraw the circuit above in terms of the impedances Z_{R1} and Z_{L1} . Also write the expressions for the impedance of each element (in terms of $R1$, $L1$ and s) next to the appropriate element in the diagram.



- b) [4 pts] Find the transfer function $H(s)$ of the circuit above for the V_{out} indicated in the circuit diagram. Rearrange your expression for $H(s)$ so that the denominator takes the form: $s + \alpha$, where α is a constant. Then, also write your transfer function in terms of $j\omega$ to give $H(j\omega)$.

Via voltage divider:
$$V_{out} = V_{in} \frac{Z_{L1}}{Z_{R1} + Z_{L1}}$$

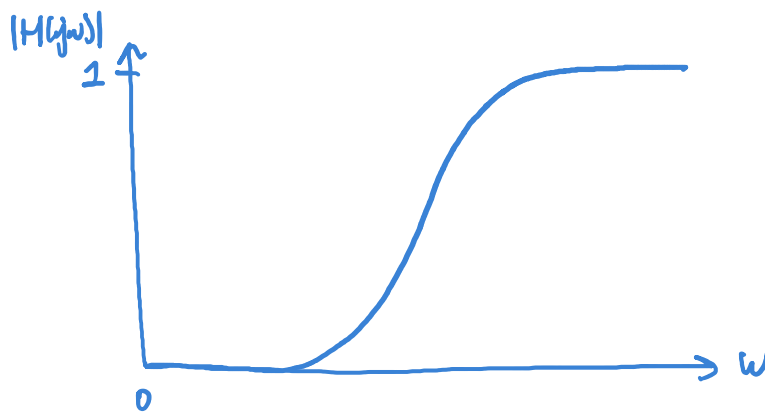
$$= V_{in} \frac{sL_1}{R_1 + sL_1} \cdot \frac{(1/L_1)}{(1/L_1)} = \frac{s}{s + R_1/L_1} V_{in}$$

$$H(s) = \frac{s}{s + R_1/L_1}; \quad H(j\omega) = \frac{j\omega}{j\omega + R_1/L_1}$$

- c) [3 pts] Find $|H(j\omega)|$ the magnitude of your transfer function in part b, in terms of R_1 , L_1 , and ω .

$$\begin{aligned}
 H(j\omega) &= \frac{j\omega}{j\omega + R_1/L_1} \rightarrow |H(j\omega)| = \sqrt{H(j\omega) \cdot H(-j\omega)} \\
 &= \sqrt{\left(\frac{j\omega}{j\omega + R_1/L_1}\right) \left(\frac{-j\omega}{-j\omega + R_1/L_1}\right)} \\
 &= \frac{\omega}{\sqrt{\omega^2 + (R_1/L_1)^2}}
 \end{aligned}$$

- d) [3 pts] Sketch a plot of the $H(j\omega)$ that you derived in part b **on a linear scale**. Show your work by writing $H(j\omega)$ when ω goes to 0 and when ω goes to ∞ for full credit.



$$\omega \rightarrow 0: H \rightarrow \left(\frac{0}{0 + R_1/L_1}\right) = 0$$

$$\omega \rightarrow \infty: H \rightarrow \frac{\infty}{\infty} \rightarrow 1$$

e) [2 pts] What kind of filter is represented by your Bode Plot in part c?

high-pass filter

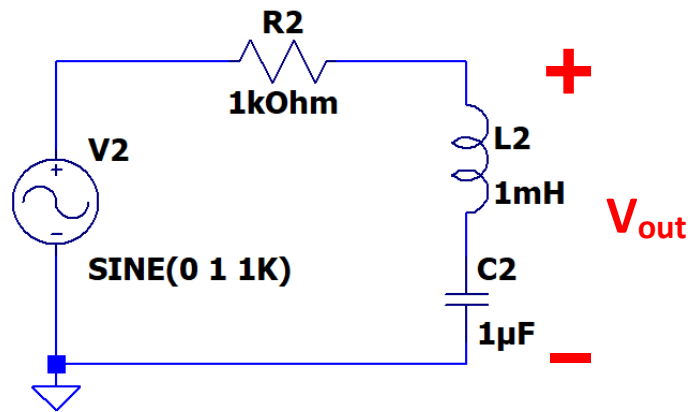
f) [2 pts] What is the corner frequency of the circuit? Give a numerical value in rad/s.

$$\omega_c = R/L_1 = \frac{1\text{ k}\Omega}{1\text{ mH}} = \frac{1000\Omega}{1 \times 10^{-3}\text{ H}} = 10^6 \text{ rad/s}$$

g) [2 pts] What is the magnitude of the **power transfer function** at the corner frequency $|H_{\text{power}}(j\omega_c)|$ in decibels?

$$H_{\text{power}}^{\text{dB}} = -3.01 \text{ dB at } \omega_c \text{ (definition)}$$

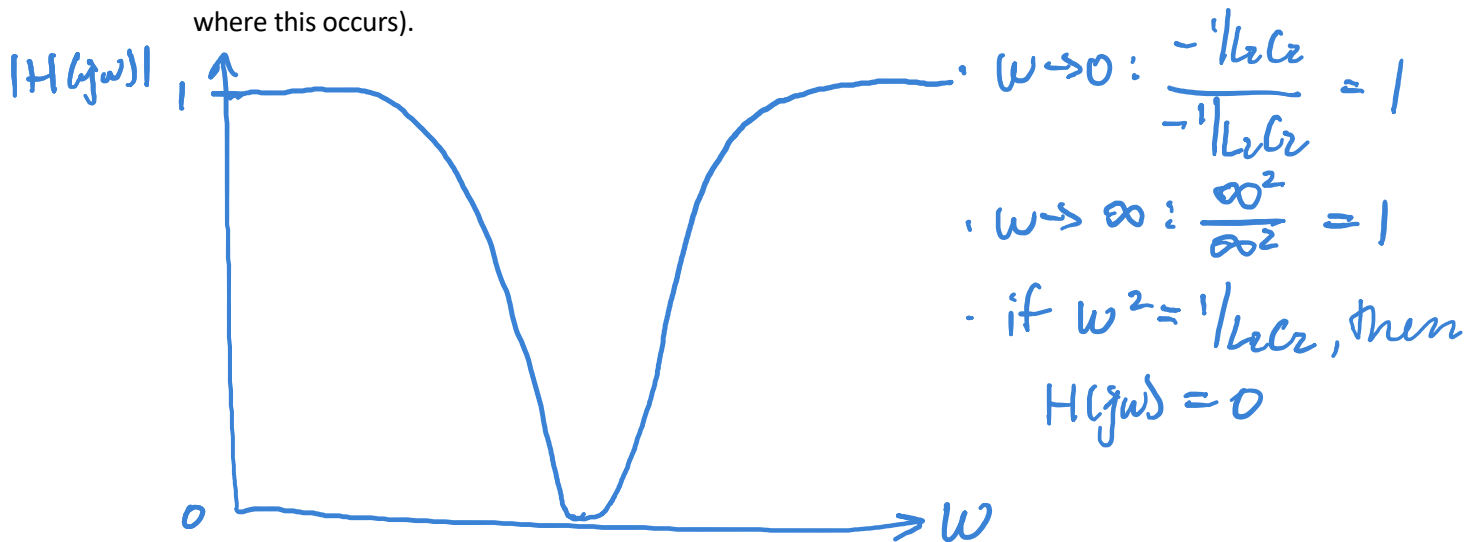
Problem 3: Concepts: 2nd Order Filters, Transfer Functions, and Bode Plots
[12 Points]



- a) [4 pts] Find the transfer function $H(s)$ of the circuit above for the V_{out} indicated in the circuit diagram. Rearrange your expression for $H(s)$ so that the denominator takes the form: $s^2 + \alpha s + \beta$, where α and β are constants. Then, also write your transfer function in terms of $j\omega$ to give $H(j\omega)$.

$$\begin{aligned}
 V_{out} &= V_2 \cdot \frac{Z_{L2} + Z_{C2}}{Z_{R2} + Z_{L2} + Z_{C2}} = V_2 \frac{sL_2 + 1/sC_2}{R_2 + sL_2 + 1/sC_2} \cdot \left(\frac{s/L_2}{s/L_2} \right) \\
 &= \frac{s^2 + 1/L_2C_2}{s^2 + s\frac{R_2}{L_2} + 1/L_2C_2} V_2 \rightarrow H(s) = \frac{s^2 + 1/L_2C_2}{s^2 + s\frac{R_2}{L_2} + 1/L_2C_2} \\
 &\quad \downarrow \\
 H(j\omega) &= \frac{-\omega^2 + 1/L_2C_2}{-\omega^2 + j\omega\frac{R_2}{L_2} + 1/L_2C_2} \\
 &= \frac{\omega^2 - 1/L_2C_2}{\omega^2 - j\omega\frac{R_2}{L_2} - 1/L_2C_2}
 \end{aligned}$$

- b) [4 pts] Sketch a plot of the $H(j\omega)$ that you derived in part a **on a linear scale**. Show your work by writing $H(j\omega)$ when ω goes to 0 and when ω goes to ∞ for full credit. If $H(j\omega)$ has a maximum or minimum between $\omega = 0$ and $\omega \rightarrow \infty$, indicate that in your plot as well (you do not need to calculate exactly where this occurs).



- c) [2 pts] What type of filter does your plot in part b represent?

band-stop or band-reject filter

- d) [2 pts] Calculate the center frequency ω_0 of this filter.

$$\omega_0 = \frac{1}{\sqrt{L_2C_2}} = \frac{1}{\sqrt{1 \times 10^{-3} \text{ H} \cdot 1 \times 10^{-6} \text{ F}}} = 31622 \text{ rad/s}$$

e) [0 pts] Circle for our records please!

Do you expect to be exempt from the final?

I am confident that I will be exempt from the final

I am hoping that I will be exempt from the final

I do not expect to be exempt from the final

Will you take the final regardless of exemption?

I plan to take the final

I do not plan to the final if I am exempt

I have not made my decision to take the final yet if I am exempt