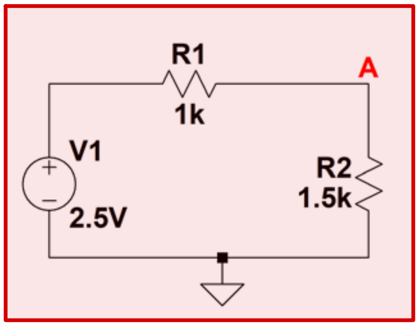
2. Design Problems (15 points)

For the two design problems below, your answers should include a clearly drawn (by hand or LTspice) schematic of the circuit and should also include handwritten work/explanation justifying your design methodology. Indicate which node/nodes satisfy the design criterion.

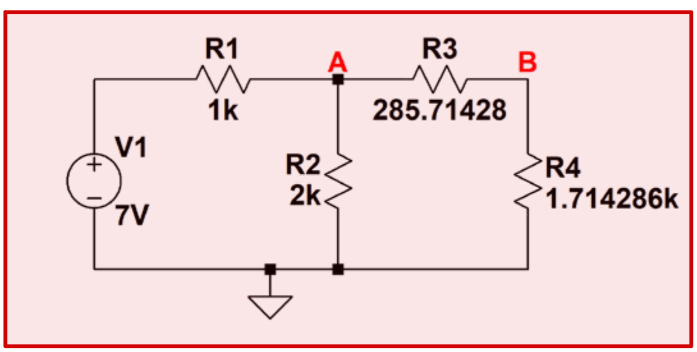
Part a) (5 points) Design a circuit such that there is a node with voltage 1.5V. Constraint: Use one voltage source of value 2.5 V.



Many correct answers possible. One simplistic approach presented.

$$V_A = V_1 \left(\frac{1.5K}{1.5K + 1K} \right) = 1.5 \text{ V}$$

Part b) (10 points) Design any circuit (any combination of resistors and source voltages) that has a node with 3.5 V and a node with 3V. Neither of those nodes can be connected to a voltage source.



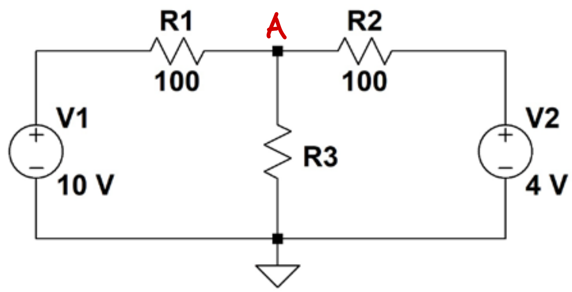
Many correct answers possible. One simplistic approach presented.

$V_A = 3.5$ V

$V_B = 3$ V

3. Nodal Analysis (20 points)

Consider the circuit shown below. The voltage across R3 is 6 V.



Part a) (10 points) Determine the value of resistance R3 using nodal analysis such that the voltage across it is 6 V. Show handwritten work.

KCL @ node A

$$\frac{V_A - V_1}{R1} + \frac{V_A - V_2}{R2} + \frac{V_A - 0}{R3} = 0$$

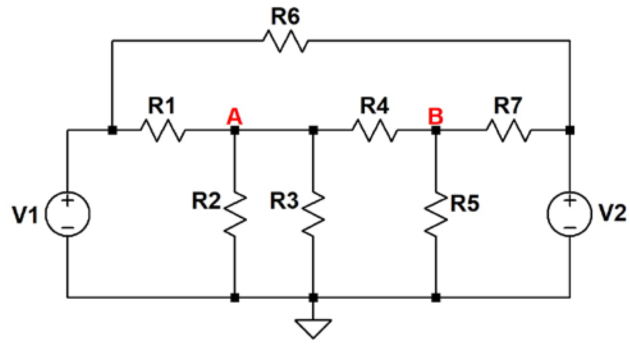
$$\Rightarrow \frac{V_A - 10}{100} + \frac{V_A - 4}{100} + \frac{V_A}{R3} = 0$$

$$\Rightarrow V_A - 10 + V_A - 4 + \frac{100 V_A}{R3} = 0$$

$$\Rightarrow \frac{100 V_A}{R_3} = 14 - 2V_A$$

$$\Rightarrow R_3 = \frac{100 V_A}{14 - 2V_A} \overset{V_A = 6\,V}{=} \frac{600}{2} = \boxed{300\ \Omega}$$

Consider a new circuit shown below. Answer part b based on new circuit.



Part b) (10 points) Use nodal analysis to find the matrix equation, $Ax = b$. All your terms should be symbolic (no numbers needed). Your final answer should be expressed as a matrix equation as shown below. V_A and V_B are nodal voltages at nodes A and B respectively.

KCL @ node A

$$\frac{V_A - V_1}{R_1} + \frac{V_A}{R_2} + \frac{V_A}{R_3} + \frac{V_A - V_B}{R_4} = 0$$

$$\Rightarrow V_A\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right] - \frac{V_B}{R_4} = \frac{V_1}{R_1}$$

KCL @ node B

$$\frac{V_B - V_2}{R_7} + \frac{V_B}{R_5} + \frac{V_B - V_A}{R_4} = 0$$

$$-\frac{V_A}{R_4} + V_B\left[\frac{1}{R_7} + \frac{1}{R_5} + \frac{1}{R_4}\right] = \frac{V_2}{R_7}$$

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R_4} \\ -\frac{1}{R_4} & \frac{1}{R_7}+\frac{1}{R_5}+\frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} V_1/R_1 \\ V_2/R_7 \end{bmatrix}$$

4. Matrix Solutions (20 points)

Using matrix reductions techniques applied in the experiments, solve (by hand) for the unknown values in the above matrix expression. You must show your matrix reduction work.

$$\begin{bmatrix} -1 & 1 & -4 \\ -2 & 2 & -5 \\ 0 & 2 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$$

Several ways to solve.

$R2 = R2 - 2 \times R1$

$$\begin{bmatrix} -1 & 1 & -4 \\ 0 & 0 & 3 \\ 0 & 2 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

Using 2nd Row $\Rightarrow$ $3x_3 = 3$ $\Rightarrow$ $x_3 = 1$

Using 3rd Row $\Rightarrow$ $2x_2 + 7x_3 = 4$

$\Rightarrow$ $2x_2 = 4 - 7 = -3$

$\Rightarrow$ $x_2 = -1.5$

Using 1st Row $\Rightarrow$ $-x_1 + x_2 - 4x_3 = 1$

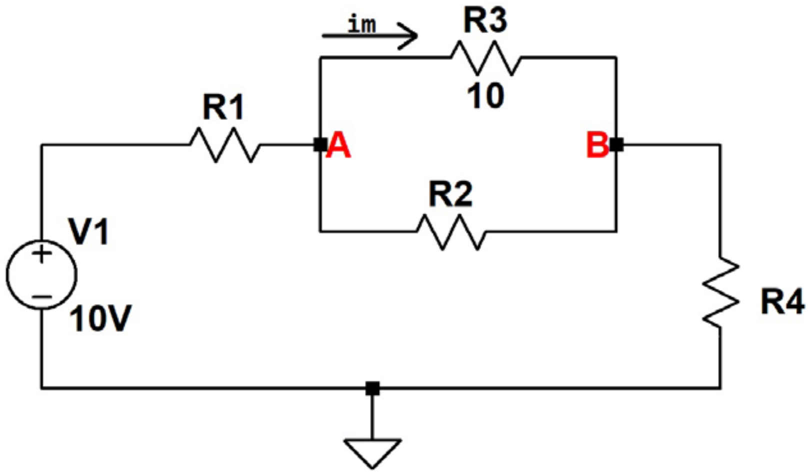
$\Rightarrow$ $-x_1 - 1.5 - 4 = 1$

$\Rightarrow$ $x_1 = -1.5 - 4 - 1 = -6.5$

$x_1 = -6.5$

5. Voltage Dividers (25 points)

Consider the circuit diagram shown below. The voltage source is a 10 V supply. R3 is 10 ohms. R1, R2, and R4 are unknown resistors. The current through R3 is im as shown.



Case 1: Given that when R4 = 0 ohms, the current through resistor R3 is 2 mA, i.e. im = 2 mA.

Case 2: Given that when R4 = 2000 ohms, current through resistor R3 is 1 mA, i.e. im = 1 mA.

Part a) (3 points) Find voltage between points A and B for each of the cases described above?

Case 1: $V_{AB} = i_m R_3 = 20\,mV$

Case 2: $V_{AB} = i_m R_3 = 10\,mV$

Part b) (2 points) What is the equivalent resistance between points A and B, RAB? (Express in terms of R2).

$$R_{AB} = R_2 \parallel R_3 = \frac{10 R_2}{10 + R_2}$$

Part c) (5 points) Using voltage divider and your answer to parts a and b, develop a relationship between R1 and R2 for case 1.

$$V_{AB} = 20mV = V_1 \left(\frac{R_{AB}}{R_1 + R_{AB} + R_4} \right)$$

10V (V_1); $R_4 \rightarrow 0\Omega$

$$\Rightarrow (R_1 + R_{AB})\,2mV = R_{AB}$$

$$\Rightarrow R_1 + \frac{10R_2}{10+R_2} = \frac{5000\,R_2}{10+R_2} \Rightarrow \boxed{10R_1 - 4990R_2 + R_1R_2 = 0}$$

equation 1

Part d) (10 points) Using voltage divider and your answer to parts a and b, develop a relationship between R1 and R2 for case 2.

$$V_{AB} = 10mV = V_1 \left(\frac{R_{AB}}{R_1 + R_{AB} + R_4} \right)$$

$V_1 \to 10V$, $R_4 \to 2000\Omega$

$$\Rightarrow \left(R_1 + \frac{10R_2}{10+R_2} + 2000 \right) 1mV = \frac{10R_2}{10+R_2}$$

$$\Rightarrow \boxed{10R_1 - 7990R_2 + R_1R_2 = -20000} \text{ equation 2}$$

Part e) (5 points) Solve the linear relationships (any method you want) derived in the previous parts, to determine the values of resistors R1 and R2 such that both case 1 and 2 are satisfied.

equation 1 − equation 2

$$\Rightarrow 3000R_2 = 20000 \Rightarrow \boxed{R_2 = 6.667\ \Omega}$$

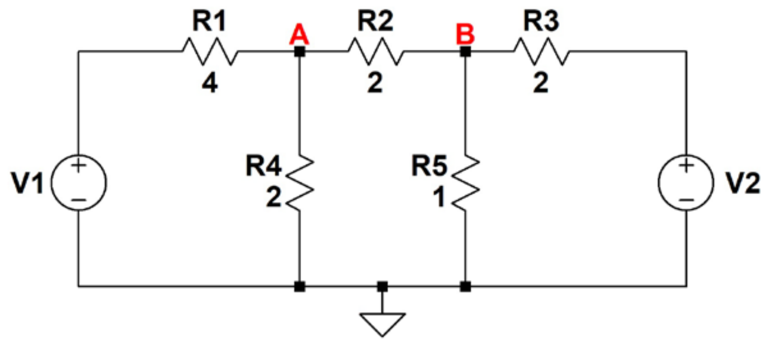
Substitute in equation 1.

$$10R_1 - 33268.33 + 6.667R_1 = 0$$

$$\Rightarrow \boxed{R_1 = 1996\ \Omega}$$

6. Nodal Analysis – Multiple Sources (20 points)

Consider the circuit shown below.



Using nodal analysis techniques, express the voltage at node A and node B in the form, $V_A = aV_1 + bV_2$ and $V_B = cV_1 + dV_2$. In other words, find values for the constants a, b, c, and d.

KCL @ node A

$$\frac{V_A - V_1}{4} + \frac{V_A}{2} + \frac{V_A - V_B}{2} = 0$$

$$\Rightarrow \quad 1.25\,V_A - 0.5\,V_B = 0.25\,V_1 \quad \text{--- (1)}$$

KCL @ node B

$$\frac{V_B - V_A}{2} + \frac{V_B}{1} + \frac{V_B - V_2}{2} = 0$$

$$-0.5\,V_A + 2\,V_B = 0.5\,V_2 \quad \text{--- (2)}$$

From (2). $2V_B = 0.5V_2 + 0.5V_A$

$$\Rightarrow \quad V_B = 0.25\,V_A + 0.25\,V_2 \quad \text{— (3)}$$

Substitute (3) in (1)

$$1.25\,V_A - 0.125\,V_A - 0.125\,V_2 = 0.25\,V_1$$

$$\Rightarrow \quad 1.125\,V_A = 0.25\,V_1 + 0.125\,V_2$$

$$\boxed{V_A = \frac{2}{9}V_1 + \frac{1}{9}V_2}$$

$\frac{2}{9} \rightarrow a$, $\frac{1}{9} \rightarrow b$

Substituting V_A in (3)

$$V_B = 0.25\left[\frac{2}{9}V_1 + \frac{1}{9}V_2\right] + 0.25\,V_2$$

$$= \frac{1}{18}V_1 + \frac{1}{36}V_2 + \frac{1}{4}V_2$$

$$\boxed{V_B = \frac{1}{18}V_1 + \frac{5}{18}V_2}$$

$\frac{1}{18} \rightarrow c$, $\frac{5}{18} \rightarrow d$