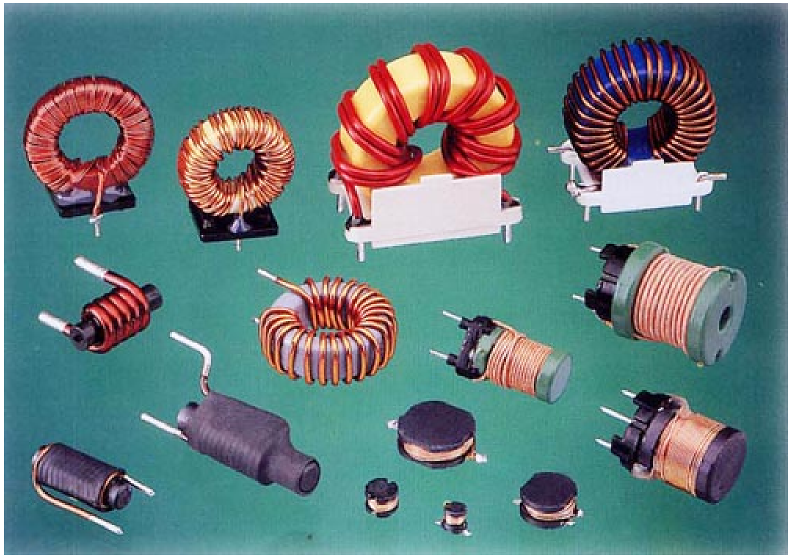
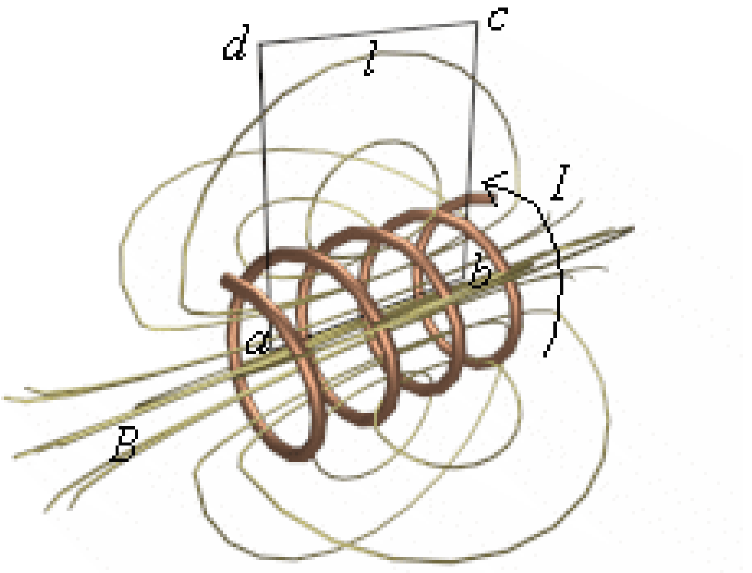


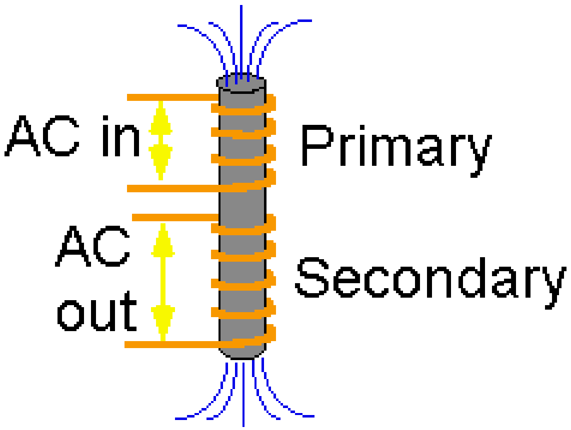


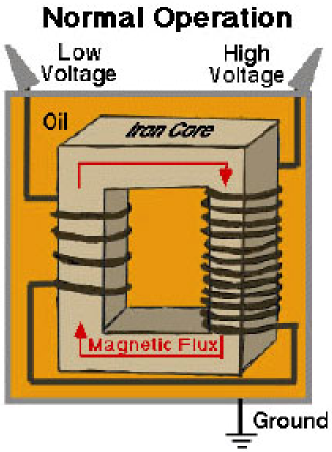
Electronic Instrumentation

Experiment 3

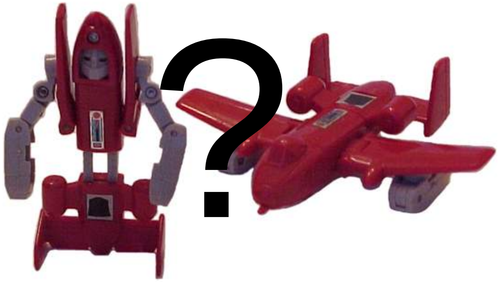
- Part A: Making an Inductor
- Part B: Measurement of Inductance
- Part C: Simulation of a Transformer
- Part D: Making a Transformer

Inductors & Transformers





- How do transformers work?
- How to make an inductor?
- How to measure inductance?
- How to make a transformer?



Part A

- Inductors Review
- Calculating Inductance
- Calculating Resistance

Inductors-Review

- General form of I-V relationship

$$V = L\frac{dI}{dt}$$

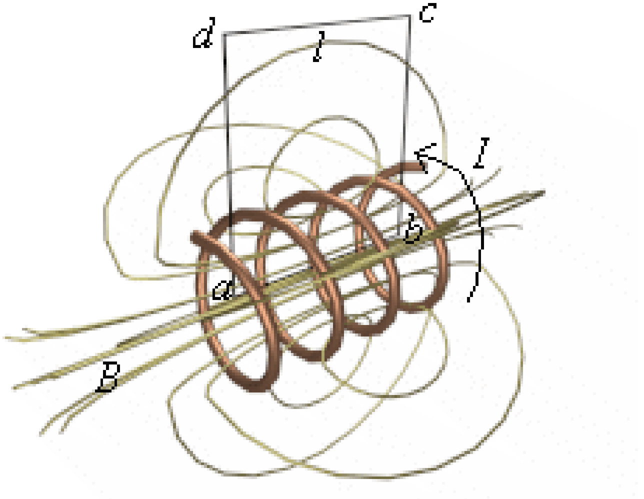
- For steady-state sine wave excitation

$$Z_L = j\omega L \qquad V = j\omega LI$$

Determining Inductance

- Calculate it from dimensions and material properties
- Measure using commercial bridge (expensive device)
- Infer inductance from response of a circuit. This latter approach is the cheapest and usually the simplest to apply. Most of the time, we can determine circuit parameters from circuit performance.

Making an Inductor

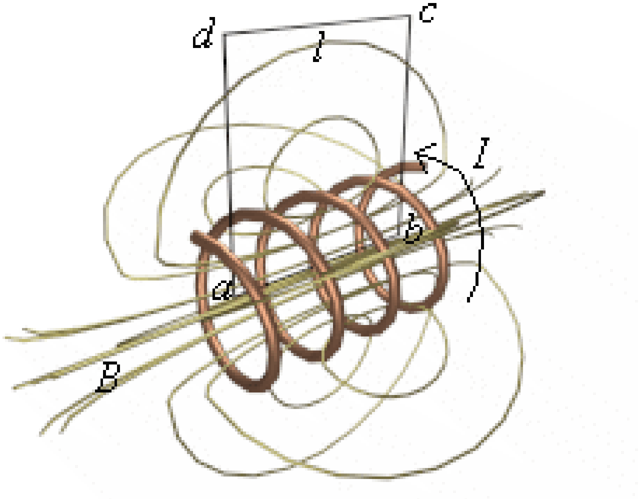


- For a simple cylindrical inductor (called a solenoid), we wind *N* turns of wire around a cylindrical form. The inductance is ideally given by

$$L = \frac{(\mu_0 N^2 \pi r_c^{\;2})}{d} Henries$$

where this expression only holds when the length *d* is very much greater than the diameter $2r_c$

Making an Inductor

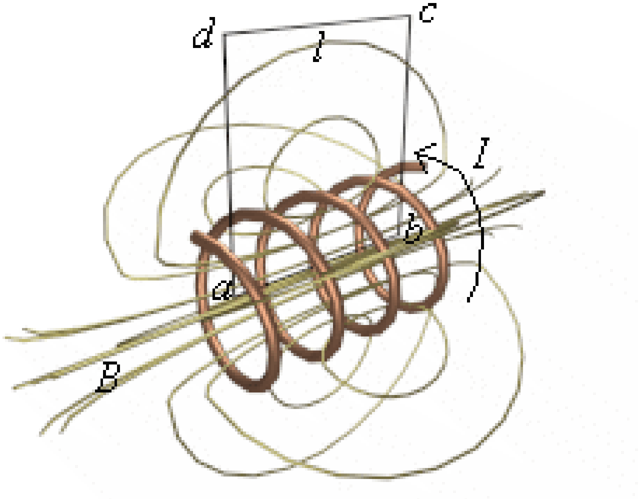


- Note that the constant $\mu_o = 4\pi \times 10^{-7}$ H/m is required to have inductance in Henries (named after Joseph Henry of Albany)
- For magnetic materials, we use μ instead, which can typically be 10^5 times larger for materials like iron
- μ is called the permeability

Some Typical Permeabilities

- Air 1.257×10^{-6} H/m
- Ferrite U M33 9.42×10^{-4} H/m
- Nickel 7.54×10^{-4} H/m
- Iron 6.28×10^{-3} H/m
- Ferrite T38 1.26×10^{-2} H/m
- Silicon GO steel 5.03×10^{-2} H/m
- supermalloy 1.26 H/m

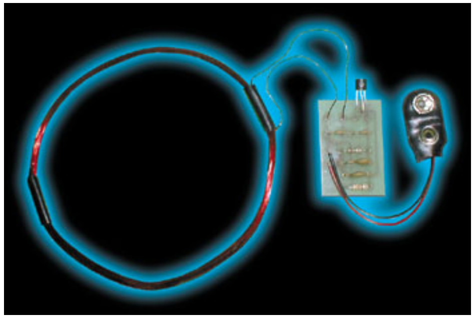
Making an Inductor



- If the coil length is much smaller than the diameter (r_w is the wire radius)

$$L \cong \mu N^2 r_c \{\ln(\frac{8r_c}{r_w}) - 2\}$$

Such a coil is used in the metal detector at the right



Calculating Resistance

- All wires have some finite resistance. Much of the time, this resistance is negligible when compared with other circuit components.
- Resistance of a wire is given by

$$R = \frac{l}{\sigma A}$$

l is the wire length

A is the wire cross sectional area (πr_w^2)

σ is the wire conductivity

Some Typical Conductivities

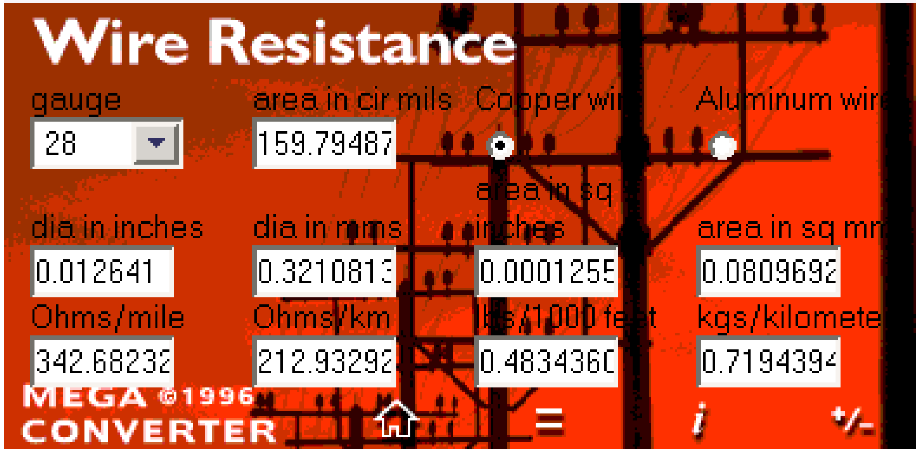
- Silver 6.17×10^7 Siemens/m
- Copper 5.8×10^7 S/m
- Aluminum 3.72×10^7 S/m
- Iron 1×10^7 S/m
- Sea Water 5 S/m
- Fresh Water 25×10^{-6} S/m
- Teflon 1×10^{-20} S/m

Siemen = 1/ohm

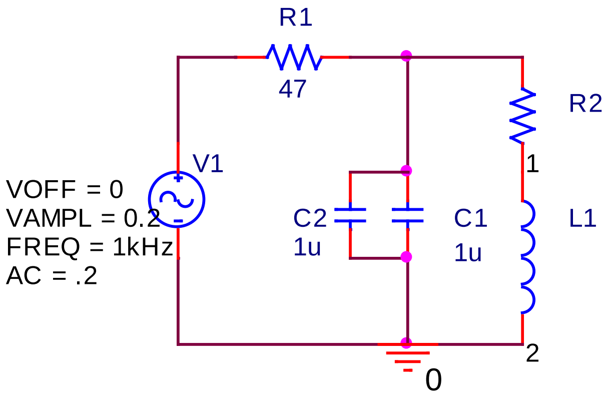
Wire Resistance

- Using the Megaconverter at http://www.megaconverter.com/Mega2/

(see course website)

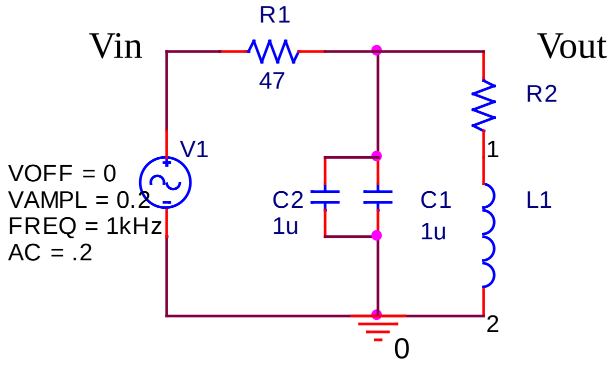


Part B: Measuring Inductance with a Circuit



- For this circuit, a resonance should occur for the parallel combination of the unknown inductor and the known capacitor. If we find this frequency, we can find the inductance.

Determining Inductance



$$\omega_0 = 1/\sqrt{LC} \qquad f_0 = 1/2\pi\sqrt{LC}$$

- Reminder—The parallel combination of L and C goes to infinity at resonance. (Assuming R2 is small.)

$$Z_{\parallel} = \frac{j\omega L\left(1/j\omega C\right)}{j\omega L + \left(1/j\omega C\right)} = \frac{j\omega L}{1-\omega^2 LC}$$

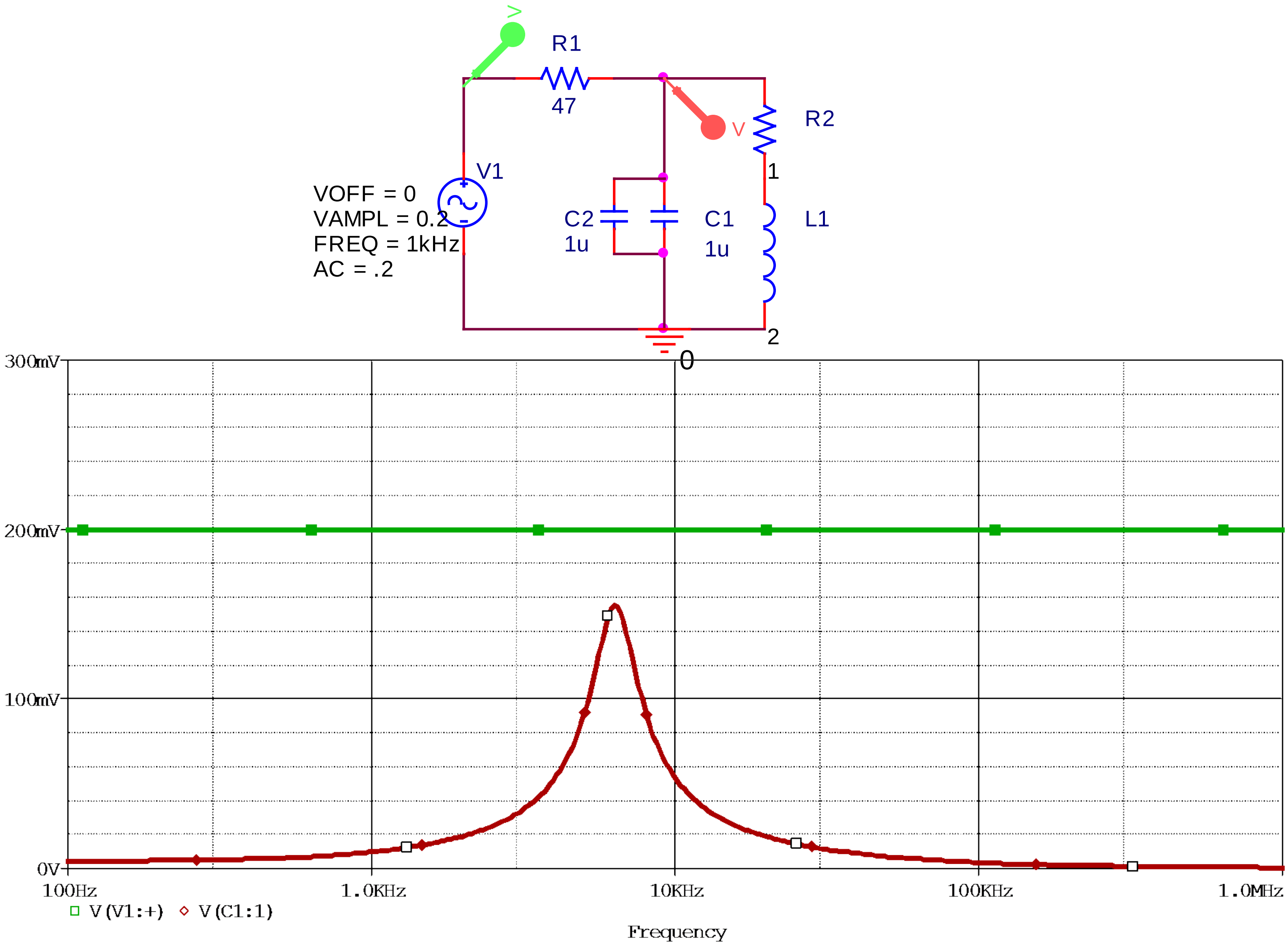
Determining Inductance

$$H = \frac{Z_{\|}}{R1 + Z_{\|}}$$

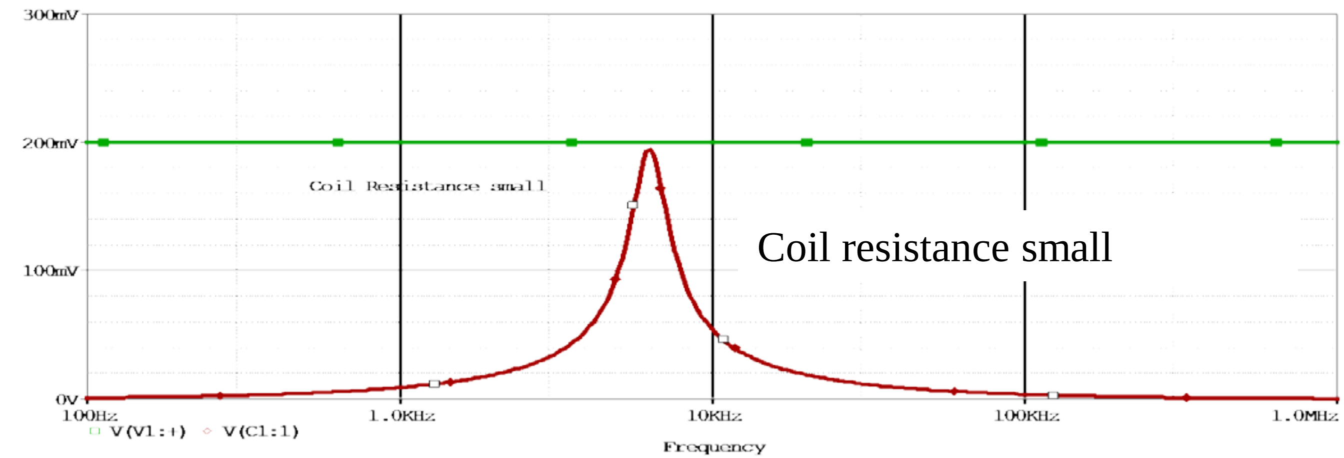
$$H = \frac{j\omega L}{R1(1-\omega^2 LC) + j\omega L}$$

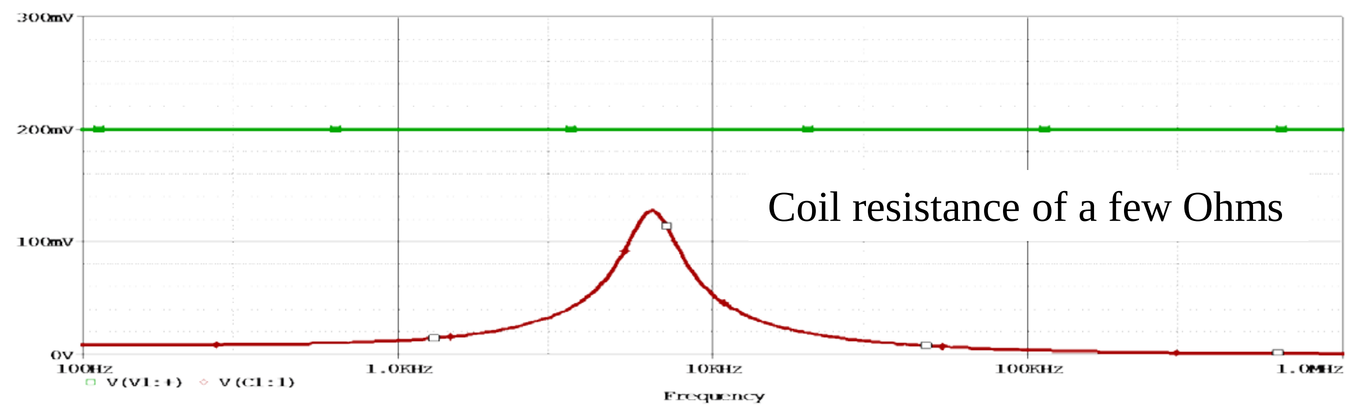
$$H_{HI} = H_{LO} = \frac{j\omega L}{R1} = small$$

$$at\ resonance,\ \omega_0, \quad H_0 = \frac{j\omega L}{j\omega L} = 1$$


R1
47
V1
VOFF = 0
VAMPL = 0.2
FREQ = 1kHz
AC = .2
C2
1u
C1
1u
R2
1
L1
2
0
300mV
200mV
100mV
0V
100Hz
1.0KHz
10KHz
100KHz
1.0MHz
V(V1:+)
V(C1:1)
Frequency

- Even 1 ohm of resistance in the coil can spoil this response somewhat



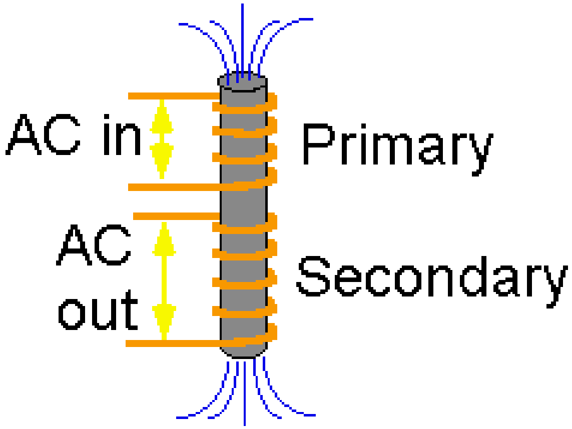


Part C

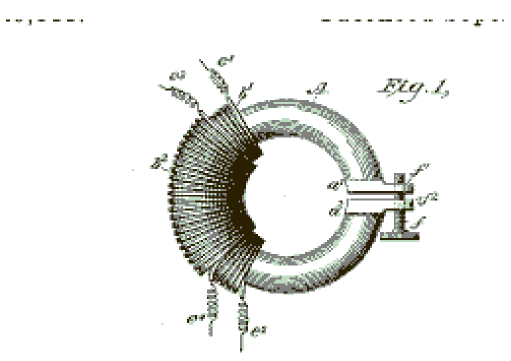
- Examples of Transformers
- Transformer Equations

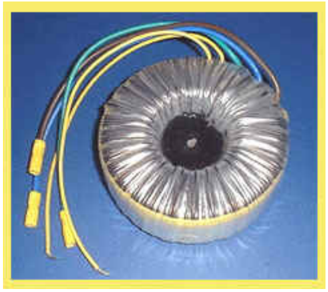
Transformers

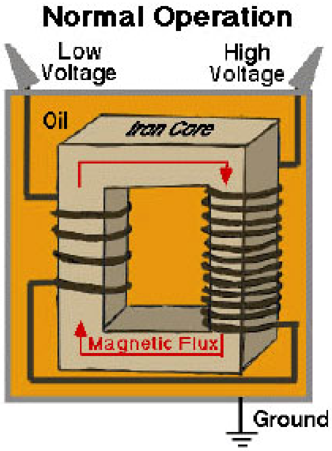
- Cylinders (solenoids)



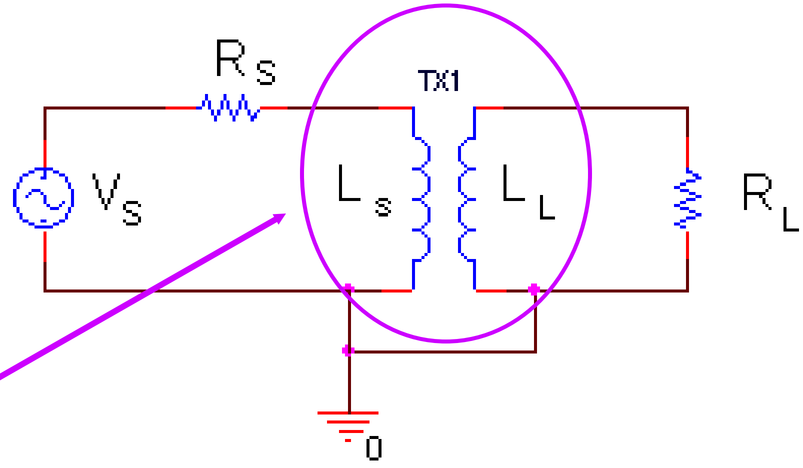
- Toroids







Transformer Equations

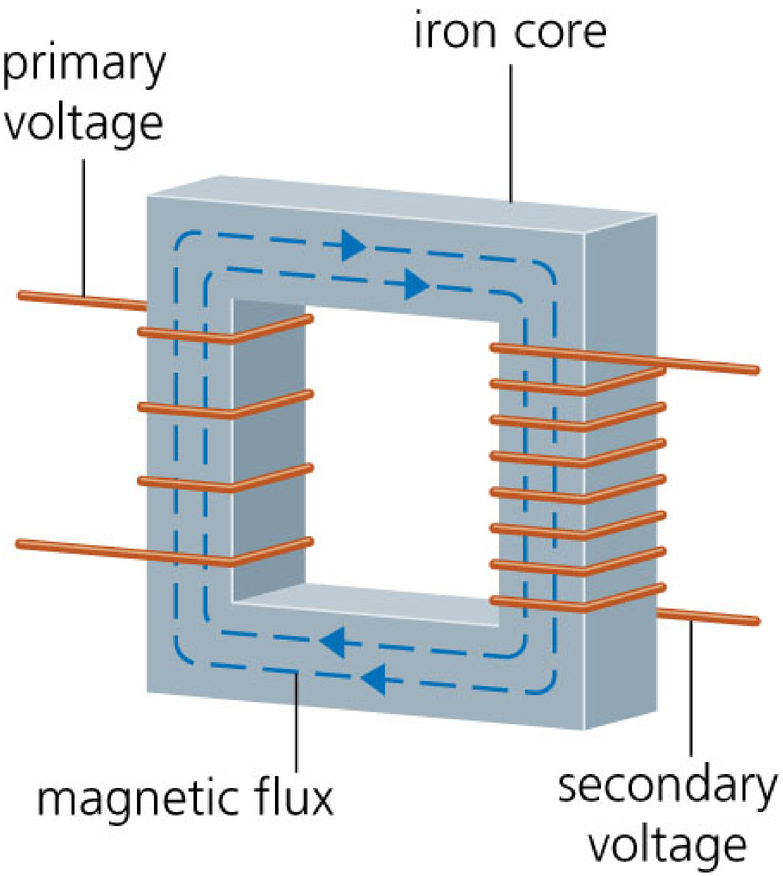


Symbol for transformer

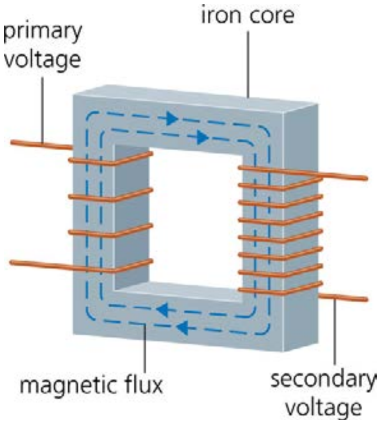
$$a = \frac{N_L}{N_S} = \frac{V_L}{V_S} = \sqrt{\frac{L_L}{L_S}} = \frac{I_S}{I_L} \qquad Z_{in} = \frac{R_L}{a^2}$$

Deriving Transformer Equations

- Note that a transformer has two inductors. One is the primary (source end) and one is the secondary (load end): L_S & L_L
- The inductors work as expected, but they also couple to one another through their mutual inductance: $M^2 = k^2 L_S L_L$



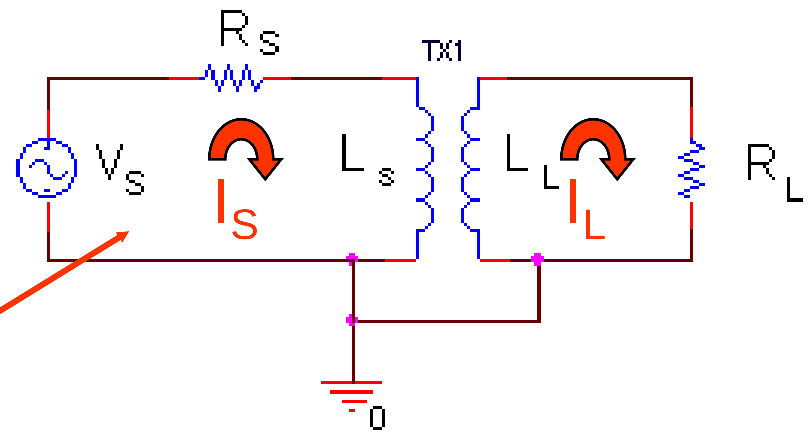
Transformers



- Assumption 1: Both Inductor Coils must have similar properties: same coil radius, same core material, and same length.

$$\frac{L_L}{L_S} = \frac{\dfrac{(\mu_0 N_L^2 \pi r_c{}^2)}{d}}{\dfrac{(\mu_0 N_S^2 \pi r_c{}^2)}{d}} = \frac{N_L^2}{N_S^2} \qquad let\ a = \frac{N_L}{N_S} \qquad \therefore\ a = \sqrt{\frac{L_L}{L_S}}$$

Transformers



Note Current Direction

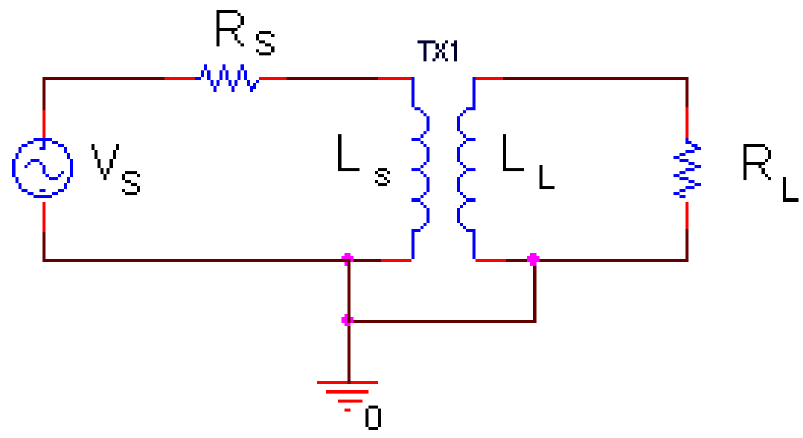
- Let the current through the primary be I_S
- Let the current through the secondary be I_L
- The voltage across the primary inductor is

$$j\omega L I_S - j\omega M I_L$$

- The voltage across the secondary inductor is

$$j\omega L I_L - j\omega M I_S$$

Transformers



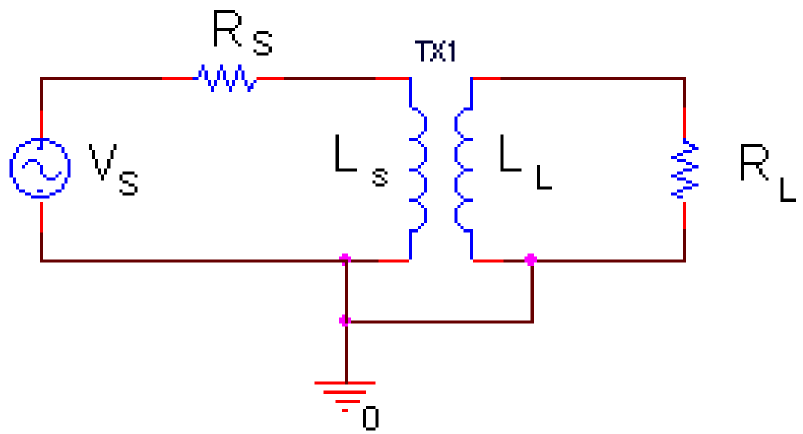
- Sum of primary voltages must equal the source

$$V_S = R_S I_S + j\omega L_S I_S - j\omega M I_L$$

- Sum of secondary voltages must equal zero

$$0 = R_L I_L + j\omega L_L I_L - j\omega M I_S$$

Transformers



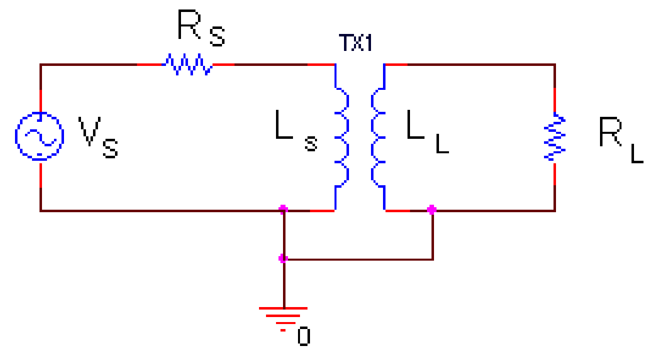
- Assumption 2: The transformer is designed such that the impedances $Z = j\omega L$ are much larger than any resistance in the circuit. Then, from the second loop equation

$$0 = R_L I_L + j\omega L_L I_L - j\omega M I_S$$

$$j\omega L_L I_L \approx j\omega M I_S \Rightarrow L_L^2 I_L^2 \approx M^2 I_S^2$$

$$\therefore \quad \frac{I_L}{I_S} \approx \frac{M}{L_L}$$

Transformers

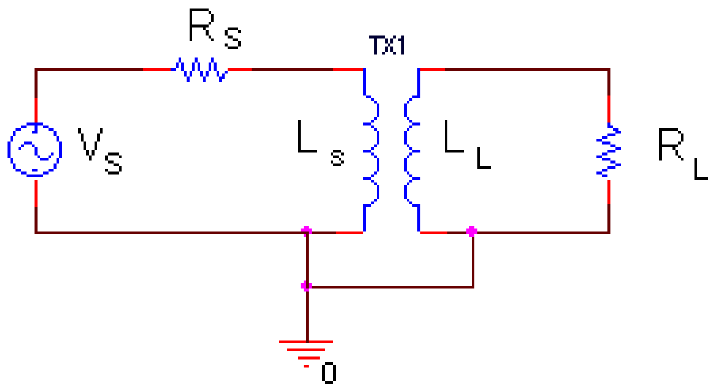


- k is the coupling coefficient
  - If k=1, there is perfect coupling.
  - k is usually a little less than 1 in a good transformer.
- Assumption 3: Assume perfect coupling (k=1)

We know $M^2=k^2 L_S L_L = L_S L_L$ and $a = \sqrt{\frac{L_L}{L_S}}$

Therefore, $\therefore \quad \frac{I_L}{I_S} \approx \frac{M}{L_L} = \frac{\sqrt{L_S L_L}}{L_L} = \sqrt{\frac{Ls}{L_L}} = \frac{1}{a}$

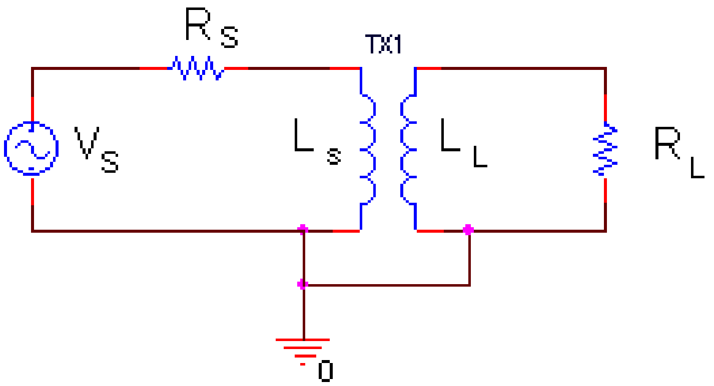
Transformers



- The input impedance of the primary winding reflects the load impedance. $Z_{L_S} = Z_{in} = Z_{total} - R_S$
- It can be determined from the loop equations
  - 1] $V_S = R_S I_S + j\omega L_S I_S - j\omega M I_L$
  - 2] $0 = R_L I_L + j\omega L_L I_L - j\omega M I_S$
- Divide by 1] I_S. Substitute 2] and M into 1]

$$Z_{IN} = {V_S}/{I_S} - R_S = j\omega L_S + {\omega^2 L_S L_L}/{\left(R_L + j\omega L_L\right)}$$

Transformers



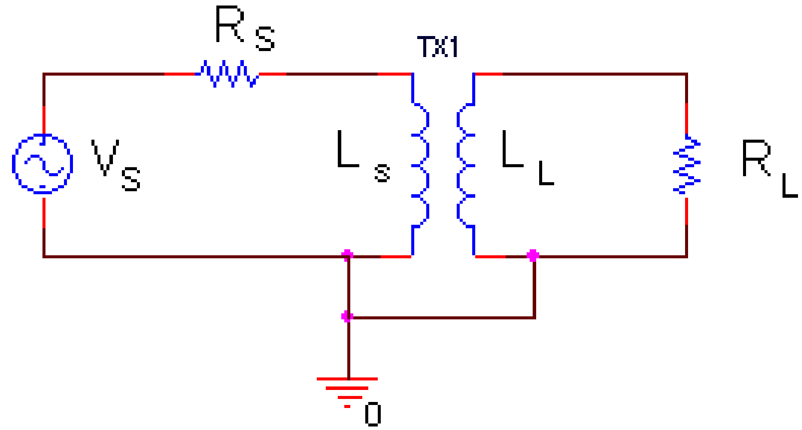
- Find a common denominator and simplify

$$Z_{IN} = \frac{j\omega L_S R_L}{j\omega L_L + R_L}$$

- By Assumption 2, R_L is small compared to the impedance of the transformer, so

$$Z_{IN} = \frac{L_S R_L}{L_L} = \frac{R_L}{a^2}$$

Transformers



- It can also be shown that the voltages across the primary and secondary terminals of the transformer are related by

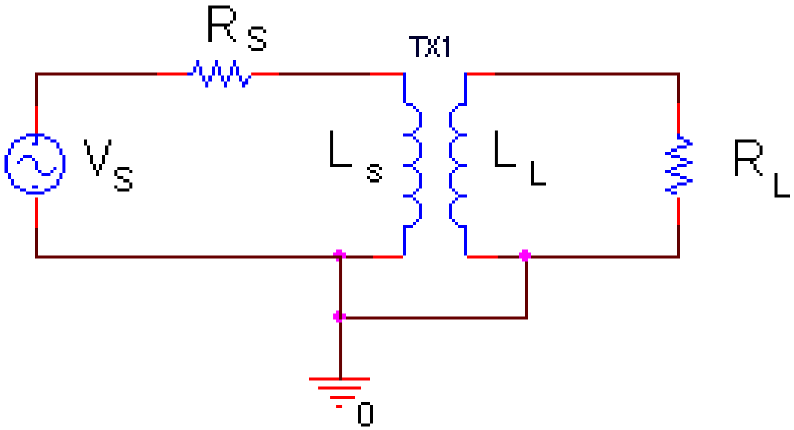
$$N_S V_L = N_L V_S$$

Note that the coil with more turns has the larger voltage.

- Detailed derivation of transformer equations

http://hibp.ecse.rpi.edu/~connor/education/transformer_notes.pdf

Transformer Equations



$$a = \frac{N_L}{N_S} = \frac{V_L}{V_S} = \sqrt{\frac{L_L}{L_S}} = \frac{I_S}{I_L} \qquad Z_{in} = \frac{R_L}{a^2}$$

Part D

- Step-up and Step-down transformers
- Build a transformer

Step-up and Step-down Transformers

Step-up Transformer	Step-down Transformer
$N_2 > N_1$	$N_2 < N_1$
$V_2 > V_1$	$V_2 < V_1$
$I_2 < I_1$	$I_2 > I_1$
$\sqrt{L_2} > \sqrt{L_1}$	$\sqrt{L_2} < \sqrt{L_1}$

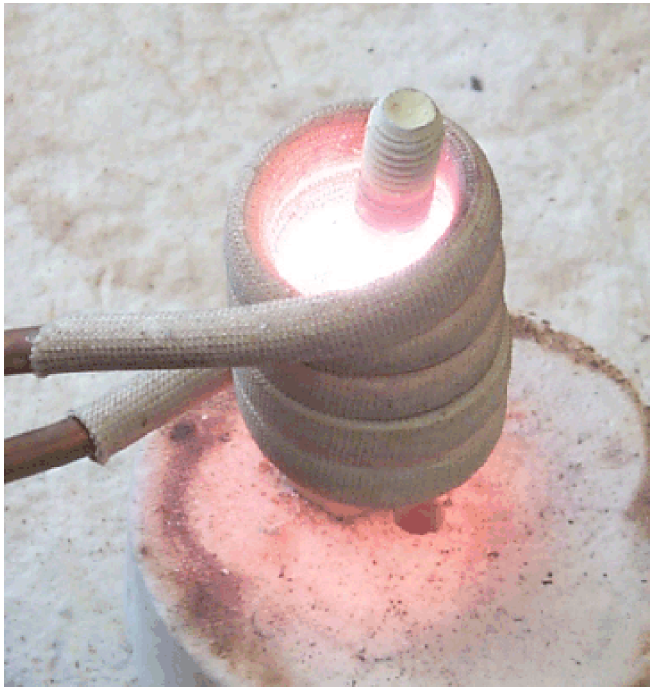
Note that power (P=VI) is conserved in both cases.

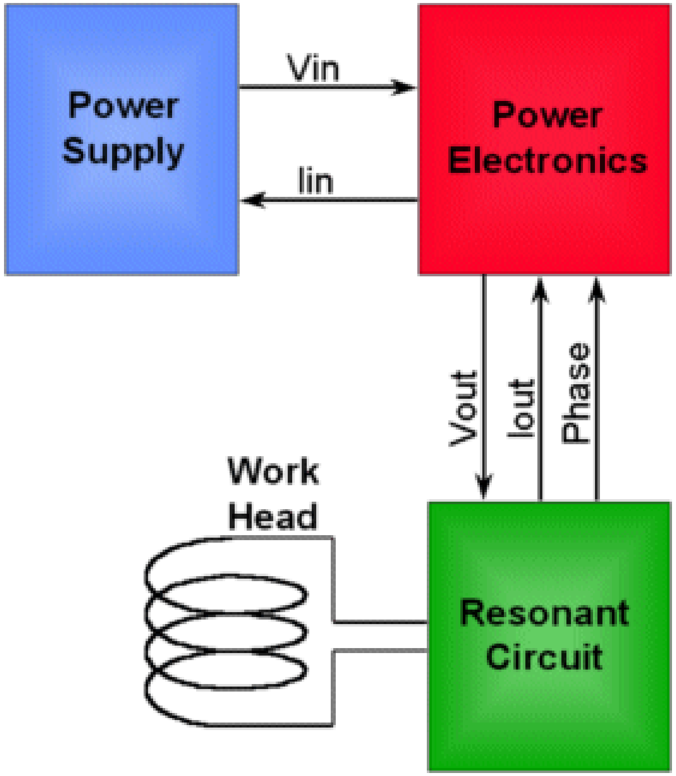
Build a Transformer

- Wind secondary coil directly over primary coil
- "Try" for half the number of turns
- At what frequencies does it work as expected with respect to voltage? When is $\omega L >> R$?

$$a = \frac{N_L}{N_S} = \frac{V_L}{V_S}$$

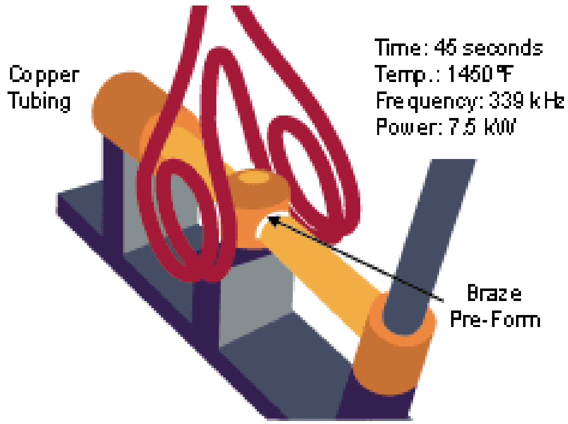
Some Interesting Inductors

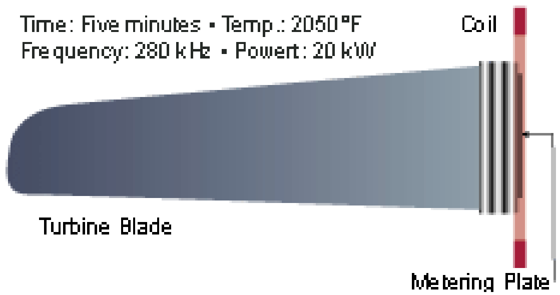




- Induction Heating

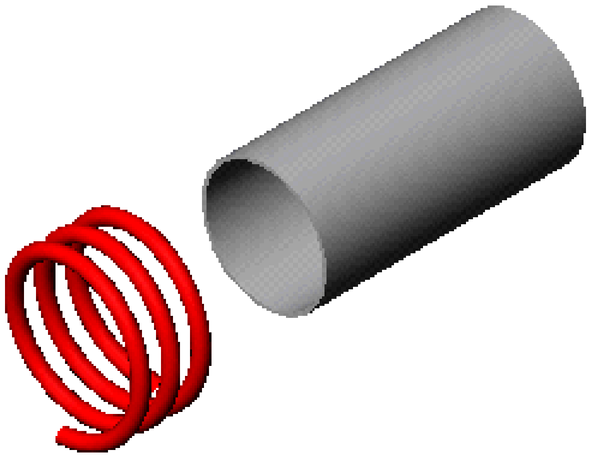
Some Interesting Inductors

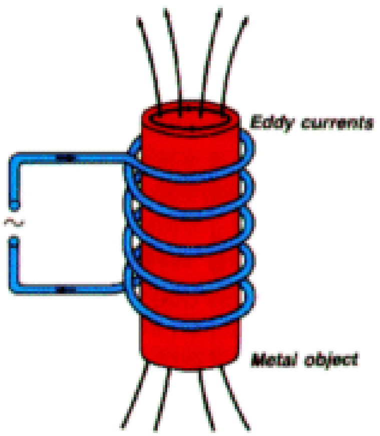




- Induction Heating in Aerospace

Some Interesting Inductors



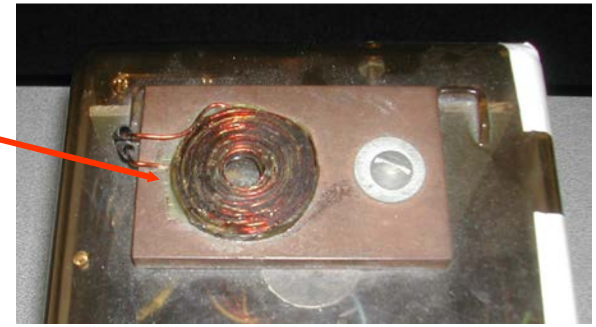


- Induction Forming

Some Interesting Inductors

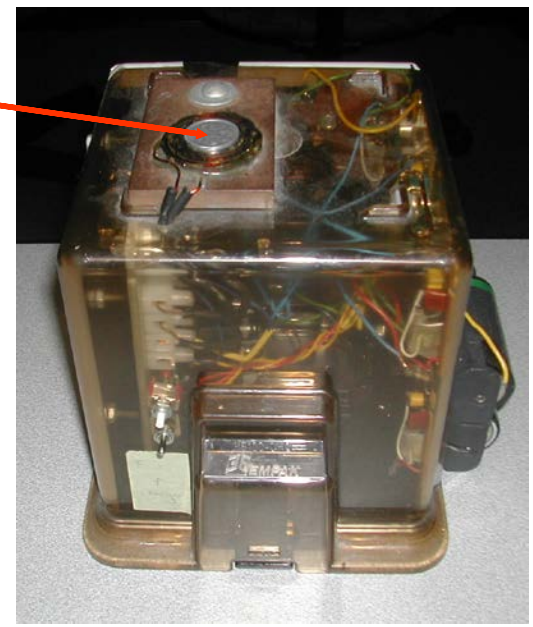
Primary

Coil



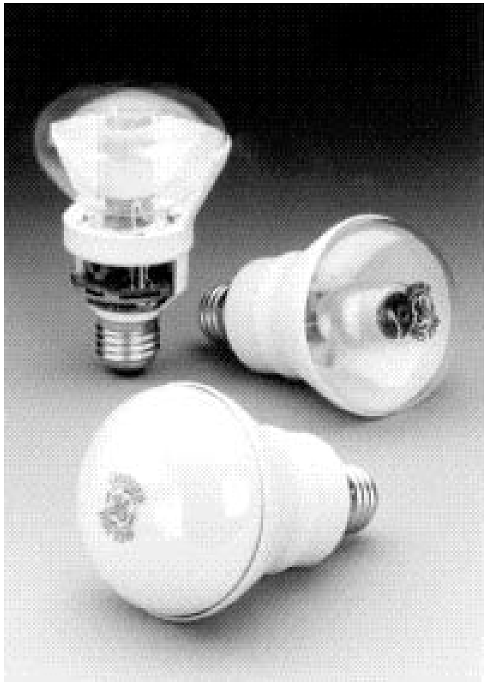
Secondary

Coil



- Coin Flipper

Some Interesting Inductors



- GE Genura Light

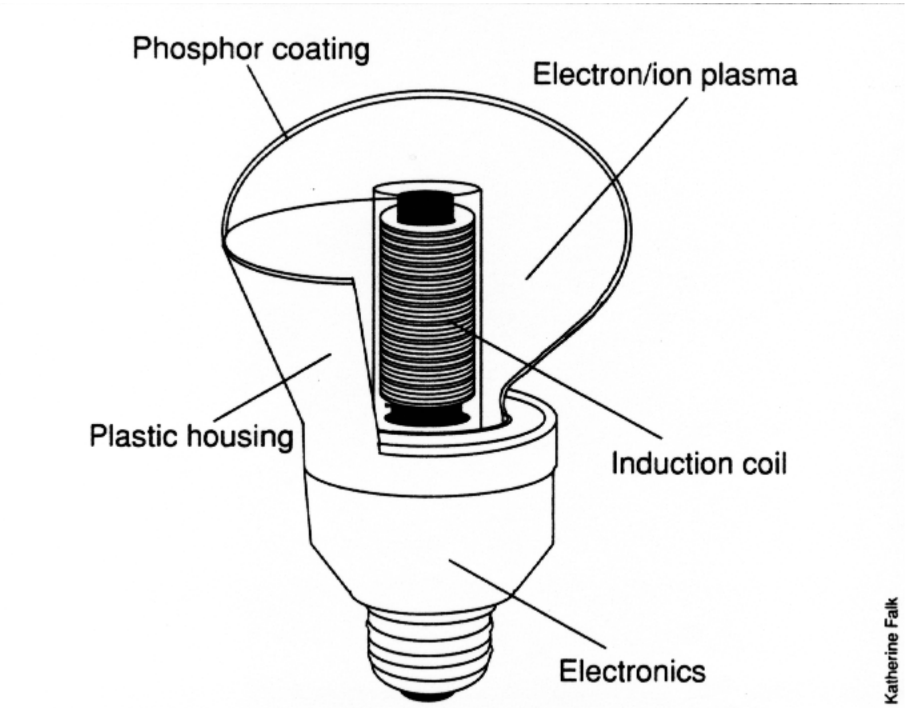
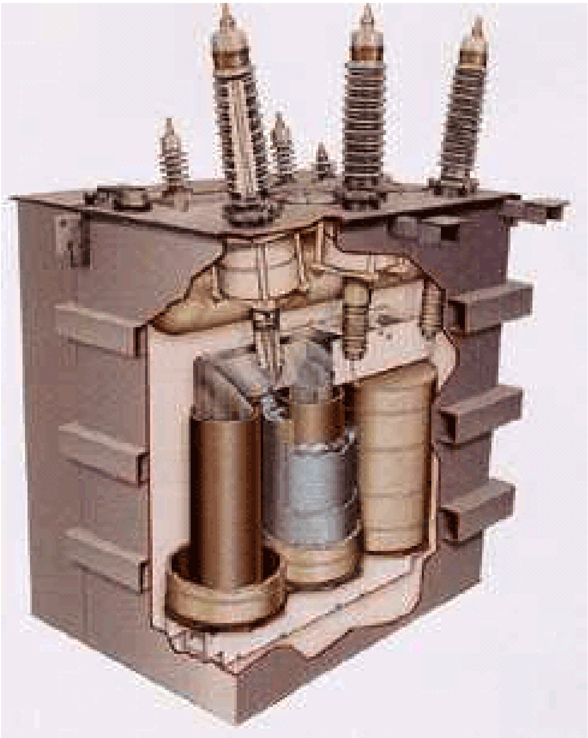


Figure 1. Schematic of the Genura induction lamp. The power supply converts ordinary 60 or 50 hertz current into high-efficiency power that is fed into an electrical coil. The coil excites a gas plasma inside the bulb, releasing UV radiation that strikes the bulb's phosphor coating and is converted into visible light.

Some Interesting Transformers

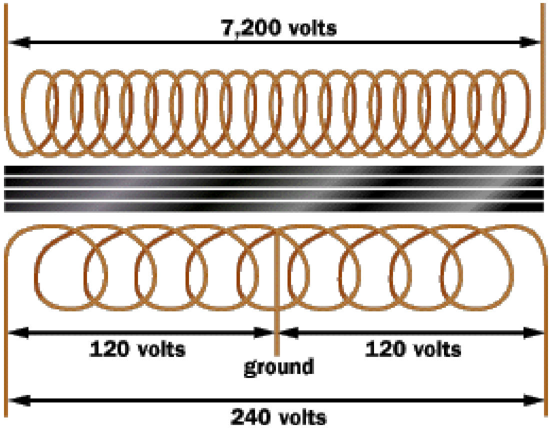
- A huge range in sizes

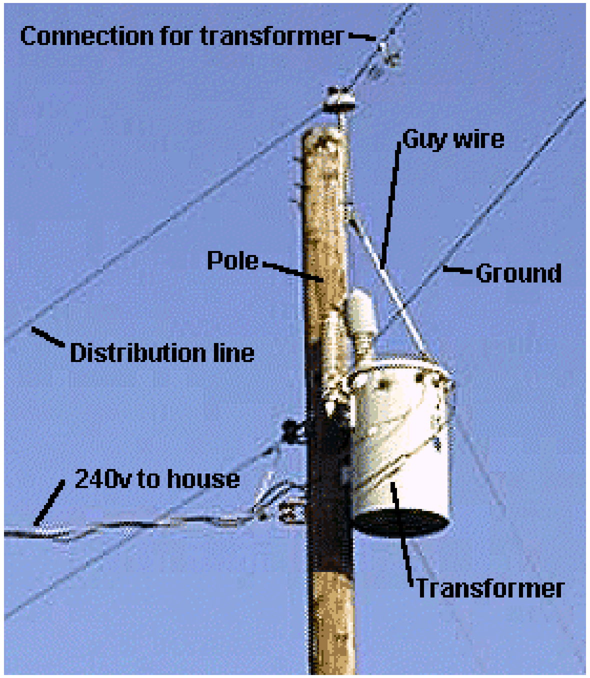
Some Interesting Transformers

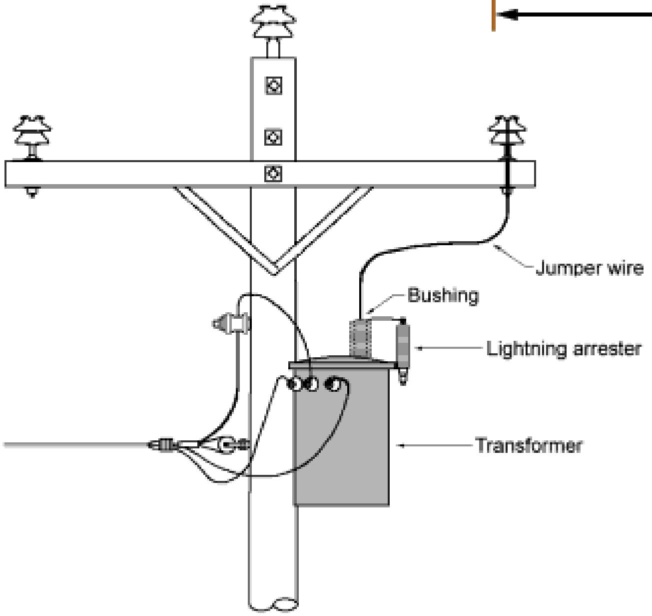


- High Temperature Superconducting Transformer

Household Power

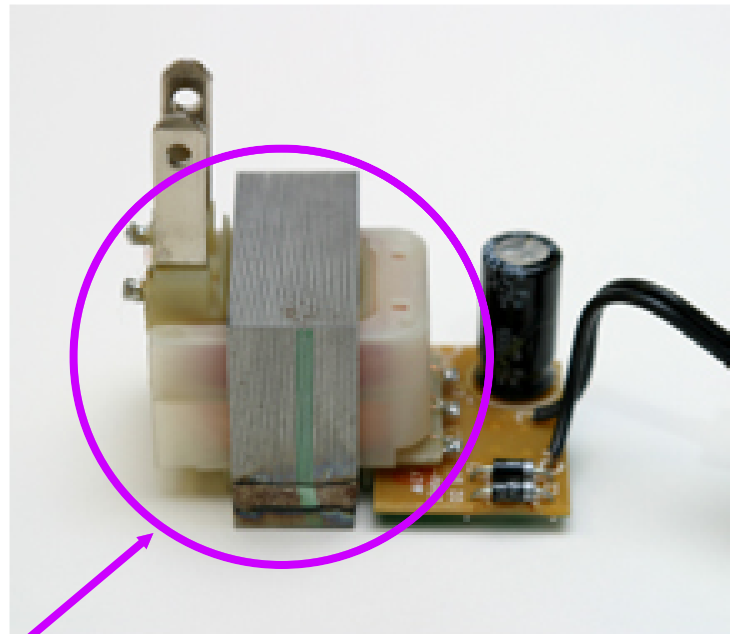






- 7200V transformed to 240V for household use

Wall Warts



Transformer