

ENGR-4300
Spring 2009
Test 2

Name: **SOLUTION**

Section: 1(MR 8:00) 2(TF 2:00) 3(MR 6:00)
(circle one)

Question I (20 points): _____

Question II (20 points): _____

Question III (17 points): _____

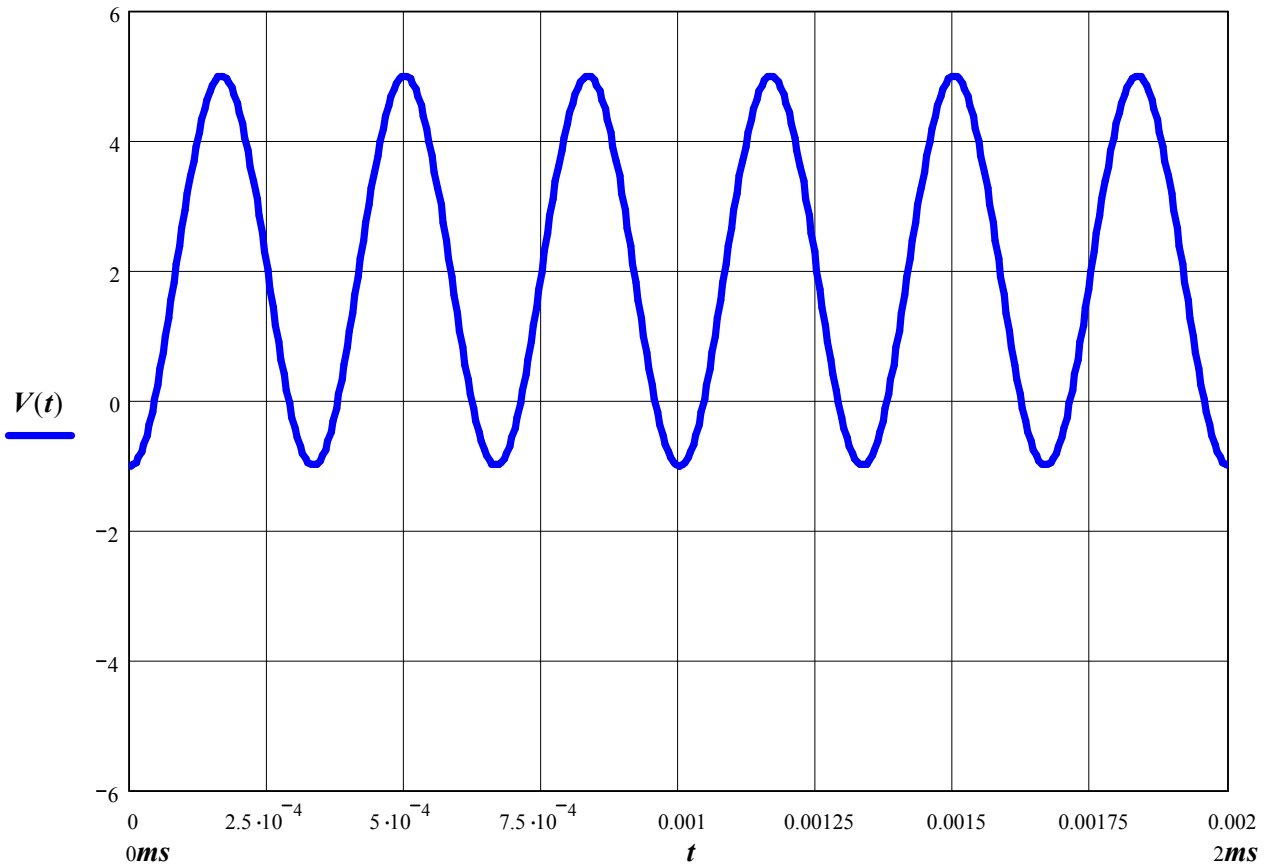
Question IV (20 points): _____

Question V (23 points): _____

Total (100 points): _____

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

Question I – Bridges and Damped Sinusoids (20 points)



1) (4 pts) Find the *period and frequency* of this signal. Include units.

$T = 1\text{ms}/3$ cycles they can also see that $\sim .33$ ms for one cycle or $2\text{ms}/6$ cycles = .33333 s
 $f = 1/T = 3\text{k}$

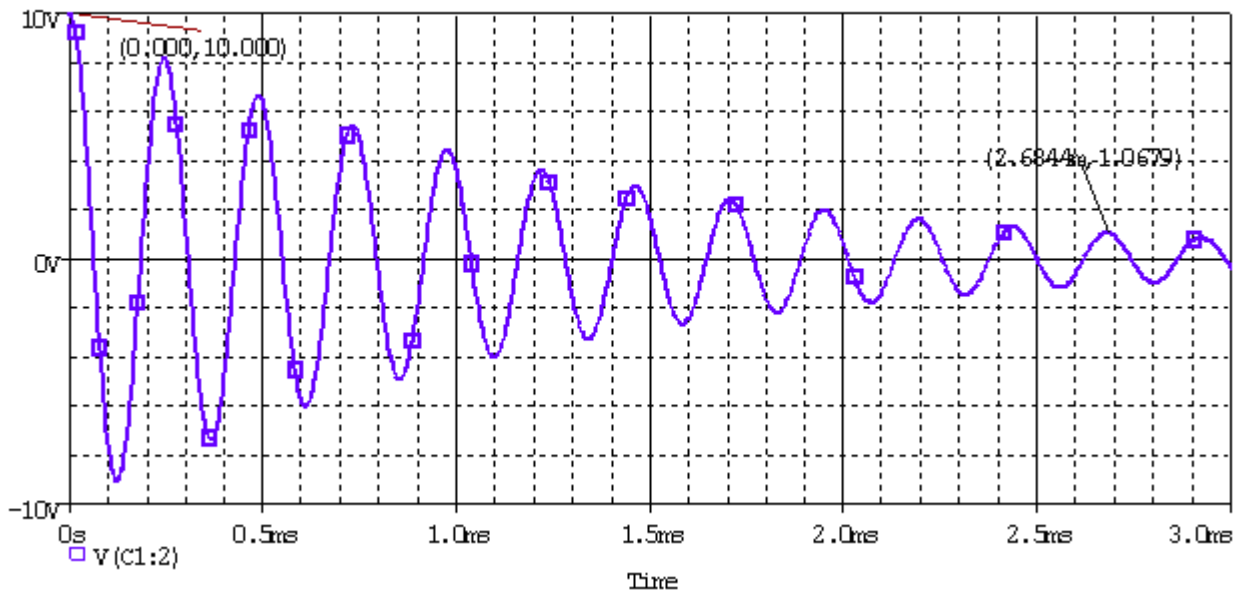
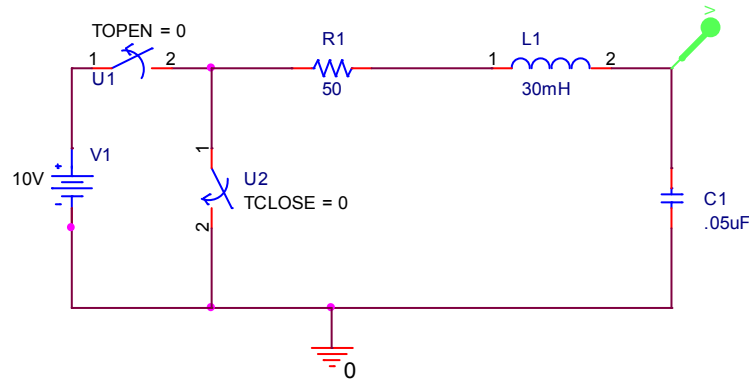
2) (2 pts) What is the value of the DC offset?

Offset = + 2V

3) (4 pts) Write the mathematical expression for this signal and its offset. In general, this is given by $X = X_0 + X_1 \sin(\omega t + \Phi_0)$ which accounts for its frequency, phase shift, and offset. (Phase shift with respect to $t=0$)

Phase shift = -90 or $-\pi/2$
 $2 + 3V \sin(2 * \pi * 3\text{k} * t - \pi/2) = 2V + 3V \sin(18,850t - \pi/2)$

The circuit below gives the simulation output below.



4) (3 pts) Find the damping constant α for this data (*use points labeled for you to three significant figures*).

$(t_0, V_0) = (0\text{ms}, 10\text{V})$

$(t_1, V_1) = (2.68\text{ ms}, 1.07\text{V})$

$V_1 = V_0 e^{-\alpha(t_1 - t_0)}$
 $\alpha = 834\text{ 1/s}$

$V_1 := 1.07\text{V}$

$V_0 := 10\text{V}$

$t_1 := 2.68\text{ms}$

$t_0 := 0\text{ms}$

$$\ln\left(\frac{V_1}{V_0}\right) = -\alpha(t_1 - t_0)$$

$$\frac{\ln\left(\frac{V_1}{V_0}\right)}{(t_1 - t_0)} = 833.928 \frac{1}{s}$$

5) (4 pts) Write the mathematical expression for this data in the form of $v(t) = A \cos(\omega t) e^{-\alpha t}$

$f = 12 \text{ cycles}/3\text{ms} = 4\text{k Hz}$ or they can find the period and inverse

$\omega = 2\pi f = 25,132 \text{ rad/s}$ or 25 K rad/s

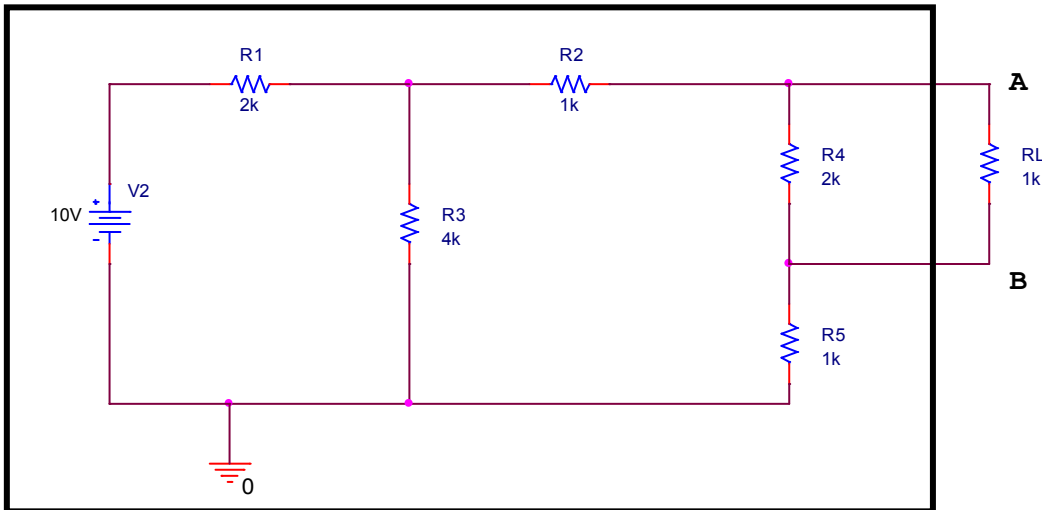
$$0 + 10V \cos(25 \text{ k} * t) e^{-834t}$$

6) (3 pt) If the resistor is completely removed from the circuit and replaced with a wire, what will happen to the **frequency and amplitude** of the simulated output? **Why?**

The frequency will remain the same.

The amplitude will no longer be dampened and it will forever oscillate sinusoidally. The frequency is not affected by the resistor. The amplitude no longer reduces with time because there is nothing to slow the transfer of energy between the potential energy and kinetic energy from the capacitor and inductor respectively. (conservation of energy)

Question II – Thevenin Equivalents (20 points)



1) (6 pts) Find the Thevenin voltage (V_{th}) of the circuit inside the rectangle.

$$R_{245} := R_2 + R_4 + R_5$$

$$R_{245} = 4 \times 10^3 \Omega$$

$$R_{2345} := \frac{R_3 \cdot R_{245}}{R_3 + R_{245}}$$

$$R_{2345} = 2 \times 10^3 \Omega$$

$$V_{2345} := \frac{10V \cdot 2k\Omega}{4k\Omega}$$

$$V_{2345} = 5V$$

$$V_4 = V_A - V_B = V_{th}$$

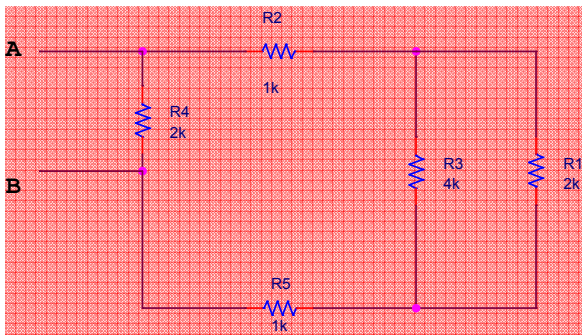
$$V_4 := V_{2345} \cdot \left(\frac{R_4}{R_2 + R_4 + R_5} \right)$$

$$V_4 = 2.5V$$

$$V_{th} := 2.5V$$

2) (6 pts) Find the Thevenin resistance.

Short the voltage supply and if it helps turn the circuit around



$$R_{13} := \frac{R_1 \cdot R_3}{R_1 + R_3}$$

$$R_{13} = 1.333 \times 10^3 \Omega$$

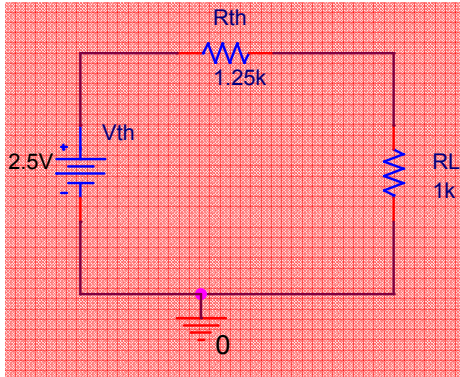
$$R_{1325} := R_2 + R_5 + R_{13}$$

$$R_{1325} = 3.333 \times 10^3 \Omega$$

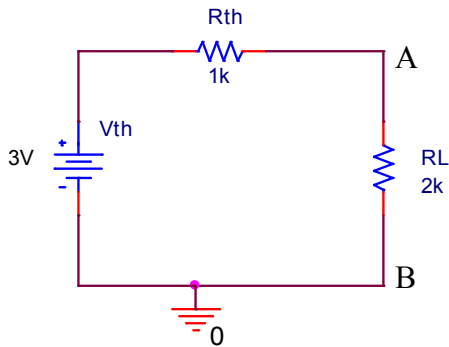
$$R_{41325} := \frac{R_4 \cdot R_{1325}}{R_4 + R_{1325}}$$

$$R_{41325} = 1.25k\Omega \quad R_{th} = 1.25k\Omega$$

3) (4 pts) Draw the Thevenin equivalent circuit with the load (1K ohms).

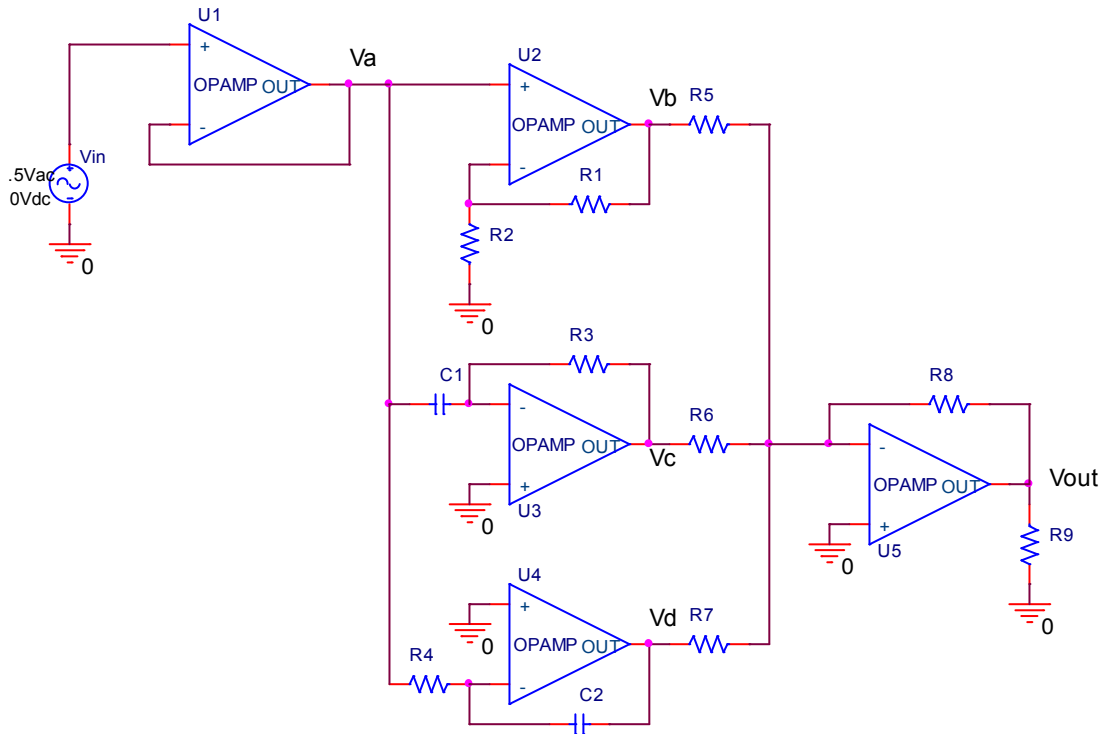


4) (4 pts) For the following Thevenin equivalent circuit (*not the one above*) find the voltage between A and B with a load of 2K ohms.



$$3 \text{ V} \cdot \frac{2 \text{ k}\Omega}{3 \text{ k}\Omega} = 2 \text{ V}$$

Question III – Op-Amp Applications (17 points)



Assume that ± 9 Volt power supplies have been properly connected to all five op-amps in the circuit above.

1) (5pt) The circuit has 5 op-amps labeled as U1 through U5. State what the op-amp circuit is for each. Choices are: 1. Follower/Buffer, 2. Inverting Amp, 3. Non-inverting Amp, 4. Differentiator, 5. Integrator, 6. Adding (Mixing) Amp, 7. Difference (Differential) Amp, 8. Miller Integrator, 9. Practical Differentiator.

U1 Circuit: FOLLOWER U2 Circuit: NON-INVERTING U3 Circuit: DIFFERENTIATOR

U4 Circuit: INTEGRATOR U5 Circuit: ADDING

2) (4pt) Determine the values relative to ground, of $V_a(t)$, $V_b(t)$, $V_c(t)$ and $V_d(t)$ as functions of $V_{in}(t)$ with $R_1 = 20k$, $R_2 = 20k$, $R_3 = 4k$, $R_4 = 3k$, $R_5 = 10k$, $R_6 = 20k$, $R_7 = 30k$, $R_8 = 10k$, $R_9 = 1k$, $C_1 = 100\mu F$ and $C_2 = 68\mu F$.

a) Voltage at point $V_a(t)$:

$$V_a(t) = V_{in}(t) \quad \text{Follower output} = \text{input}$$

b) Voltage at point $V_b(t)$:

$$V_b(t) = \left(1 + \frac{R_1}{R_2}\right)V_a(t) = \left(1 + \frac{20k}{20k}\right)V_{in}(t) = 2V_{in}(t)$$

c) Voltage at point Vc(t):

$$V_c(t) = -(R3 \cdot C1) \frac{dV_a(t)}{dt} = (-4k \cdot 100\mu) \frac{dV_{in}(t)}{dt} = (-0.4) \frac{dV_{in}(t)}{dt}$$

d) Voltage at point Vd(t):

$$V_d(t) = -\frac{1}{R4 \cdot C2} \int V_a(t) dt = -\frac{1}{3k \cdot 68\mu} \int V_{in}(t) dt = -4.902 \int V_{in}(t) dt$$

3) (3pt) Determine the output voltage, Vout(t), as a function of Vb(t), Vc(t) and Vd(t).

$$\begin{aligned} V_{out}(t) &= -\frac{R8}{R5} V_b(t) - \frac{R8}{R6} V_c(t) - \frac{R8}{R7} V_d(t) \\ &= -\frac{10k}{10k} [V_b(t)] - \frac{10k}{20k} V_c(t) - \frac{10k}{30k} [V_d(t)] \\ &= -V_b(t) - 0.5V_c(t) - 0.333V_d(t) \end{aligned}$$

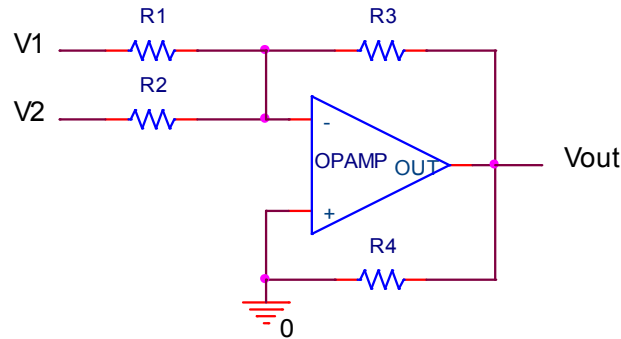
4) (4pt) Now find Vout(t) as a function of Vin(t).

$$\begin{aligned} V_{out}(t) &= -V_b(t) - 0.5V_c(t) - 0.333V_d(t) \\ &= -[2V_{in}(t)] - 0.5[-0.4 \frac{dV_{in}(t)}{dt}] - 0.333[-4.902 \int V_{in}(t) dt] \\ &= -2V_{in}(t) + 0.2 \frac{dV_{in}(t)}{dt} + 1.634 \int V_{in}(t) dt \end{aligned}$$

5) (1pt) What is a practical use for a voltage follower/buffer op-amp?

1. Prevent a load or the circuit being driven from changing the characteristics of the previous stage
2. Provide a high input impedance for a circuit's output
3. Provide a power gain but not a voltage gain

Question IV – Op-Amp Analysis (20 points)



Assume that ± 9 Volt power supplies have been properly connected to the op-amp in the circuit above.

1) (2pt) What op-amp circuit given on your crib sheet does this circuit most closely represent?

Adder (Mixing) Amplifier

2) (3pt) What feedback connection is necessary before the golden rule concerning the voltages on the two input terminals can be applied?

The op-amp must have negative feedback from its output to the negative input terminal. This can be a plain wire or a resistor.

3) (10pt) Using the two golden rules of op-amp analysis, **derive** an expression for V_{out} in terms of V_2 and the appropriate resistor values, **when V_1 has been set to 0Volts**. You must use Ohm's Law and current laws to describe how the op-amp functions by the golden rules.

$V_+ = V_- = 0V$ since V_+ is grounded. No current will flow into V_- and since $V_1 = 0V$, no current will flow through R_1 ($\Delta V=0$). Therefore all the current through R_2 must go through R_3 .

$$\frac{V_2 - 0}{R_2} = \frac{0 - V_{out}}{R_3}$$

$$V_{out} = -\frac{R_3}{R_2} V_2$$

4) (4pt) Remove R4, set $V_2 = V_1$ (short the 2 inputs together), and **derive** the expression for V_{out} in terms of V_1 and the resistors using the rules for op-amp analysis.

$$V_+ = V_- = 0V \quad R_{12} = R_1 \parallel R_2 = R_1 R_2 / (R_1 + R_2)$$

The current through R12 equals the current through R3 (no current into the op-amp V- input)

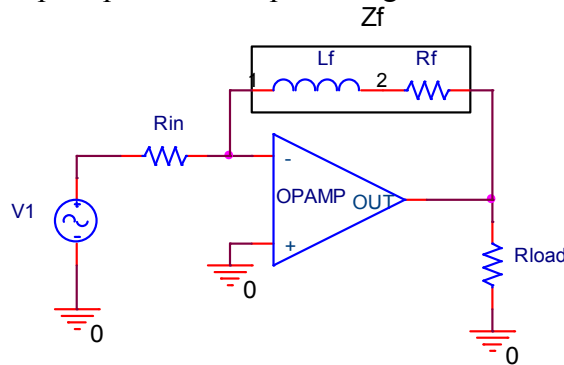
$$\frac{V_1 - 0}{R_{12}} = \frac{0 - V_{out}}{R_3}$$
$$V_{out} = -\frac{R_3}{R_{12}} V_1 = -R_3 \frac{R_1 + R_2}{R_1 \cdot R_2} V_1$$

5) (1pt) What does R4 model or represent in this circuit?

R4 represent the load resistor on the op-amp or the load being driven by the op-amp such as a motor, speaker, or LED

Question V – Op-Amp Integrators and Differentiators (23 points)

In the circuit below, $R_{in} = 200\Omega$, $L_f = 200\text{mH}$ and $R_f = 200\Omega$. Assume that power supplies have been properly connected to an ideal op-amp and the output voltage is limited to $\pm 10\text{V}$



1) (4pt) Find the transfer function $H(j\omega) = V_{out}(j\omega)/V_1(j\omega)$ for this circuit. (Substitute in the values provided for the components.)

$$H(j\omega) = \frac{V_{out}}{V_1} = -\frac{Z_f}{Z_{in}} = -\frac{j\omega L_f + R_f}{R_{in}} = -\frac{j\omega 0.2 + 200}{200} = -\frac{200 + j(0.2)\omega}{200} = -1 - j0.001\omega$$

2) (2pt) What function is this circuit designed to perform at high frequencies ($f \geq 2\text{kHz}$)?

$$H(j\omega) = \frac{V_{out}}{V_1} = -\frac{j\omega L_f}{R_{in}} = -\frac{j\omega 0.2}{200} = -j0.001\omega \quad \text{for } \omega \gg \frac{R_f}{L_f}$$

Differentiation (with gain of -0.001)

3) (2pt) What function is this circuit designed to perform at low frequencies ($f \leq 10\text{Hz}$)?

$$H(j\omega) = \frac{V_{out}}{V_1} = -\frac{R_f}{R_{in}} = -\frac{200}{200} = -1 \quad \text{for } \omega \ll \frac{R_f}{L_f}$$

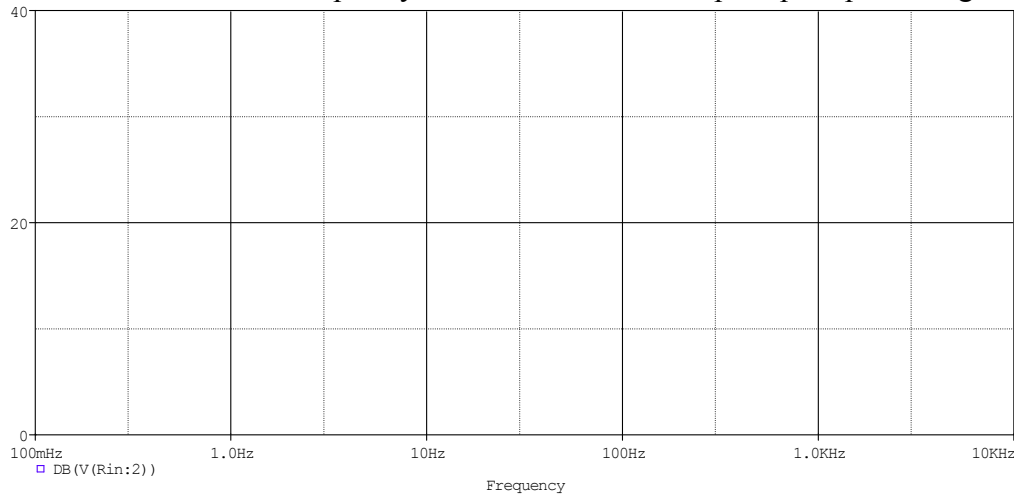
Inverting amplifier (with gain of -1)

4) (4pt) What is the corner frequency (in Hz) for the circuit where it transitions from its low frequency performance to its high frequency performance?

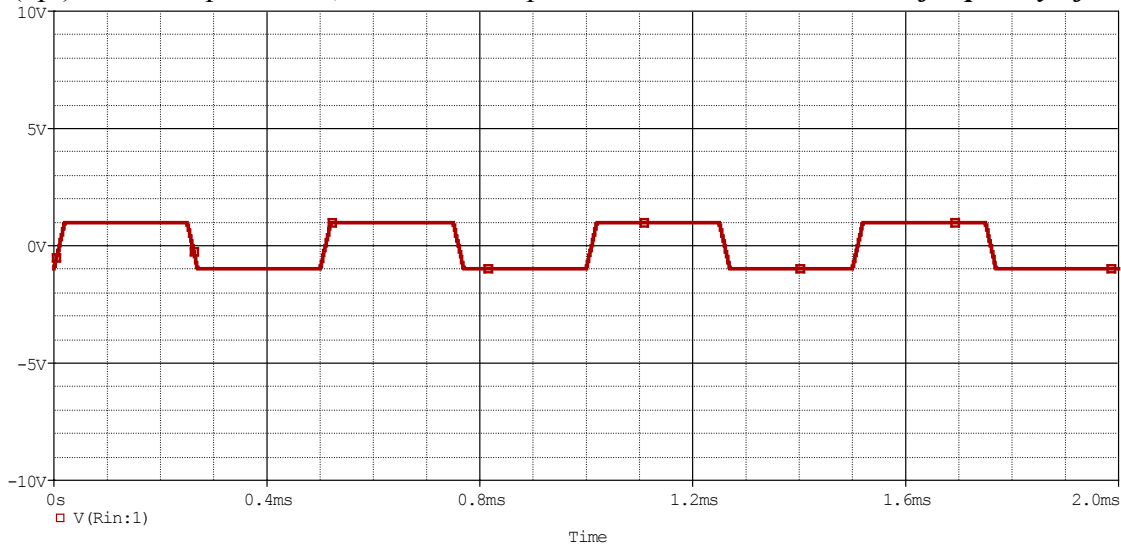
$$\omega_c \Rightarrow |R_f| = |j\omega L_f|$$

$$\omega_c = R_f/L_f = 200/0.2 = 1\text{krad} \quad f_c = \omega_c/2\pi = 159.2\text{Hz}$$

5) (4pt) Sketch the bode plot of $|H(j\omega)|$, $20\log(\text{magnitude})$ vs. \log of frequency, of the transfer function from 0.1 to 10kHz. For simplicity assume that the ideal op-amp output voltage is unlimited.



6) (4pt) For the input below, sketch the output on the same axis. *Note the frequency of the input signal.*



7) (2pt) For the input below, sketch the output on the same axis. *Note the frequency of the input signal.*

