ENGR-2300

Electronic Instrumentation

Quiz 1

Spring 2020

Name SOLUTIONS

Section ___

Question I (20 points) ___________

Question II (20 points) ___________

Question III (20 points) ___________

Question IV (20 points) ___________

LMS Question (20 points) (graded on LMS)

Total (80 points) ______________

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification. Unless otherwise stated in a problem, provide 3 significant digits in answers. Read the entire quiz before answering any questions. Also it may be easier to answer parts of questions out of order.

Standard Resistor Values (±5%)						
1.0	10	100	1.0K	10K	100K	1.0M
1.1	11	110	1.1K	11K	110K	1.1M
1.2	12	120	1.2K	12K	120K	1.2M
1.3	13	130	1.3K	13K	130K	1.3M
1.5	15	150	1.5K	15K	150K	1.5M
1.6	16	160	1.6K	16K	160K	1.6M
1.8	18	180	1.8K	18K	180K	1.8M
2.0	20	200	2.0K	20K	200K	2.0M
2.2	22	220	2.2K	22K	220K	2.2M
2.4	24	240	2.4K	24K	240K	2.4M
2.7	27	270	2.7K	27K	270K	2.7M
3.0	30	300	3.0K	30K	300K	3.0M
3.3	33	330	3.3K	33K	330K	3.3M
3.6	36	360	3.6K	36K	360K	3.6M
3.9	39	390	3.9K	39K	390K	3.9M
4.3	43	430	4.3K	43K	430K	4.3M
4.7	47	470	4.7K	47K	470K	4.7M
5.1	51	510	5.1K	51K	510K	5.1M
5.6	56	560	5.6K	56K	560K	5.6M
6.2	62	620	6.2K	62K	620K	6.2M
6.8	68	680	6.8K	68K	680K	6.8M
7.5	75	750	7.5K	75K	750K	7.5M
8.2	82	820	8.2K	82K	820K	8.2M
9.1	91	910	9.1K	91K	910K	9.1M

Type	R_{int} (Ω)	V_{oc} (V)	Capacity[a] continuous, to 1V/cell				Size (in)	Weight (gm)	Connec[b]	Comments
			(mAh)	@ (mA)	(mAh)	@ (mA)				
9V "1604"										
Le Clanche	35	9	300	1	160	10	0.65x1x1.9	35	S	
Heavy Duty	35	9	400	1	180	10	"	40	S	
Alkaline	2	9	500	1	470	10	"	55	S	280mAh@100mA
Lithium	18	9	1000	25	950	80	"	38	S	Kodak Li-MnO_2

I. Voltage Dividers and Battery Characteristics (20 points)

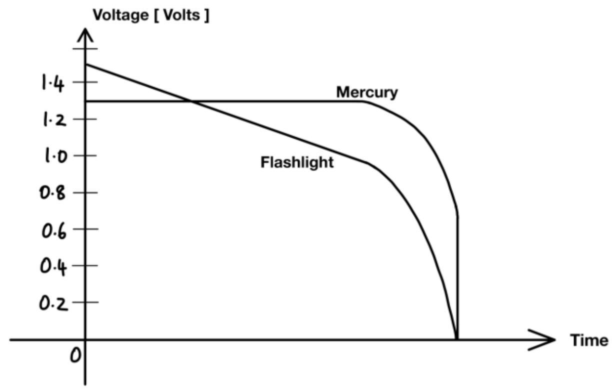
As stated on the cover page: Round answers to 3 significant digits. Show formulas first and show your work. No credit will be given for numbers that appear without justification.

a. The voltage-time curves for two types of batteries, flashlight and Mercury, are shown below.

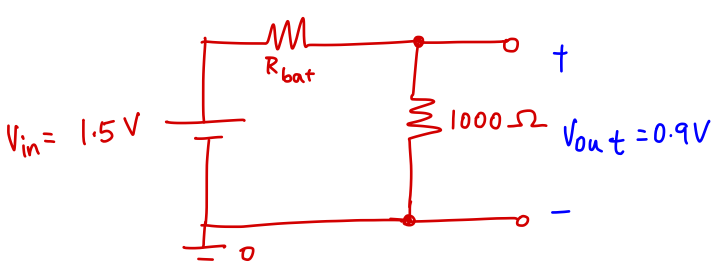
i. Interpret the behavior in terms of the respective internal resistances. For this, you need to specifically comment about how their internal resistances change over time. (2pts)

Initially, the internal resistance of flashlight
battery increases rapidly -
at a much faster rate compared to
that of the Mercury battery.

As time increases, just before the batteries are completely dead, the internal resistance of Mercury battery increases sharply.



ii. A voltmeter whose resistance is 1000 Ω measures the voltage of a worn-out 1.5V flashlight battery (same one used in the previous part) as 0.9V. What is the internal resistance of the battery? (3pts)

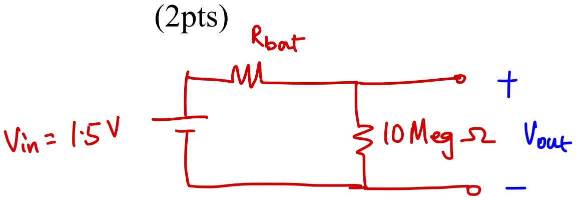


$$V_{out} = V_{in}\left(\frac{1000}{1000 + R_{bat}}\right)$$

$$\Rightarrow 900 + 0.9\,R_{bat} = 1500$$

$$\Rightarrow \boxed{R_{bat} = 666.67\,\Omega}$$

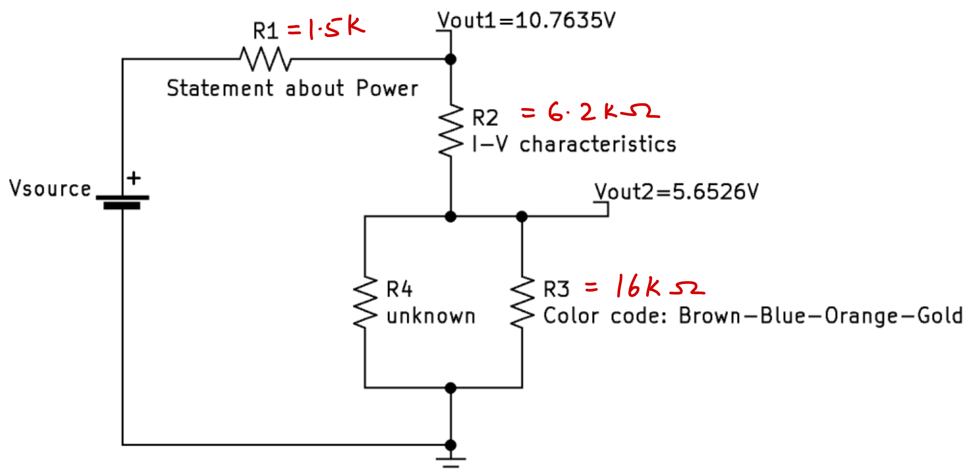
iii. If the flashlight battery from the previous part had been measured using a vacuum tube voltmeter with a resistance of 10 MegΩ, what voltage would have been read? (2pts)



$$R_{bat} << 10\,Meg\,\Omega$$

$$\Rightarrow \boxed{V_{out} \approx V_{in} = 1.5\,V}$$

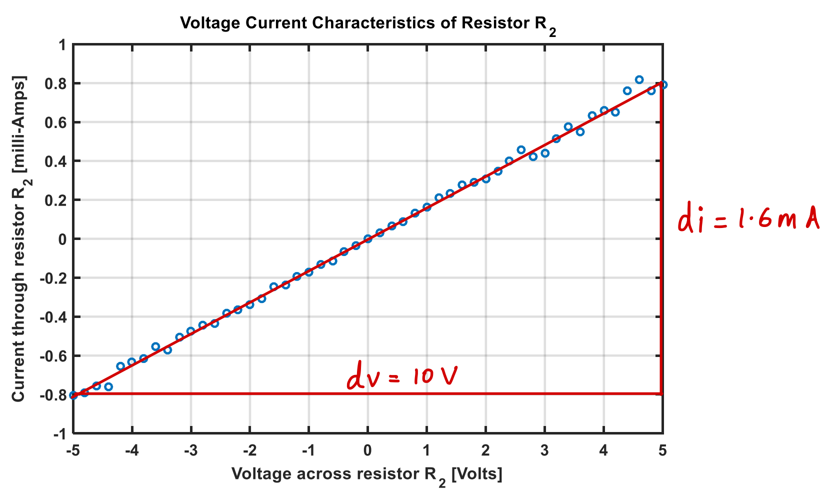
b. Consider the circuit shown below. The DC voltage of the source, Vsource, is unknown. We do know a few details about the resistors that are used in the circuit and the voltages Vout1 and Vout2 measured at points shown in the circuit to be 10.7635V and 5.6526V respectively.



i. The power absorbed by resistor R1 is 4.3 mW when 1.693 mA flows through it. What is the value of R1? (1pt) *Use the closest standard resistor value; refer to information on "Standard Resistor Values" at the beginning of the quiz.*

$R1 = \frac{P1}{I_1^2} = \boxed{1.5\ k\Omega}$

ii. The voltage-current characteristics of resistor R2 is provided to the right. Determine the value of R2. (2pts) *Use the closest standard resistor value; refer to information on "Standard Resistor Values" at the beginning of the quiz.*



$R2 = \frac{dv}{di} = \frac{10}{1.6m} = 6.25\ k\Omega$

USE $\boxed{6.2\ k\Omega}$ as standard resistor

iii. The color code of resistor R3 is Brown-Blue-Orange-Gold. What is the value of resistor R3? (1pt)

$R3 = \boxed{16\ k\Omega} \pm 5\%$

iv. Given Vout1=10.7635V and Vout2 = 5.6526 V and the answers from the previous questions, determine the value of Vsource? (4pts) Hint: First find current through R2.

Voltage across R2 = $V_{out1} - V_{out2} = 5.111\ V$

Current through R2 = $\frac{5.111}{R2} = 0.824\ mA$ or $824.4\ \mu A$

This is the same current through R1

$\Rightarrow$ Voltage across R1 = $(1.5\ k\Omega)(0.824\ mA) = 1.237\ V$

$V_{source} = V_{R1} + V_{out1} = 12\ V$

or $= V_{R1} + V_{R2} + V_{out2} = 12\ V$

Vsource = $\boxed{12\ V}$

v. Using all the information you have so far, determine how much current flows through resistor R4? (5pts)

Current through R3 = $\frac{V_{out2}}{R_3} = 0.3533\ mA$ or $353.3\ \mu A$

Current through R4 = Current through R2 − Current through R3

$= 824.4\ \mu A - 353.3\ \mu A$

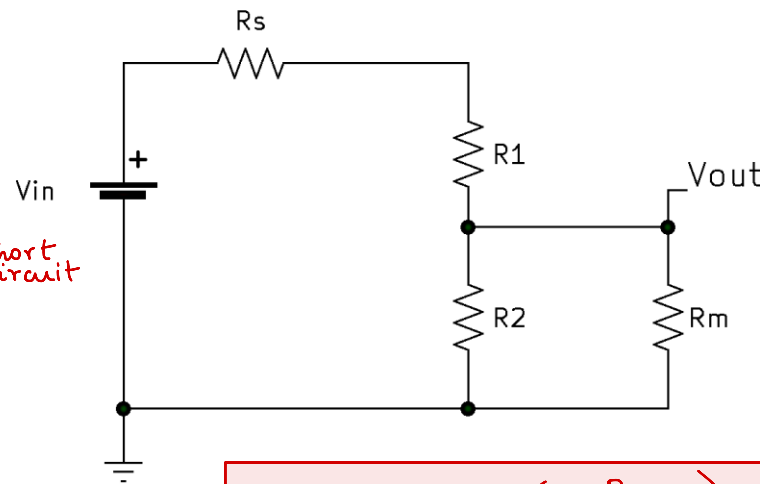
$= 471.1\ \mu A$

Optional: $R4 = \frac{V_{out2}}{I_{R4}} = 12\ k\Omega$

Current through R4 = $\boxed{0.471\ mA \text{ or } 471.1\ \mu A}$

II. Design Problem on Minimizing Measurement Error (20 points)

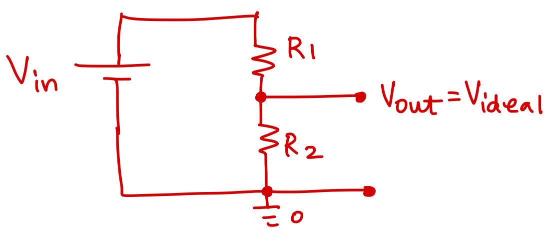
A voltage divider is connected to a source and a voltmeter as shown below. Ideally, Rs = 0 and Rm = ∞. However, for one practical circuit, Rs ≤ 125Ω and Rm ≥ 10kΩ. The overall goal of this problem is to select R1 and R2 to minimize the error introduced by Rs and Rm when it is desired that $\frac{V_{out}}{V_{in}} = 0.75$. For simplicity use $a = \frac{R2}{R1+R2}$



i. Determine an expression for Vout in terms of Vin, R1, and R2 under ideal conditions, i.e. Rs = 0 and Rm = ∞. Denote this output voltage as Videal. (3pts)

short circuit

open circuit



$$\boxed{V_{ideal} = V_{in}\left(\frac{R_2}{R_1+R_2}\right)}$$

$$\boxed{\underline{\text{or}} \quad = a\,V_{in}, \quad \text{where} \quad a = \frac{R_2}{R_1+R_2}}$$

ii. Determine an expression for Vout in terms of Vin, R1 , R2, Rs, and Rm under practical conditions. Denote this output voltage as Vpract. (4pts)

$$\text{Let} \quad R_p = R_2 \,||\, R_m = \frac{R_2 R_m}{R_2 + R_m}$$

$$V_{out} = V_{pract} = V_{in}\left(\frac{R_p}{R_p + R_1 + R_s}\right) = \boxed{V_{in}\left[\frac{R_2 R_m}{R_2 R_m + (R_2+R_m)(R_1+R_s)}\right]}$$

<u>Or</u> in terms of a →

$$\boxed{V_{pract} = V_{in}\left[\frac{a R_2 R_m}{(a R_s + R_2)(R_m + R_2) - a R_2^2}\right]}$$

iii. Find and expression for measurement error 'e'. Define $e = \frac{Videal - Vpract}{Videal}$. Simplify such that there are no fractions in the numerator or denominator. (5pts)

$$e = \frac{V_{ideal} - V_{pract}}{V_{ideal}} = \frac{a V_{in} - V_{pract}}{a V_{in}} = 1 - \frac{V_{pract}}{a V_{in}}$$

$$= \boxed{1 - \frac{R_2 R_m}{(aR_s + R_2)(R_m + R_2) - aR_2^2}}$$

OR

$$e = 1 - \frac{V_{pract}}{V_{ideal}} = \boxed{1 - \frac{R_m(R_1 + R_2)}{R_2 R_m + (R_2 + R_m)(R_1 + R_s)}}$$

iv. Describe how you would analytically minimize this error. You just need to describe your steps; not actually go through them. (4pts)

We can use calculus and set $\frac{de}{dR_2} = 0$ to find the best value of R_2.

OR

We can use spreadsheet computer progam to find the best value of R_2 that minimizes e.

v. It turns out that the error is minimized when R2 = $\sqrt{3 \times Rm \times Rs}$. To obtain the desired $\frac{V_{out}}{V_{in}} = 0.75$ and using the worst case values of Rm and Rs provided in the problem statement, find the values of R1 and R2 that will satisfy this design problem. (4pts)

Worst case: $R_m = 10k\Omega$ and $R_s = 125\Omega$

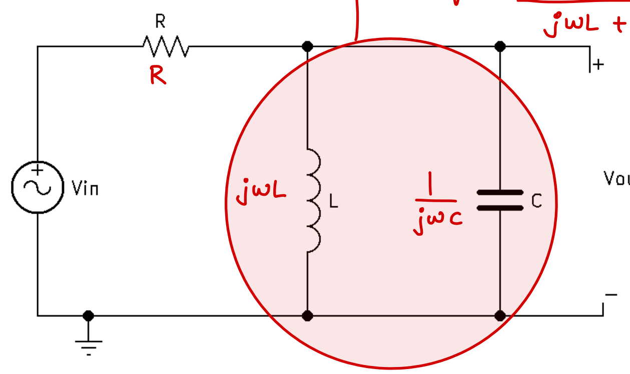
$$\Rightarrow R_2 = \sqrt{3 \times 10K \times 125} = \boxed{1936.5\ \Omega}$$

$$V_{out} = a V_{in} \Rightarrow a = \frac{V_{out}}{V_{in}} = 0.75$$

$$a = \frac{R_2}{R_1 + R_2} \Rightarrow R_1 = \frac{R_2(1-a)}{a} = \boxed{645.5\ \Omega}$$

III. Filters & Transfer Functions (20 points) For this problem assume AC Steady State.

a. **Find the transfer function** of the circuit shown. Simplify such that there are no fractions in the numerator or denominator of the transfer function. $H(j\omega) = V_{out}(j\omega)/V_{in}(j\omega)$ (4pts)



$$Z_{eq} = \frac{j\omega L \times \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1-\omega^2 LC}$$

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_{eq}}{Z_{eq} + R}$$

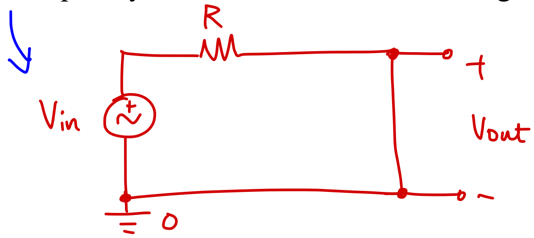
$$= \frac{\frac{j\omega L}{1-\omega^2 LC}}{\frac{j\omega L}{1-\omega^2 LC} + R} = \boxed{\frac{j\omega L}{R(1-\omega^2 LC) + j\omega L}}$$

b. Determine the magnitude and phase of the transfer function for the circuit for very small frequency and for very high frequency. Do not take this to 0 or infinite Hz. (4pts)

Low frequency	High frequency
$\lvert H(j\omega)\rvert = \boxed{\frac{j\omega L}{R}}$	$\lvert H(j\omega)\rvert = \frac{j\omega L}{-\omega^2 RLC} = \boxed{\frac{-j}{\omega RC}}$
$\angle H(j\omega) = \angle \frac{j\omega L}{R} = \boxed{90^\circ}$	$\angle H(j\omega) = \angle \frac{-j}{\omega RC} = \boxed{-90^\circ}$

c. Redraw the circuit and simplify the circuit for operation at low and high frequency. For this part you take it to extremes, low frequency is dc operation. High is approaching infinity. (2pts)

Low Frequency High Frequency



Same circuit for both

d. Corner frequency of a filter, f_c, is defined as the frequency at which the magnitude of the transfer function, $|H(j\omega)|$, is 0.707, or $\frac{1}{\sqrt{2}}$. Alternatively, this can be expressed as the frequency at which the power at the output is half of the power at the input.
Derive the corner frequency or frequencies for this circuit when R=10 Ω, L=2H, and C=0.5F ? (5 pts)

$$H(j\omega) = \frac{j\omega L}{R(1-\omega^2 LC) + j\omega L} \Rightarrow |H(j\omega)| = \frac{|j\omega L|}{|R(1-\omega^2 LC) + j\omega L|}$$

$$= \frac{\omega L}{\sqrt{R^2(1-\omega^2 LC)^2 + \omega^2 L^2}} \overset{\text{set}}{=} \frac{1}{\sqrt{2}}$$

Substituting R, L, C values

$$= \frac{2\omega_c}{\sqrt{100(1-\omega_c^2)^2 + 4\omega_c^2}} = \frac{1}{\sqrt{2}}$$

$$8\omega^2 = 100(1-\omega_c^2)^2 + 4\omega_c^2 \Rightarrow 100\omega_c^4 - 204\omega_c^2 + 100 = 0$$

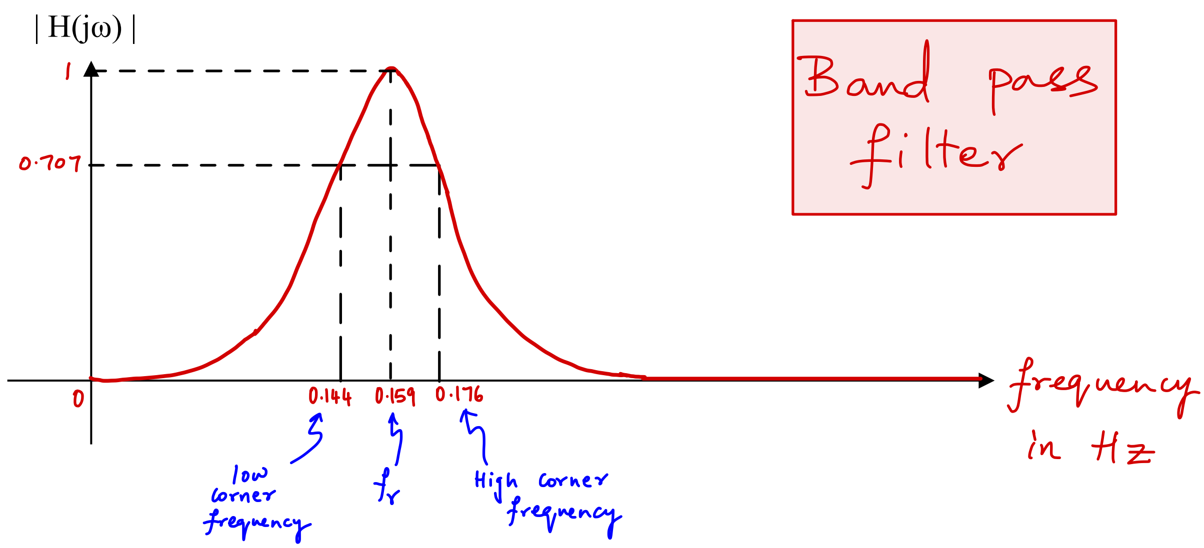
$$\omega_c^2 = \frac{204 \pm \sqrt{204^2 - 4\times 10^4}}{200} = 1.221,\ 0.819$$

$$\Rightarrow \omega_c = 1.1050,\ 0.9050 \Rightarrow \boxed{f_c = \frac{\omega_c}{2\pi} = 0.176\text{ Hz and } 0.144\text{ Hz}}$$

e. Resonant frequency of a filter, f_r, is defined as the frequency at which the magnitude of the transfer function, $|H(j\omega)|$, is at a maximum or minimum, depending on the type of filter. Determine the resonant frequency of the circuit, f_r using the same R, L, and C values as the previous part and the equation from the crib sheet. (2 pts)

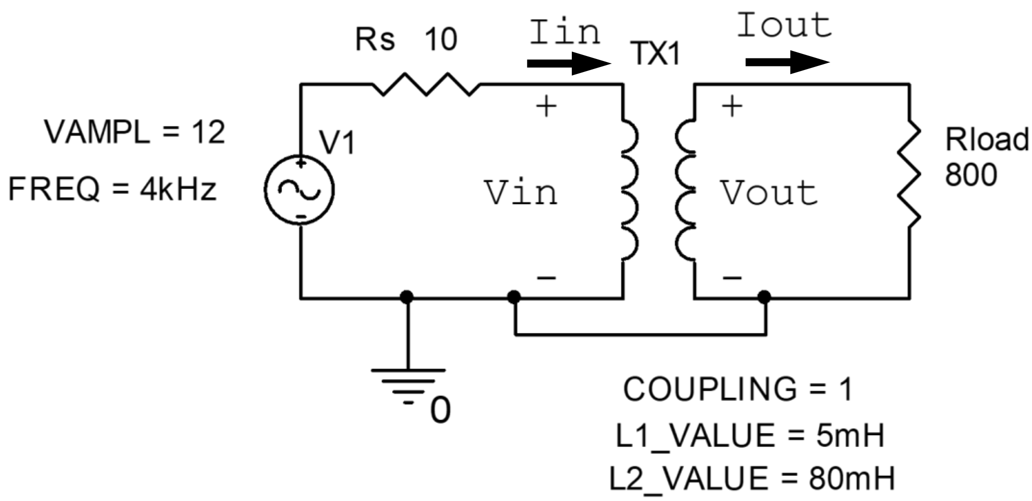
$$f_r = \frac{1}{2\pi\sqrt{LC}} = \boxed{0.159\ Hz}$$

f. Provide a rough sketch for the AC sweep analysis ($|H(j\omega)|$ vs. frequency) for this filter. Clearly mark the corner frequency and resonant frequency. Label the x-axis. **What type of filter** have we been working with for this problem? (3pts)



Band pass filter

IV – Phasors and Transformers (20 points)



a. Assume an ideal transformer with full coupling.

i. For the given information, determine the turns ratio, a. And determine the ratios Vout/Vin, Iout/Iin and the transformer input impedance Rin. (Rin is Vin/Iin) (6 pts)

$$a = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{80}{5}} = 4$$

$$\frac{V_{out}}{V_{in}} = a = 4$$

$$\frac{I_{out}}{I_{in}} = \frac{1}{a} = \frac{1}{4}$$

$$R_{in} = \frac{R_{Load}}{a^2} = 50\,\Omega$$

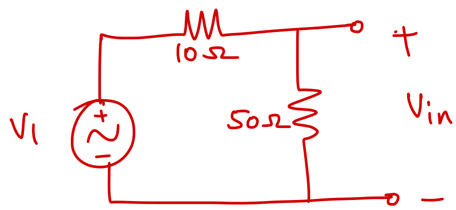
a= 4 (2 pts)

Vout/Vin= 4 (1 pt)

Iout/Iin= 0.25 (1 pt)

Rin= 50Ω (2 pts)

ii. ***Solve for Vin*** (voltage across the input terminals of the ideal transformer) ***and Vout,*** the voltage across the output terminals and the of the ideal transformer. **Assume the phase of V1 is zero degrees and give the answer in the form of v(t)=V₁Cos(ωt+θ₁)** (3 pts)



$$|V_{in}| = \frac{50}{50+10} \times 12 = 10\,V$$

$$\angle V_{in} = \angle V_1 = 0^\circ$$

$$\omega = 2\pi \times 4\times 10^3 = 8000\pi \text{ rad/s}$$

$$V_{out} = 4 \times V_{in}$$

Vin(t)= $10\cos(8000\pi t + 0^\circ)$ V

Vout(t)= $40\cos(8000\pi t + 0^\circ)$ V

iii. Above you were told to assume that the transformer is ideal. For that to be valid, the impedance of the primary inductor should be much larger than the source resistance. Is that valid in this case? Explain or justify. Would it be valid at if the signal source was at 60Hz? (3pts)

@ 4 KHz

$$|j\omega L_1| = 8000\pi \times 5\times 10^{-3} = 125.66\,\Omega$$

2pts

$125.66\,\Omega >> 10\,\Omega \Rightarrow$ Reasonably true, So Yes valid.

@ 60 Hz

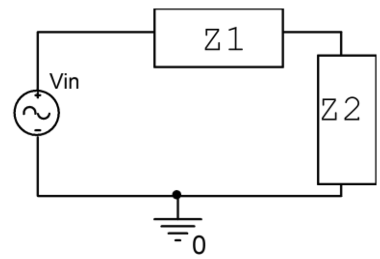
$$|j\omega L_1| = 2\pi \times 60 \times 5\times 10^{-3} = 1.885\,\Omega$$

$1.885\,\Omega$ is much less than $10\,\Omega$

1pt

No

b. Phasors: This circuit shown has 2 complex impedances, Z1 and Z2, connected as shown.
Given: $Vin = 5V\angle 45^o$ and the voltage across Z2 is measured to be $Vz2 = 3V\angle 30^o$



i. Write Vin and Vz2 in Cartesian form. (2pts)

$$V_{in} = 5\cos(45^\circ) + j\,5\sin(45^\circ)$$
$$= \boxed{3.54 + j\,3.54}$$

$$V_{z2} = 3\cos(30^\circ) + j\,3\sin(30^\circ)$$
$$= \boxed{2.6 + j\,1.5}$$

ii. Determine Vz1, the voltage across Z1 in Cartesian and polar form (3pts)

$$V_{z1} = V_{in} - V_{z2} = \boxed{0.9374 + j2.04}$$ Cartesian

$$= \boxed{2.241 \angle 65.32^\circ}$$ polar

iii. If Z2 is a 1kΩ resistor, and only a resistor, what is the current through Z2 in both polar and Cartesian form? (3pts)

$$I_{z2} = \frac{V_{z2}}{Z_2} = \frac{3\angle 30^\circ}{1K} = \boxed{3\angle 30^\circ \text{ mA}}$$ 1pt

$$= \frac{2.6 + j1.5}{1K} = \boxed{(2.6 + j\,1.5) \text{ mA}}$$ 2pt