

ENGR-4300

Fall 2008

Test 1

Name **SOLUTION**

Section: 1(MR 8:00) 2(TR 2:00) 3(MR 6:00)
(circle one)

Question I (20 points) _____

Question II (20 points) _____

Question III (20 points) _____

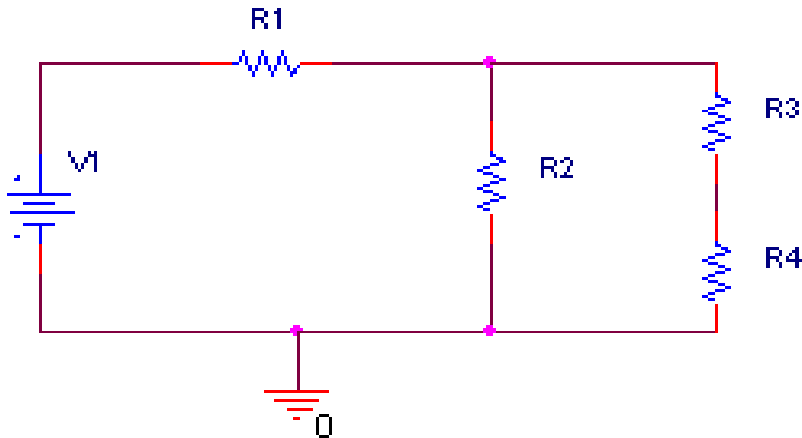
Question IV (20 points) _____

Question V (20 points) _____

Total (100 points) _____

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

Question I. Resistive circuits (20 points)



Given: $V_1=5$ volts. $R_1= 2000\Omega$, $R_2= 1000\Omega$, $R_3= 500\Omega$, $R_4= 400\Omega$

1) (7 pts) Find the total resistance of the circuit.

$$\begin{aligned}
 R_T &= R_1 + R_2 // (R_3 + R_4) \\
 (R_3 + R_4) &= .5K + .4K = .9K \\
 R_{234} &= R_2 // (R_3 + R_4) = (1K * .9K) / (1K + .9K) = 0.474K \\
 R_T &= 2K + 0.474K = \mathbf{2.474 \text{ Kohms}}
 \end{aligned}$$

2) (5 pts) Find the voltage across R_1 . (Note that this is not referenced to ground.)

$$\begin{aligned}
 I_T &= V_T / R_T = 5V / 2.474K = 2.02 \text{ mA} \\
 V_1 &= I_T * R_1 = 2.02m * 2K = \mathbf{4.04 \text{ V}}
 \end{aligned}$$

3) (6 pts) Find the current through R_4 .

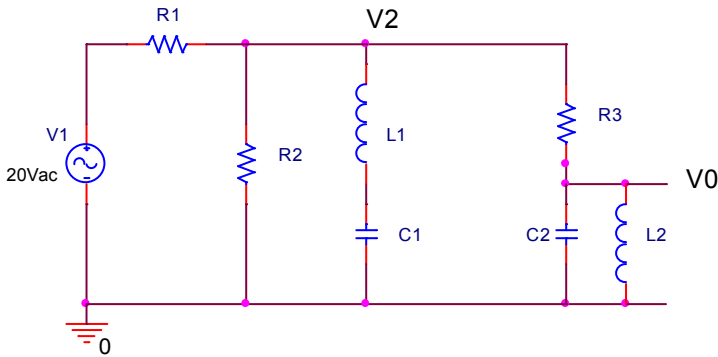
$$\begin{aligned}
 V_2 &= V_T - V_1 = 5 - 4.04 = 0.96 \text{ V} \\
 I_2 &= V_2 / R_2 = 0.96 / 1K = 0.96 \text{ mA} \\
 I_3 = I_4 &= I_T - I_2 = 2.02m - 0.96m = \mathbf{1.06 \text{ mA}}
 \end{aligned}$$

4) (2 pts) Which resistor has the largest voltage drop across it?

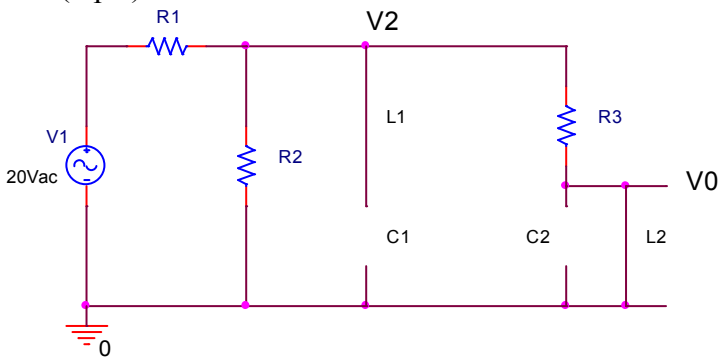
$$V_{R_2} = 5 - 4.04 = 0.96V \quad V_{R_3} < V_{R_2} \ \& \ V_{R_4} < V_{R_2} \ \Rightarrow \mathbf{R_1 \text{ has largest voltage drop}}$$

Question II – Filters (20 points)

You are given the following circuit. The input at V1 is a 20V AC signal. Leave all answers for 1) – 4) in terms of R1, R2, R3, C1, C2, L1, and L2. V0 and V2 are measured with respect to ground.

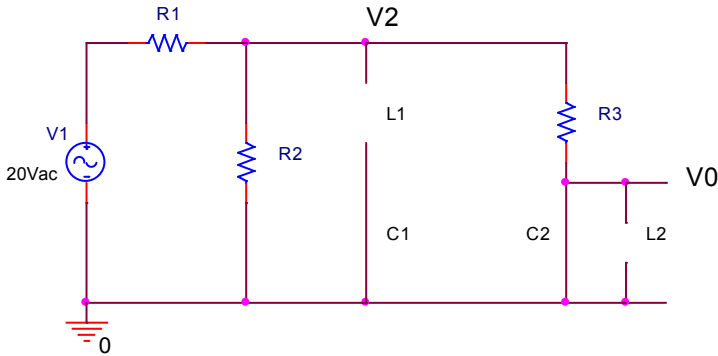


1) Redraw the circuit for $\omega \rightarrow 0$ with appropriate simplifications for the impedances and find V0 and V2. (5 pts)



$$V_0 = 0 \quad V_2 = \left(\frac{\frac{R_2 R_3}{R_2 + R_3}}{\frac{R_2 R_3}{R_2 + R_3} + R_1} \right) 20V = \frac{20R_2 R_3}{R_2 R_3 + R_1(R_2 + R_3)}$$

2) Redraw the circuit for $\omega \rightarrow \infty$ (high frequencies) with appropriate simplifications for the impedances and find V0 and V2. (5 pts)



$$V_0 = 0 \quad V_2 = \left(\frac{\frac{R_2 R_3}{R_2 + R_3}}{\frac{R_2 R_3}{R_2 + R_3} + R_1} \right) 20V = \frac{20R_2 R_3}{R_2 R_3 + R_1(R_2 + R_3)}$$

3) At what value of ω (greater than 0 and less than ∞) would you expect to find V_0 minimized? (3 pts)

At $\omega = \frac{1}{\sqrt{L_1 C_1}}$ the voltage at $V_2 = 0$ when $Z_{L_1 C_1} = 0$ and V_0 is also 0

4) At what value of ω (greater than 0 and less than ∞) would you expect to find V_0 maximized? (3 pts)

At $\omega = \frac{1}{\sqrt{L_2 C_2}}$ the voltage at V_0 is at a maximum when $Z_{L_2 C_2} = \infty$

5) What special name is given to the ω values in 3) and 4)? (2 pts)

The resonant frequency of the LC (either parallel or series).

6) TRUE or FALSE: If V_1 had a DC offset and ω is neither the value in 3) nor 4), the output V_0 would also have a DC offset. (2 pts)

Question III – Transfer Functions (20 points)

1) For $\omega = 10$, a circuit has a transfer function $H(j10) = \frac{2 + j5}{-8 - j6}$. Find the magnitude and phase of $H(j10)$. (4 pts)

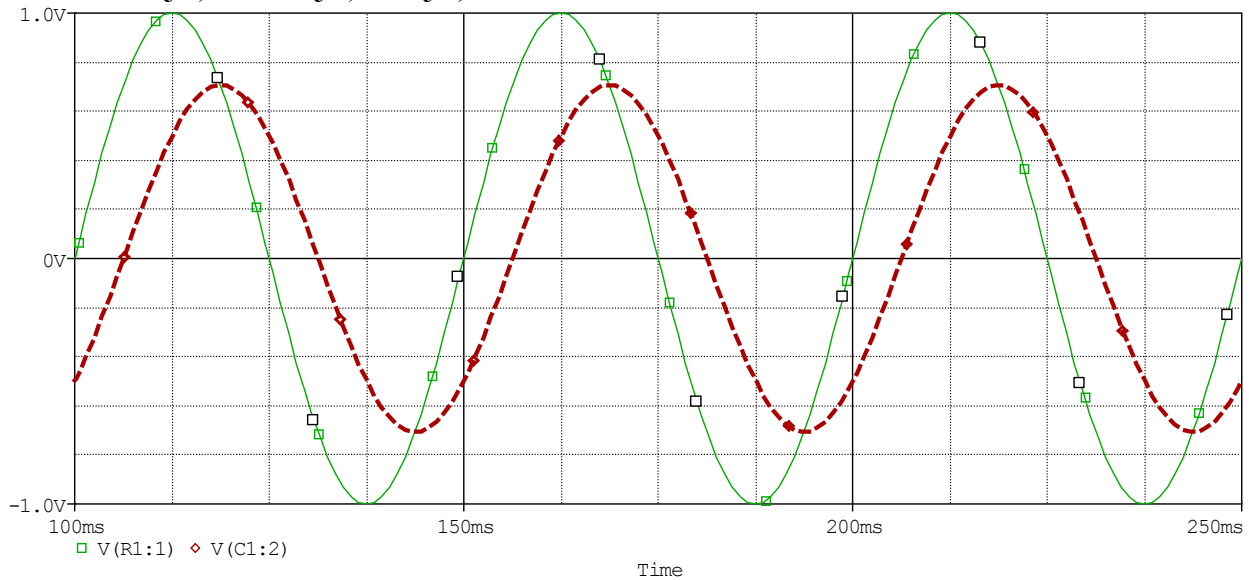
$$|H(j\omega)| = \frac{\sqrt{2^2 + 5^2}}{\sqrt{8^2 + 6^2}} = 0.5385$$

$$\angle H(j\omega) = \angle(2 + j5) - \angle(-8 - j6) = \arctan\left(\frac{5}{2}\right) - \arctan\left(\frac{-6}{-8}\right) = 68.2^\circ - (-143.1^\circ) = 211.3^\circ = -148.7^\circ$$

2) For an $H(j5) = 0.75/\pi/3$ and $V_{in}(t) = 2\cos(5t - \pi/2)$, find $V_{out}(t)$. (6 pts)

$$V_{out} = H \times V_{in} = |H| \times |V_{in}| \angle(\underline{H} + \underline{V_{in}}) = (0.75)(2)\cos(5t - \pi/2 + \pi/3) = 1.5\cos(5t - \pi/6)$$

The plot below shows the input (solid thin line) and output (dashed wide line) of a circuit whose transform is $H(j\omega) = V_{out}(j\omega)/V_{in}(j\omega)$. Note that the time axis is from 100ms to 250ms.



3) Determine ω (radians/s) for the input and output signals? (2 pts)

$$\omega = 2\pi/T = 2\pi/50\text{ms} = 125.7\text{radians/s}$$

4) Determine the magnitude of $H(j\omega)$ at this frequency. (3 pts)

$$|H| = V_{out}/V_{in} = 0.707/1.0 = 0.707$$

5) Determine the phase of $H(j\omega)$ at this frequency? (3 pts)

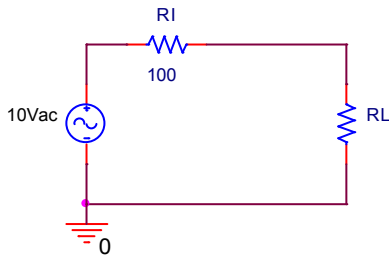
$$\angle H = \angle V_{out} - \angle V_{in} = (-2\pi/8) - 0 = -\pi/4$$

6) If $H(j\omega)$ is a 1st order LPF, ω is most likely: (2 pts)

a. the resonant frequency b. in the pass band c. in the stop band d. the corner frequency

With magnitude down by $\frac{1}{\sqrt{2}}$ and phase shifted by $-\pi/4 \Rightarrow$ corner frequency

Question IV – Signals, Transformers and Inductors (20 points)



1) Given the circuit above, for what value of R_L will the power in R_L be maximized? (2 pts)

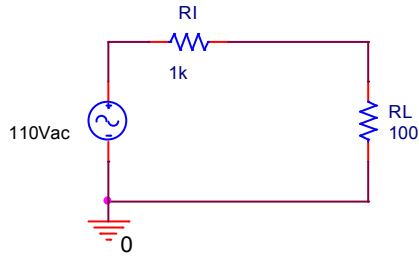
$$R_L = R_I = 100\Omega$$

2) What is the power dissipated in R_I and in R_L ? (4 pts)

$$(10V)(100)/(100 + 100) = 5V$$
$$P_{R_L} = 5^2/100 = 0.25W \quad P_{R_I} = 5^2/100 = 0.25W$$

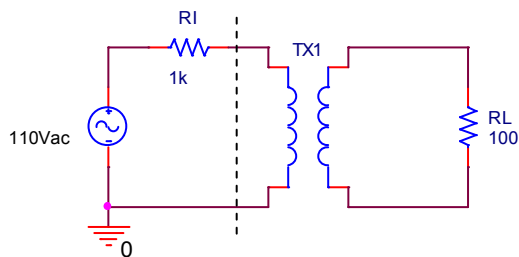
3) What is the efficiency of the circuit in terms of the power dissipated in R_L over the total power dissipated in the circuit $\left(\frac{\text{Power in } R_L}{\text{Total Power}}\right)$? (2 pts)

$$\text{Eff} = 0.25/(0.25 + 0.25) = 0.5 = 50\%$$



4) Now with $R_I = 1k\Omega$ and $R_L = 100\Omega$, assume $P_{R_L} = 1.0W$ and $P_{R_I} = 10.0W$. What is the efficiency of this circuit $\left(\frac{\text{Power in } R_L}{\text{Total Power}}\right)$? (2 pts)

$$\text{Eff} = 1.0/(1.0 + 10.0) = 0.091 = 9.1\%$$



5) An ideal transformer is added to improve the efficiency as shown above. The desire is to have Z_{in} of the transformer at the dashed line equal to R_I ($1k\Omega$). What turns ratio 'a' will do this? (5 pts)

$$Z_{in} = R_L/a^2 \quad a^2 = R_L/Z_{in} = 100/1000 = 0.1 \quad a = 0.316$$

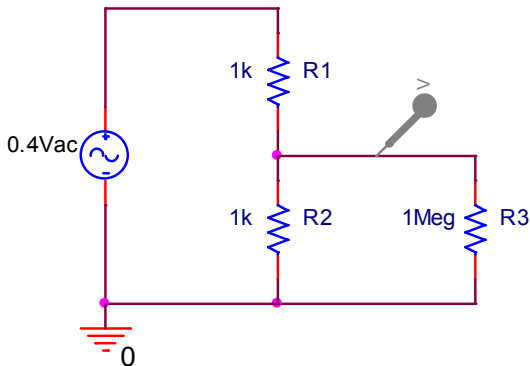
6) **TRUE** or **FALSE**: the expected efficiency of the modified circuit in 5) is 50%. (1 pt)

7) For the transformer with turns ratio $a = 5$, if $L_1 = 10mH$ what is L_2 ? (4 pts)

$$a = \sqrt{\frac{L_2}{L_1}} \quad L_2 = L_1 a^2 = 10mH \cdot 5^2 = 250mH$$

Question V – Instrumentation, PSpice and components (20 points)

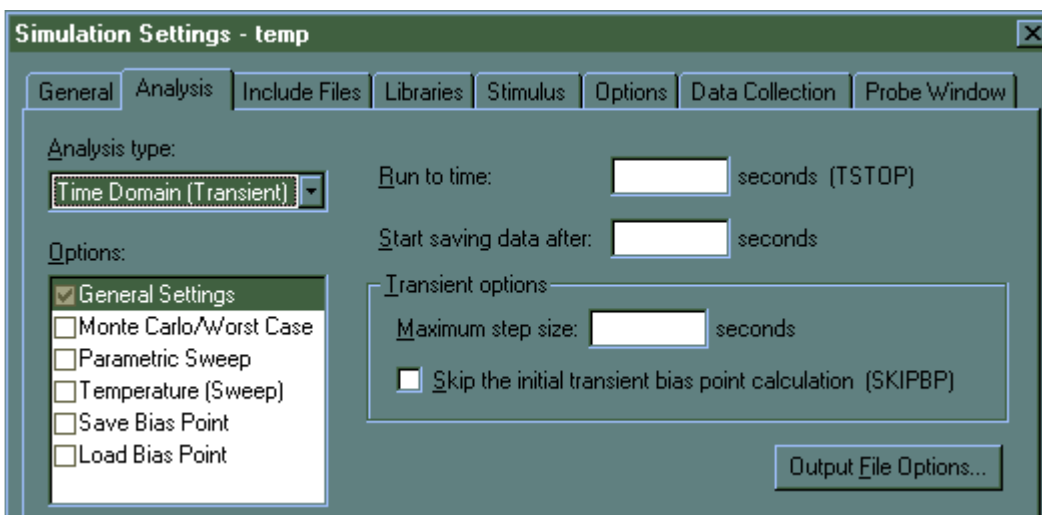
In experiment 1, you wired a circuit similar to the one below using real components and in PSpice:



1) (6 pts) What does this circuit do? What do R1, R2, and R3 represent?

The circuit is a voltage divider with the function generator as source and a measuring device attached to the second resistor. R1 and R2 are the resistors in the voltage divider, and R3 is the impedance of the measuring device (DMM or IOBoard oscilloscope input).

2) (3 pts) Suppose V1 has a frequency of 0.5kHz, an amplitude of 400mV, and a DC offset of 0. You want to run a transient analysis. How would you set up the simulation screen below to show exactly 3 smooth cycles of the output wave? (Fill in the three empty boxes.)



Notes on 2): Anything is acceptable for run time and start time as long as the difference is 6ms (3x1/500). The maximum step size should be somewhere in the range from about 0.06ms (100 points) to 6us (1000 points). Ex: 6ms 0s 0.05ms

3) (3 pts) If you set up this transient simulation, what would be the frequency (in Hz) and the approximate amplitude (in mV) of the signal at the point of the voltage probe?

$$\underline{f = 0.5\text{kHz} \quad V_{\text{probe}} = 200\text{mV}}$$

4) (5 pts) If you replaced R2 with a resistor with color code orange-black-red-gold, what would be the frequency (in Hz) and the approximate amplitude (in mV) of the signal at the point of the voltage probe?

The resistance of the new resistor is $30 \times 100 = 3\text{K ohms}$.
 $V_{R2} = V_1 \times R_2 / (R_1 + R_2) = 400 \times 3\text{K} / (1\text{K} + 3\text{K}) = 300\text{mV}$
The frequency will not change.

$$\underline{f = 0.5 \text{ Hz} \quad V_{\text{probe}} = 300\text{mV}}$$

5) (3 pts) To get an IOBoard function generator to output the signal in the circuit above, the voltage must be specified as: a. 0.4V b. 0.2Vp-p c. 0.8V d. 0.8Vp-p e. none of these

d. 0.8Vp-p