ENGR-4300
Fall 2007
Test 1

Name ______SOLUTION__________

Section ____

Question I (20 points) ___________

Question II (22 points) ___________

Question III (20 points) ___________

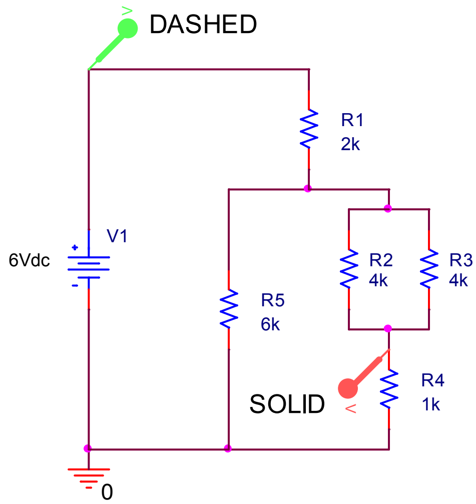
Question IV (20 points) ___________

Question V (18 points) ___________

Total (100 points) ______________

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

Question I. Resistive circuits (20 points)



R1 = 2kΩ R2 = 4kΩ R3 = 4kΩ R4 = 1kΩ R5 = 6kΩ

1) Find the total resistance of the above circuit. (5 pts)

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{4k \cdot 4k}{4k + 4k} = 2k$$

$$R_{234} = R_{23} + R_4 = 2k + 1k = 3k$$

$$R_{2345} = \frac{R_{234} R_5}{R_{234} + R_5} = \frac{3k \cdot 6k}{3k + 6k} = 2k$$

$$R_{TOTAL} = R_{2345} + R_1 = 2k + 2k = 4k$$

2) Find the voltage across R4. (5 pts)

$$V_{R5} = V_{R2345} = 6\frac{R_{2345}}{R_{2345} + R_1} = 6\frac{2k}{2k + 2k} = 3V$$

$$V_{R4} = V_{R5}\frac{R_4}{R_4 + R_{23}} = 3\frac{1k}{1k + 2k} = 1V$$

3) Find the current through R3. (5 pts)

$$I_{R234} = \frac{V_{R234}}{R_{234}} = \frac{3}{3k} = 1mA$$

since R2 = R3, current will be split equally between both branches:

$$I_{R3} = \tfrac{1}{2} I_{R234} = 0.5mA$$

4) If the DC voltage source is replaced by a sinusoidal AC voltage source of the same amplitude and frequency of 1kHz which of the following time graphs would best represent the output of the voltage probes labeled "SOLID" and "DASHED" on the above schematic? (3 pts) ***<u>In one short phrase in the space to the right of the graphs, write why the other graphs are incorrect.</u>*** (2 pts)

Check box of correct graph

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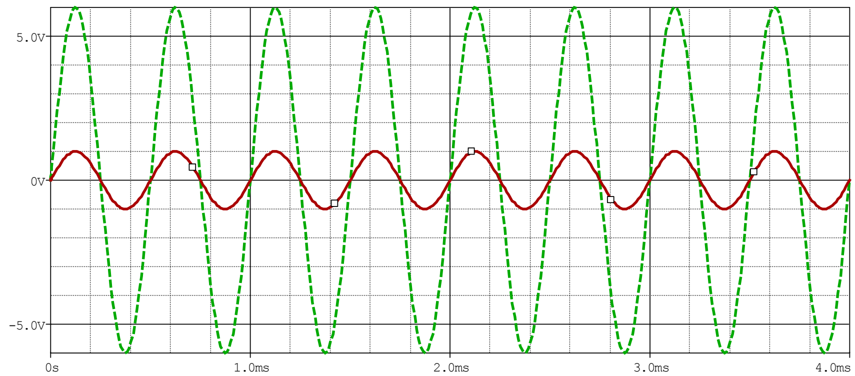
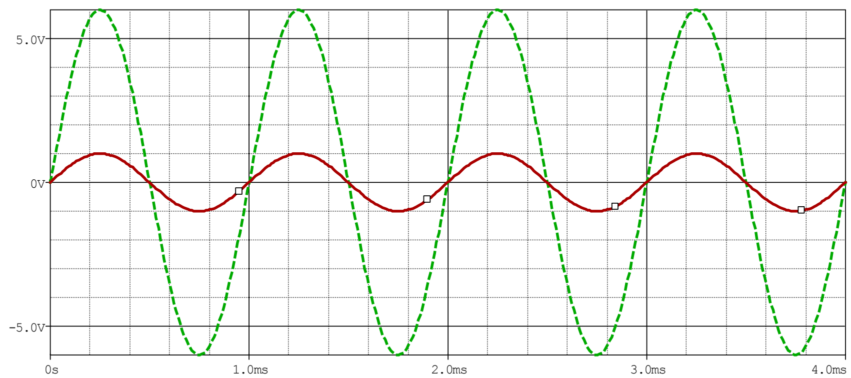


Wrong frequency

☒ X



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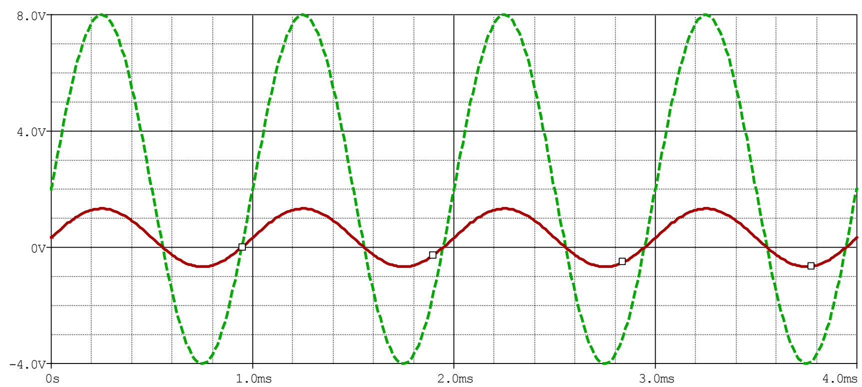
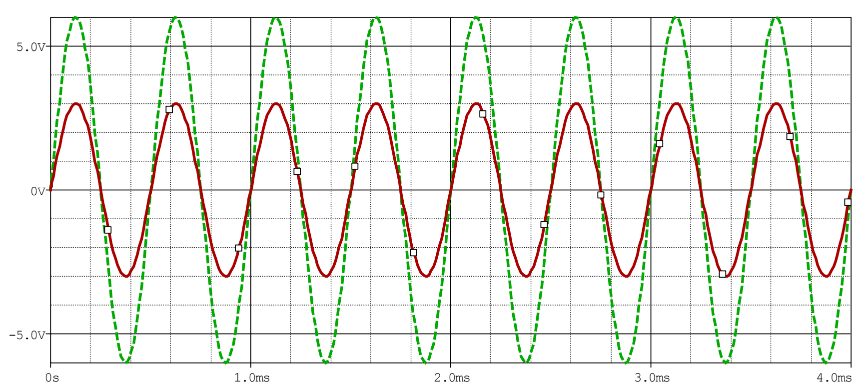


Input sin shouldn't have offset

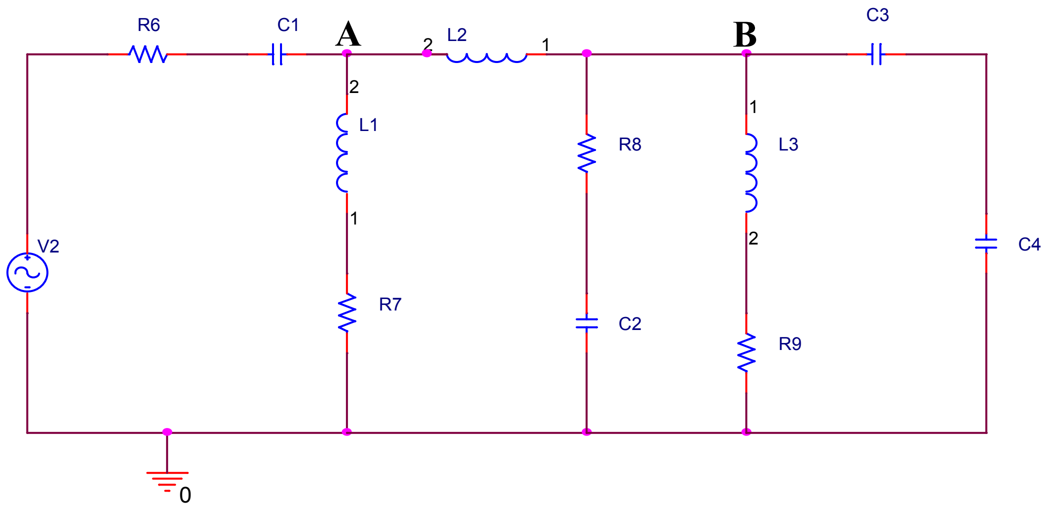
☐



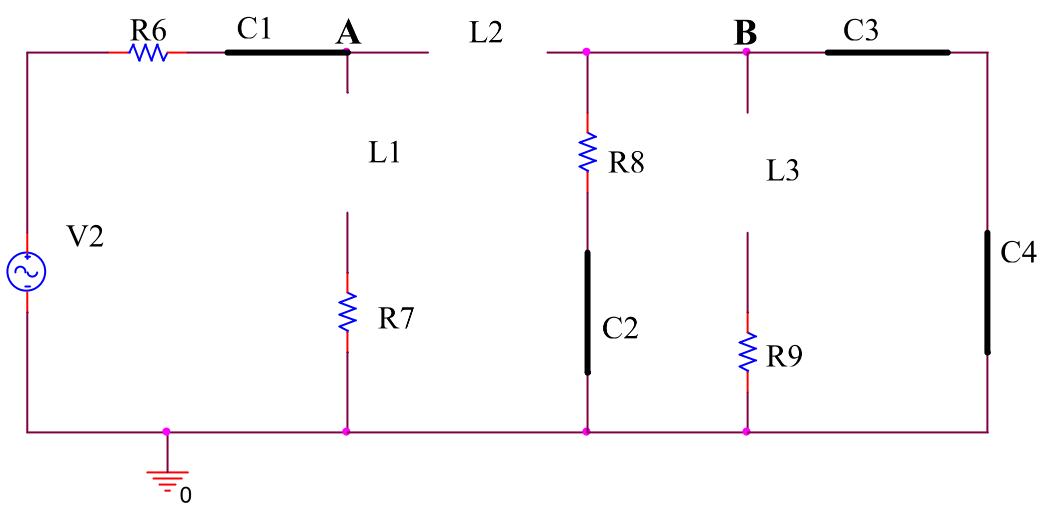
Wrong frequency and amplitude

Question II – Filters (22 points)

You are given the following circuit. The input at V2 has the following properties:
VAMPL = 500mV, FREQ = 1kHz, VOFF = 0V.



1) To investigate the behavior of this circuit at high frequencies redraw the circuit and replace the components with their high frequency equivalents. (5 pts)



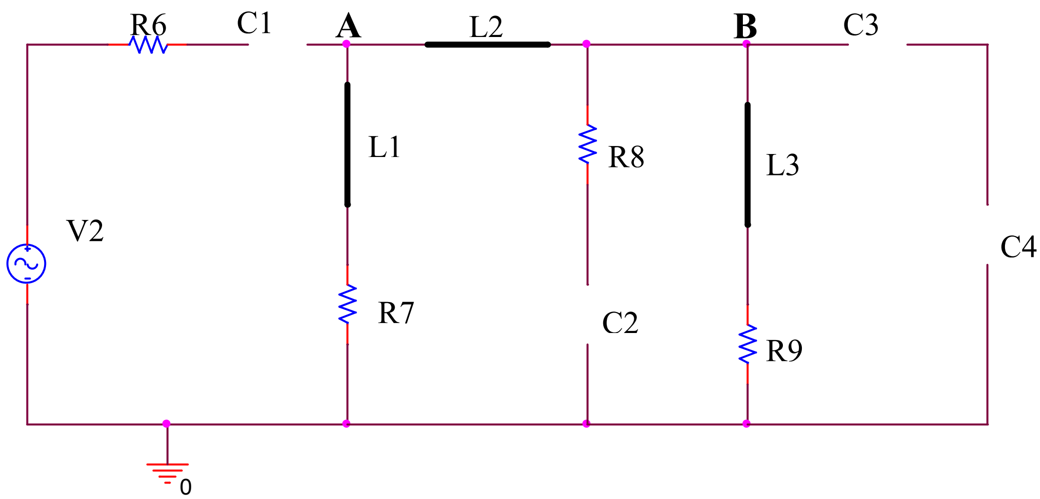
2) What is the amplitude of the voltage at point A at high frequencies? (2 pts)

V_A = V2 = 500 mV

3) What is the amplitude of the voltage at point B at high frequencies? (2 pts)

V_B = 0V

4) To investigate the behavior of this circuit at low frequencies redraw the circuit and replace the components with their low frequency equivalents. (5 pts)



5) What is the amplitude of the voltage at point A at low frequencies? (2 pts)

$V_A = 0V$

6) What is the amplitude of the voltage at point B at low frequencies? (2 pts)

$V_B = 0V$

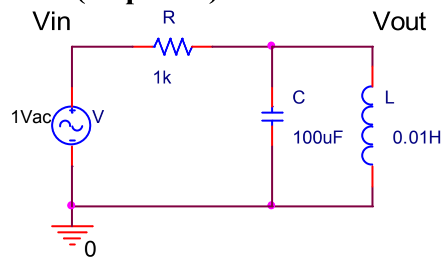
7) What type of filter could this be at point A (circle one)? (2 pts)

Low Pass High Pass Neither

8) What type of filter could this be at point B (circle one)? (2 pts)

Low Pass High Pass Neither

Question III – Transfer Functions (20 points)



1) What is the transfer function [H(jω)=Vout/Vin] for the circuit? You must simplify. (6 pts)

$$H(j\omega) = \frac{\frac{j\omega L \times 1/j\omega C}{j\omega L \times 1/j\omega C}}{R + \frac{j\omega L \times 1/j\omega C}{j\omega L \times 1/j\omega C}} = \frac{j\omega L \times 1/j\omega C}{R(j\omega L + 1/j\omega C) + j\omega L \times 1/j\omega C} = \frac{j\omega L}{R(-\omega^2 LC + 1) + j\omega L}$$

$$= \frac{j\omega L}{R - \omega^2 LRC + j\omega L} = \frac{j(0.01)\omega}{-10^{-3}\omega^2 + j(0.01)\omega + 1000}$$

2) What is the simplified transfer function of the circuit at low frequencies? (3 pts)

$$H(j\omega) = \frac{j\omega L}{R - \omega^2 LRC + j\omega L} = \frac{j\omega L}{R} = \frac{j\omega}{100{,}000} \quad \text{as } \omega \text{ gets very small (but not 0)}$$

3) What is the simplified transfer function of the circuit at high frequencies? (3 pts)

$$H(j\omega) = \frac{j\omega L}{R - \omega^2 LRC + j\omega L} = \frac{j\omega L}{-\omega^2 LRC} = \frac{\text{- j}}{\omega \text{RC}} = \frac{\text{-10j}}{\omega} \quad \text{as } \omega \text{ gets very large (but not } \infty\text{)}$$

4) Find the frequency ω_0 where the impedance of the parallel combination of the inductor and capacitor has a maximum value. (3 pts)

At resonance, the parallel combination of Z_L & Z_C is infinite:

$$\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(0.1H)(100uF)}} = \sqrt{\frac{1}{10^{-2} \times 10^{-4}}} = \sqrt{10^6} = 1000 \text{ rad/s}$$

5) What is the magnitude and phase of the transfer function H(jω) at this frequency? (3 pts)

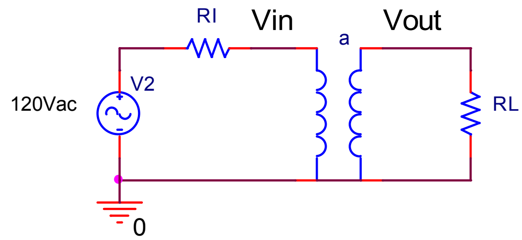
$$\left|H(j\omega)\right|_{\omega=\omega_0} = \left|\frac{j\omega L}{R - \omega^2 LRC + j\omega L}\right| = \left|\frac{j\omega L}{j\omega L}\right| = 1$$

$$\angle H(j\omega)\Big|_{\omega=\omega_0} = \arctan(\tfrac{0}{1}) = 0 \text{ rad}$$

6) Find a frequency ω where the phase of H(jω) is close to +π/2 (within 5%). (2 pts)

Any small ω < 935 rad/s (Hi Q filter has a very fast phase transition from +π/2 to -π/2)

Question IV – Signals, Transformers and Inductors (20 points)



1) Given the circuit above, assume an ideal transformer with full coupling and RI = RL = 100Ω. If RI can be neglected, what turns ratio **a** will produce an output voltage Vout = 6V and what is the corresponding dissipated power in RL? (6 pts)

a = Vout/Vin = 6/120 = 0.05

$P = V^2/R = 6^2/100 = 0.36W$

2) What is the effective Zin seen looking into the source side of the transformer due to RL, assuming **a** = 5? (4 pts)

$Zin = R/a^2 = 100/5^2 = 4\Omega$

3) For the conditions in 2) above, what is the current from the 120Vac source? RI must not be neglected. (4 pts)

$I = V/R_{total} = 120/(100 + 4) = 1.15A$

4) For the same circuit in 2), how much of the 120Vac is across RI and how much is across the transformer input (source side)? (4 pts)

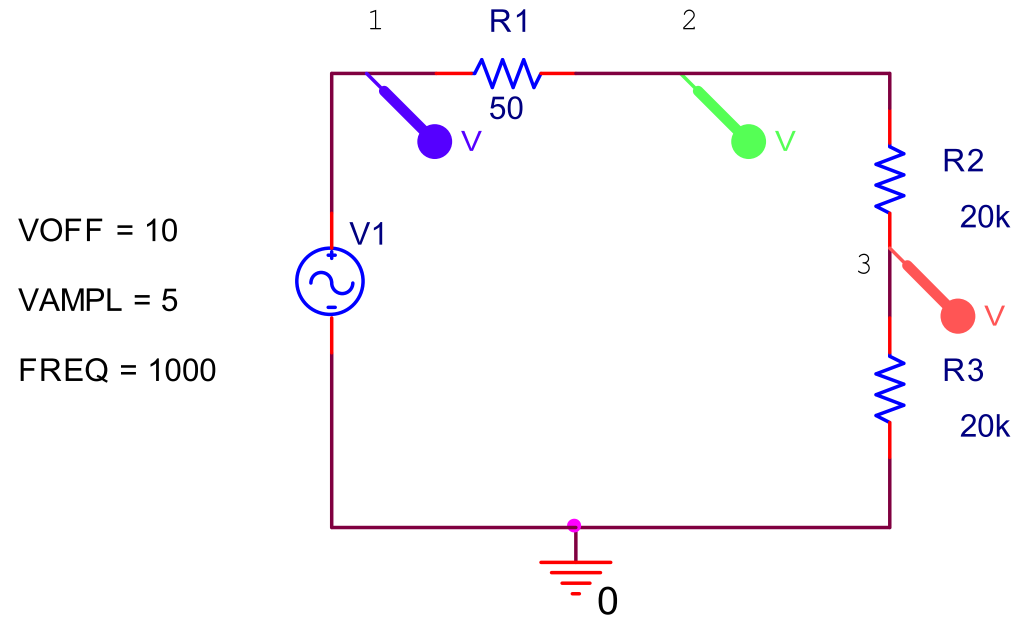
$V_{RI} = 120xRI/(RI + Zin) = 120x(100/104) = 115.4V$

$V_{Zin} = 120xZin/(RI + Zin) = 120x(4/104) = 4.6V$
Also = 120 - 115.4 = 4.6V

5) TRUE or FALSE: A transformer can only be used to produce an effective value Rin that is less than the original RL. (2 pts) **a** can be less than 1

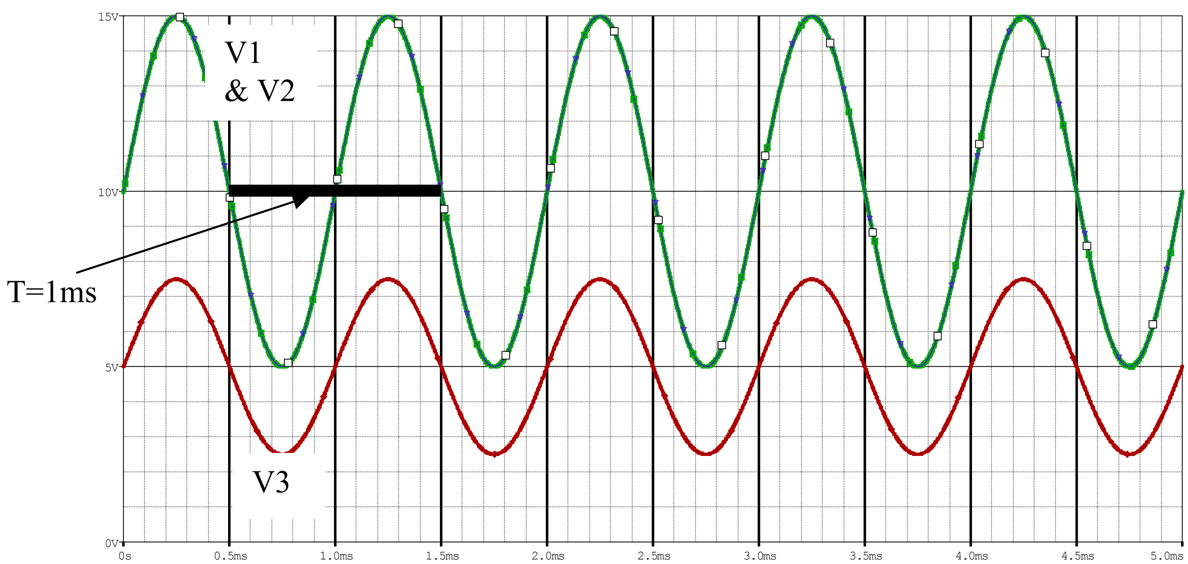
Question V – Instrumentation, PSpice and components (18 points)

The following circuit is analyzed using PSpice



Note that there are three voltage markers used in this circuit (denoted by the numbers 1, 2 and 3).

1) On the plot below, label the voltages observed for each of these three positions with the appropriate number. There are three voltage traces shown but two are essentially identical. Also, explain your answer. (7 pts)



2) On the plot above, find and label the period of the signal. (4 pts)

3) Mathematically, which of the following expressions provides the best description of the voltage produced by the source? (4 pts)

a. $V(t) = 10 + 5\sin(2000\pi t)$
b. $V(t) = 5 + 10\sin(2000\pi t)$
c. $V(t) = 10 + 5\cos(2000\pi t)$
d. $V(t) = 10 + 5\sin(1000t)$
e. $V(t) = 5 + 2.5\cos(1000t)$
f. $V(t) = 10 + 10\cos(1000t)$
g. $V(t) = 5 + 2.5\sin(2000\pi t)$
h. $V(t) = 5 + 2.5\sin(1000t)$

4) How would you expect the signal on probe 3 in 1) to change if the 20k resistors were replaced with 20M resistors and the signal were observed on the IOboard oscilloscope? (Remember, the IOboard scope is not an ideal measurement instrument.) (3 pts)

The IOboard scope input has an input impedance of about 1MΩ. That resistance in parallel with the 20MΩ resistance will have a parallel combination substantially less than 20MΩ (in fact less than 1MΩ). The output voltage across R3 will now be closer to 1/20 of the input voltage.