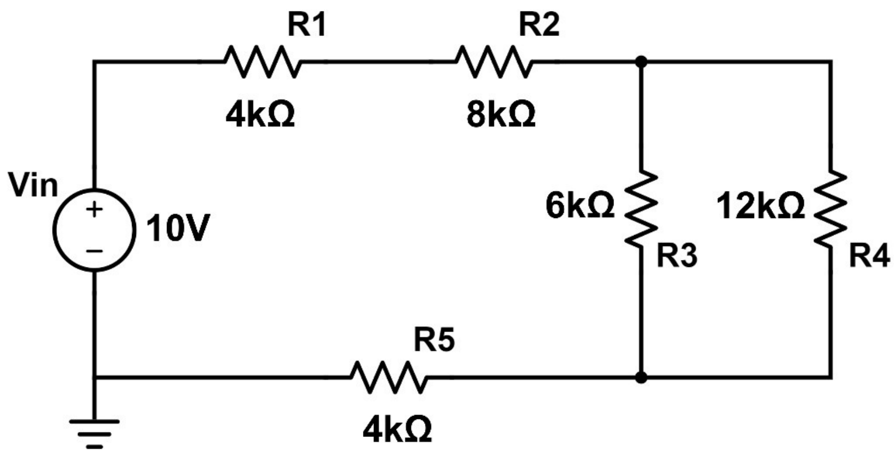
Quiz 1

Question 1: Voltage Dividers and the Effect of Measurement Devices



Q1.1) Find the voltage across R3. [4 points: 1 pt for $R_3||R_4$; 2 pts for voltage divider equation; 1 pt for correct numerical answer]

$$V_{R3} = V_{in}\frac{R_3||R_4}{R_1 + R_2 + R_3||R_4 + R_5}; \qquad R_3||R_4 = \frac{6\text{k}\Omega * 12\text{k}\Omega}{6\text{k}\Omega + 12\text{k}\Omega} = 4\text{k}\Omega$$

$$V_{R3} = 10\text{V}\frac{4\text{k}\Omega}{4\text{k}\Omega + 8\text{k}\Omega + 4\text{k}\Omega + 4\text{k}\Omega} = 10V\frac{4\text{k}\Omega}{20\text{k}\Omega} = \mathbf{2V}$$

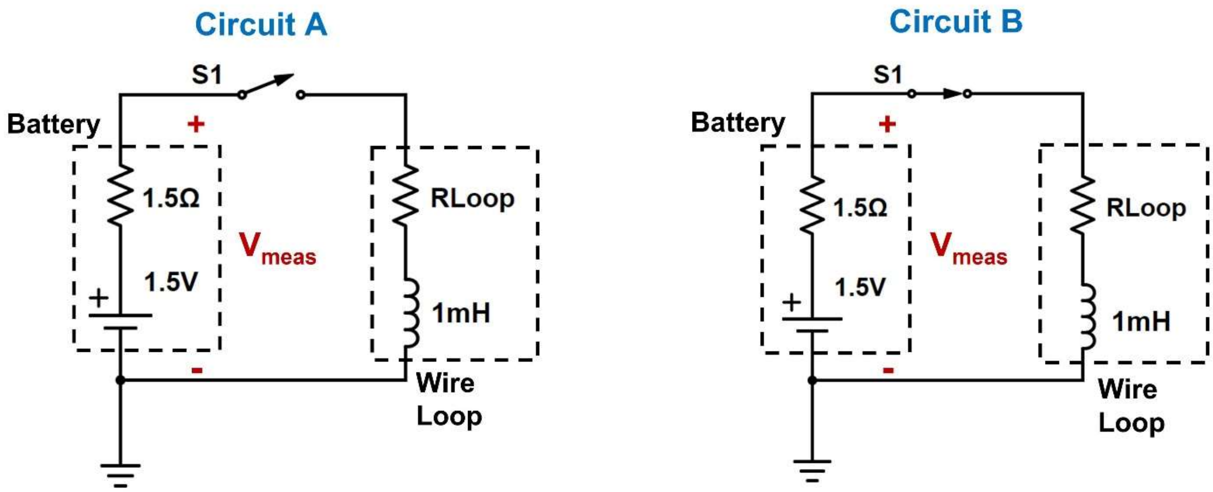
Concepts tested: combining parallel resistances, using voltage divider equation

Q1.2) Find the current through R4. [3 points: 2 for Ohm's law; 1 for correct numerical answer]

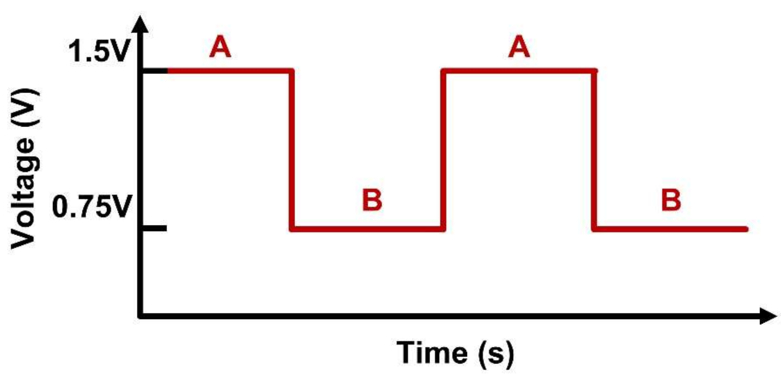
$$I_{R4} = \frac{V_{R4}}{R_4} = \frac{2V}{12\text{k}\Omega} = \frac{1}{6}\text{mA} = \mathbf{0.167mA}$$

Concepts tested: voltage across resistors in parallel, Ohm's Law

Q1.3) Beakman's motor consists of a spinning wire loop whose leads are sanded in such a way that the circuit is open for half of a cycle (circuit A) and closed for half a cycle (circuit B).



If the voltage across the battery is measured while the motor is operating, the plot below is observed on the oscilloscope: when the circuit is open (A), 1.5V is measured and when the circuit is closed (B), 0.75V is measured.



If the battery voltage is 1.5V, the battery's internal resistance is 1.5Ω, and the inductance of the wire loop is 1mH, what is the value of RLoop, the resistance of the wire loop? [4 points: 3 for voltage divider equation (2 for correct concept, 1 for equation); 1 for correct numerical answer]

$$0.75\text{V} = 1.5\text{V}\frac{R_{Loop}}{R_{Loop} + 1.5\Omega} \rightarrow \boldsymbol{R_{Loop} = 1.5\Omega}$$

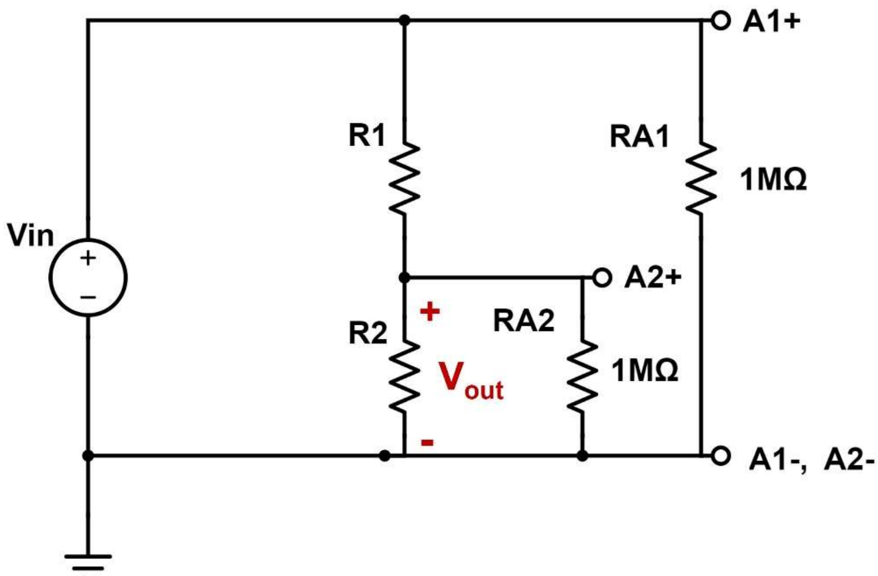
Concepts tested: voltage divider

Q1.4) How much power is delivered to the wire loop when the circuit is closed (circuit B)? [3 points: 2 for correct formula; 1 for correct numerical answer]

$$P = \frac{V_{RLoop}^2}{R_{Loop}} = \frac{\left(V_{Bat,A} - V_{Bat,B}\right)^2}{R_{Loop}} = \frac{(0.75V)^2}{1.5\Omega} = \mathbf{0.375W}$$

Concepts tested: electric power

Q1.5) Below is the circuit diagram for the measurement of the input voltage ($V_{in} = V_{RA1}$) and output voltage ($V_{out} = V_{R2}$) of a voltage divider circuit. A1+ and A1- are the differential leads of channel 1, which has input impedance RA1 = 1MΩ on the M2K and A2+ and A2- are the leads for channel 2, which has input impedance RA2 = 1MΩ.



Under which conditions do we need to be concerned that the input impedance of channel 2 (RA2) will noticeably affect the voltage measurement across R2? [3 points: 1 for identifying voltage divider; 1 for identifying significance of parallel resistance; 1 for conditions for R2]

Since RA2 is in parallel with R2, the measurement of V_{out} will be affected if R2 is large enough such that 1/R2 is of a similar order of magnitude to 1/RA2. If this is the case, the equivalent parallel resistance of R2 and RA2 will differ enough from the value of R2 that the distribution of Vin that's between R1 and R2||RA2 will change.

Concepts tested: voltage divider, effect of measurement devices on the measurement (resistances in parallel)

Q1.6) Why don't we need to be concerned about the effect of the input impedance of channel 1 (RA1) on the measurement of V_{in}? [3 points: 1 for identifying that RA1 is in parallel with R1+R2 and Vin; 2 for identifying that VRA1 will always be the same as Vin because they are in parallel]

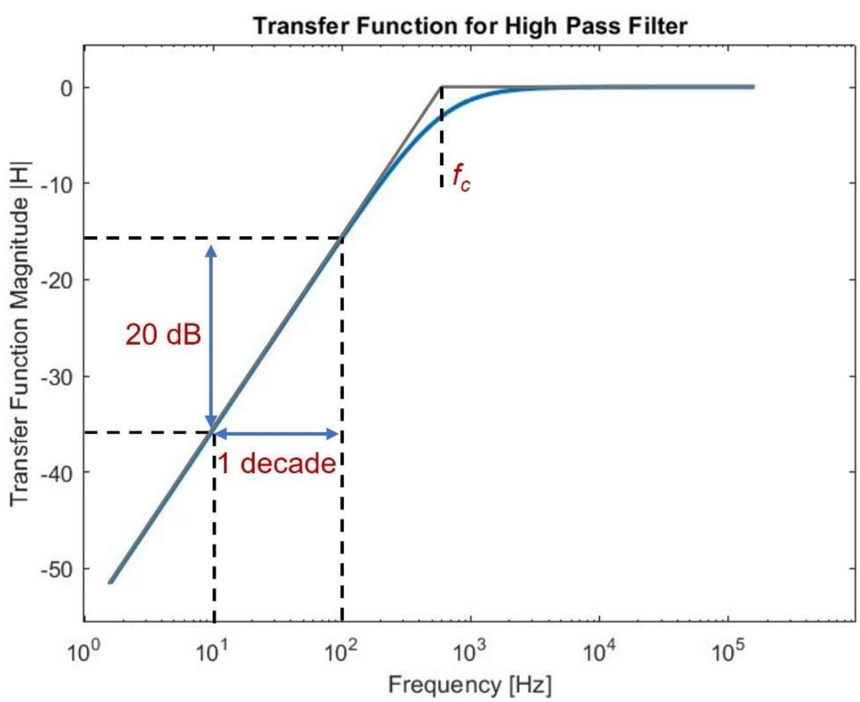
Since RA1 is in parallel with Vin and R1+R2, no matter the value of RA1 the voltage drop across RA1 and R1+R2 will always be Vin (resistors in parallel have the same voltage drop).

Concepts tested: resistances in parallel (same voltage drop)

Question 2: Filters

It is often necessary to filter noise at specific frequencies out of an audio signal. One example is the noise at 60 Hz, generated by the main AC wall plug power in a building. If the cable that carries an audio signal is not properly shielded and is near a power cable, a humming sound at 60 Hz will be picked up by the audio cable. If the audio has already been recorded and the 60 Hz hum is present, it is necessary to use a filter to remove it. Although a high quality factor band-stop filter, which rejects a very narrow range of frequencies, is typically used, we can use the knowledge learned up to this point in this course to construct a simple filter to remove 60Hz noise.

The filters explored so far in this course are called first-order filters and consist of a resistor and a single capacitor or inductor. As shown in the plot below (example only), below the corner frequency, first-order filters attenuate a signal at a rate of 20 dB/decade, stated in other words, the magnitude of the transfer function changes at a rate of 20 dB per every power of 10 in frequency.

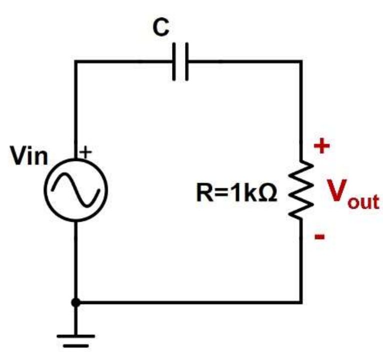


Q2.1) To filter out the 60 Hz noise, we want to design a high pass filter whose attenuation at f = 60 Hz is 40 dB (i.e., H = -40 dB at 60 Hz). Assuming that the corner frequency f_c is located where the linear approximation of the filter at low frequencies crosses 0 dB (as in the plot above), what corner frequency (in Hz) should the high pass filter we are designing have? [4 points: 2 pts for using roll-off of 20dB/decade line; 1 pt for calculating number of decades; 1 pt for correct numerical answer]

If H = -40dB at 60Hz and the corner frequency f_c is located where the linear approximation of the transfer function at low frequencies crosses 0dB, the transfer function must increase 40dB from its value at 60Hz to the corner frequency. Because the filter roll-off is 20dB/decade, the increase from -40dB to 0dB spans 2 decades in frequency, so the corner frequency is located at 60Hz * 10^2 = **6 kHz =** $\boldsymbol{f_c}$.

Concepts tested: interpreting magnitude Bode Plots, corner frequency

Q2.2) To implement our high-pass filter, we will use a design with a resistor R = 1 kΩ and a capacitor C, as shown below.



Using the value determined for the corner frequency in Q2.1, calculate the capacitance C required for the filter. [3 points: 2 for correct formula; 1 for correct numerical answer for C (if Q2.1 was correct)]

The formula for the corner frequency of a first-order filter with a resistor and a capacitor is $f_c = \frac{1}{2\pi R}$. Solving for C with f_c = 6kHz and R = 1kΩ, $C = \frac{1}{2\pi f_c R} = \frac{1}{2\pi * 6kHz * 1k\Omega} = \mathbf{26.5\ nF}$

Concepts tested: using the corner frequency of an RC filter

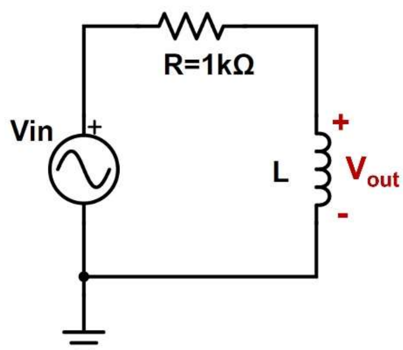
Q2.3) Write the transfer function $H(j\omega)$ for this filter and then find the expression for its magnitude $|H(j\omega)|$. [4 points: 2 pts for correct transfer function; 2 pts for correct magnitude]

$$H(j\omega) = \frac{R}{R + \frac{1}{j\omega C}}$$

$$|H(j\omega)| = \sqrt{H(-j\omega)H(j\omega)} = \sqrt{\left(\frac{R}{R - \frac{1}{j\omega C}}\right)\left(\frac{R}{R + \frac{1}{j\omega C}}\right)} = \frac{\omega RC}{\sqrt{\omega^2 R^2 C^2 + 1}}$$

Concepts tested: transfer function, voltage divider, impedance of capacitor, magnitude of a complex function

Q2.4) Alternatively, we could implement the filter using a resistor R = 1 kΩ and inductor L in series, as shown below. What value of L is required to implement a high-pass filter with the same corner frequency as calculated in Q2.1? [3 points: 2 for correct formula; 1 for correct numerical answer for C (if Q2.1 was correct)]



The formula for the corner frequency of a first-order filter with a resistor and an inductor is $f_c = \frac{R}{2\pi L}$. Solving for L with f_c = 6kHz and R = 1kΩ, $L = \frac{R}{2\pi f_c} = \frac{1k\Omega}{2\pi * 6kHz} = \mathbf{26.5\ mH}$

Concepts tested: using the corner frequency of an RL filter

Q2.5) Write the transfer function *H*(*jω*) and the magnitude of the transfer function |*H*(*jω*)| for this filter as well. [2 points: 2 pts for correct transfer function; 2 pts for correct magnitude]

$$H(j\omega) = \frac{j\omega L}{j\omega L + R}$$

$$|H(j\omega)| = \sqrt{H(-j\omega)H(j\omega)} = \sqrt{\left(\frac{-j\omega L}{-j\omega L + R}\right)\left(\frac{j\omega L}{j\omega L + R}\right)} = \frac{\omega L}{\sqrt{\omega^2 L^2 + R^2}}$$

Concepts tested: transfer function, voltage divider, impedance of inductor, magnitude of a complex function

Q2.6) What property of capacitors and inductors (as opposed to resistors) allows us to use them to make filters? [2 points]

A frequency-dependent impedance.

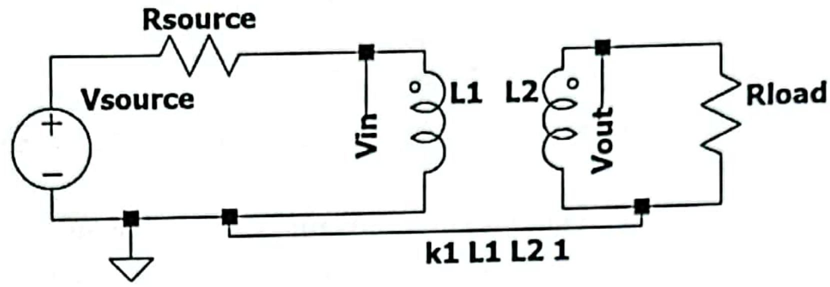
Concepts tested: significance of frequency-dependent impedance of capacitors and inductors

Q2.6) OR Given a choice between using a capacitor or an inductor in a high-pass filter for use at very high frequencies, which would you choose and why? [2 points]

It's usually preferable to choose a capacitor, since an inductor has a non-negligible parasitic capacitance at high frequencies due to the capacitance between the windings.

Concepts tested: circuit models of non-ideal capacitors and inductors

III. Phasors and Transformers (20 points)



Assume L1 and L2 form an ideal transformer with full coupling. The transformer has these specifications: **a=3, L1= 5mH**

a. (2pts) Determine the value of L2 that will allow the transformer to match the given a and L1 values:

$a = \sqrt{\frac{L_2}{L_1}} \rightarrow 3 = \sqrt{\frac{L_2}{5mH}} \rightarrow L_2 = 45mH$

L2= 45 mH

b. (4 pts) Determine the ratios Vout/Vin, and Iout/Iin

$\frac{V_{out}}{V_{in}} = a \qquad \frac{I_{out}}{I_{in}} = \frac{1}{a}$

Vout/Vin= 3

Iout/Iin= 1/3

c. (3 pts) Find the value of Rload that results in Rin = 3kΩ (Rin is Vin/Iin)

$Z_{in} = 3k\Omega = \frac{Z_{Load}}{a^2}$

Rload= 27 kΩ

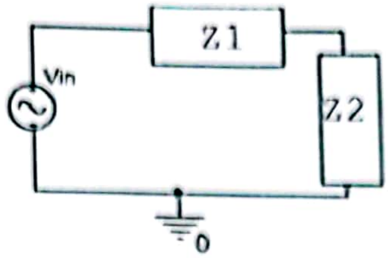
$Z_{load} = (3k\Omega)a^2 = 27k\Omega$

d. (3 pts) Assume that the transformer above is an air core transformer operating at a sufficiently high frequency for normal operation. Now suppose that we replace the air core with a ferrite core similar to the one used in Experiment 3. Will this change the Vout/Vin ratio? Why or why not?

No. It will not change L_2/L_1 and therefore will not change a.

e. (2 pts) Suppose that the coupling coefficient between the two coils is decreased. What effect will this have on Vout/Vin?

It will decrease Vout / Vin.



This circuit shown above has 2 complex impedances, Z1 and Z2, connected as shown.

f. (3 pts) Suppose that Z1 consists of a 1 microfarad capacitor and Z2 consists of a ~~1~~ 2 millihenry inductor. If Vin is a 5V sinusoidal voltage with frequency 5 kHz, what is the input impedance of the circuit?

$$Z_1 = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 5 \cdot 10^{3} \cdot 1 \cdot 10^{-6}} = -j\,31.83$$

$$Z_2 = j2(2\pi)(5 \cdot 10^{3})(10^{-3}) = j\,62.83$$

$$Z_{in} = Z_1 + Z_2 = j\,31\ \Omega$$

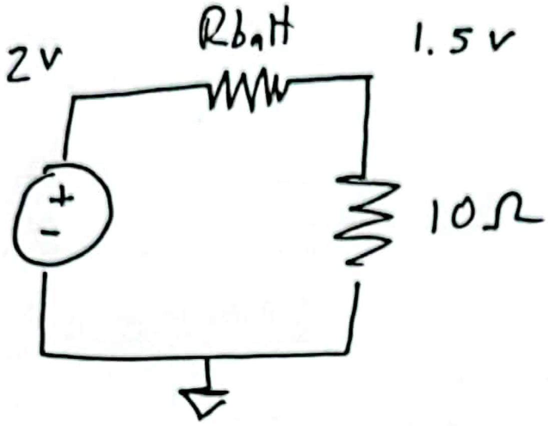
g.) (3 pts) What does the phase of the input impedance approach as the frequency of Vin becomes very high?

$$Z_{in} = \frac{1}{j\omega C} + j\omega L$$

As $\omega \to \infty$, the $\frac{1}{j\omega C}$ term approaches zero, leaving the $j\omega L$ term which has a $+90°$ phase.

IV. Concepts and miscellaneous (20 points)

a.) (3 pts) Suppose that you have a battery with an open circuit voltage of 2V and an unknown battery resistance. You create a circuit that discharges the battery through a 10-ohm resistor. Draw a circuit diagram of this, including the battery resistance Rbatt.

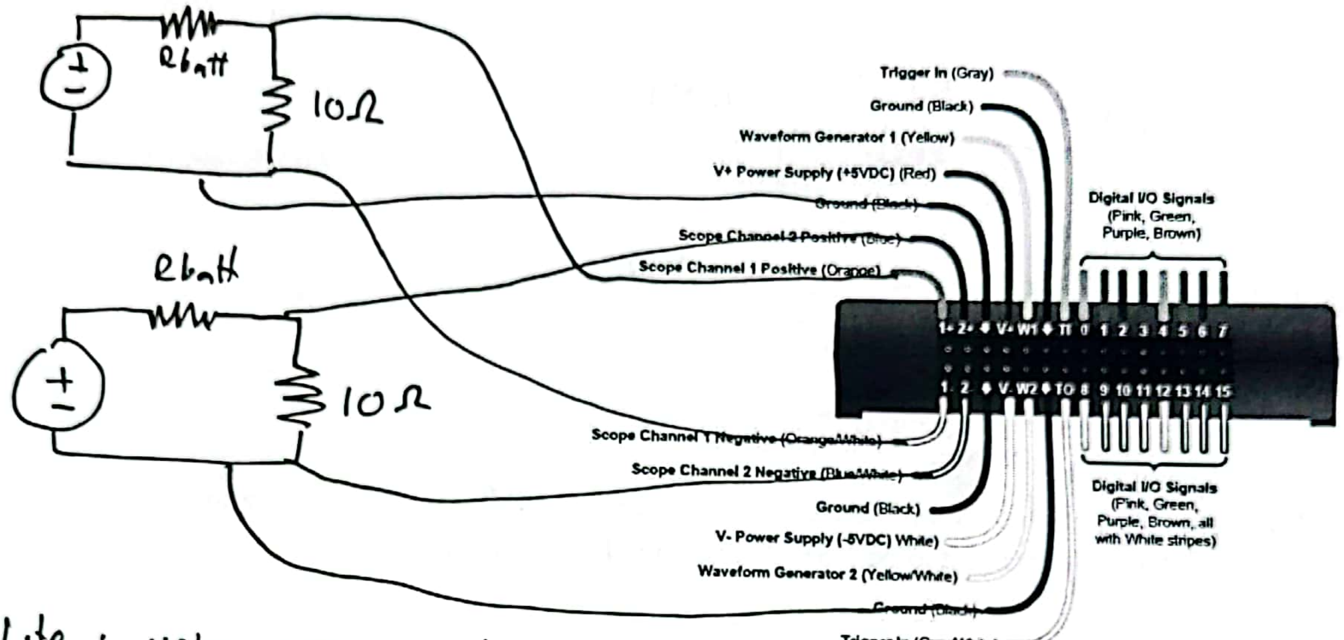


b.) (3 pts) Suppose that you measure the battery voltage while it is discharging into the 10 ohm resistor and find it to be 1.5V. What is the battery's resistance?

$$1.5V = 2 \frac{10\Omega}{R_{batt} + 10\Omega}$$

$$R_{batt} = 3.33 \ V$$

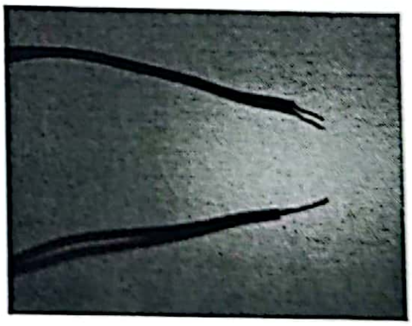
c.) (6 pts) Suppose that you wished to use your Analog Discovery Board or M2K Board to test the resistance of two batteries at the same time by discharging each one into a 10 ohm resistor in a separate circuit. Create a diagram of the two discharging circuits below, then show how you would connect the board to the circuits by drawing lines between the AD2/MDK board and the circuits. Note that the M2K and the Analog Discovery 2 have the same wiring and colors.



Note: using one ground or both grounds is acceptable.

Miscellaneous Questions

d.) (1 pt) Is the image shown at the right
1. a short circuit?
2. an open circuit?



e) (1 pt) Is the image shown at the right a
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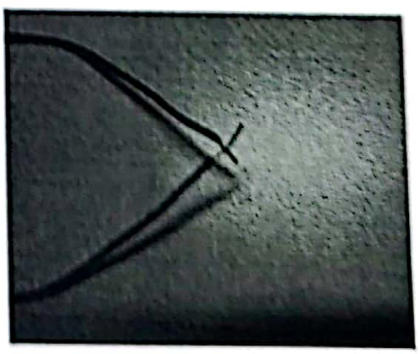
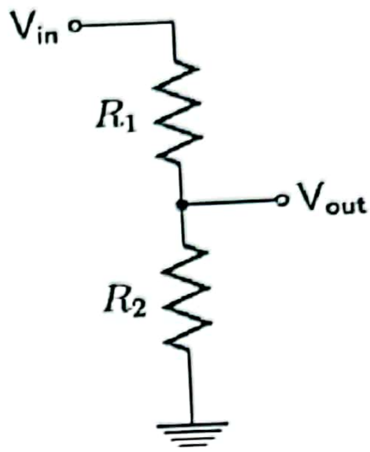


f.) (1 pt) In the standard voltage divider configuration shown at the right, resistor R1 is much smaller than resistor R2. Is the power dissipated in R1



1. Much greater than the power dissipated in R2
2. Much less than the power dissipated in R2
3. About the same as the power dissipated in R2

g.) (1 pt) A capacitor works by:

1.) Storing energy in an electric field
2.) Storing energy in a magnetic field
3.) Storing energy in a resistance

h.) (1 pt) An inductor works by:

4.) Storing energy in an electric field
5.) Storing energy in a magnetic field
6.) Storing energy in a resistance

i.) (2 pt) Explain why an inductor has a series resistance.

the wire used has a resistance per unit length.

j.) (1 pt) Name one of the teaching assistants or undergraduate student assistance for the course.

- Arpan Mukherjee
- Saad Ali Bin Reza
- Sudipta Paul
- Md. Shamim Islam
- Yufei Zhang
- Zephan D' Cruz
- Zeyu "Bennett" Song