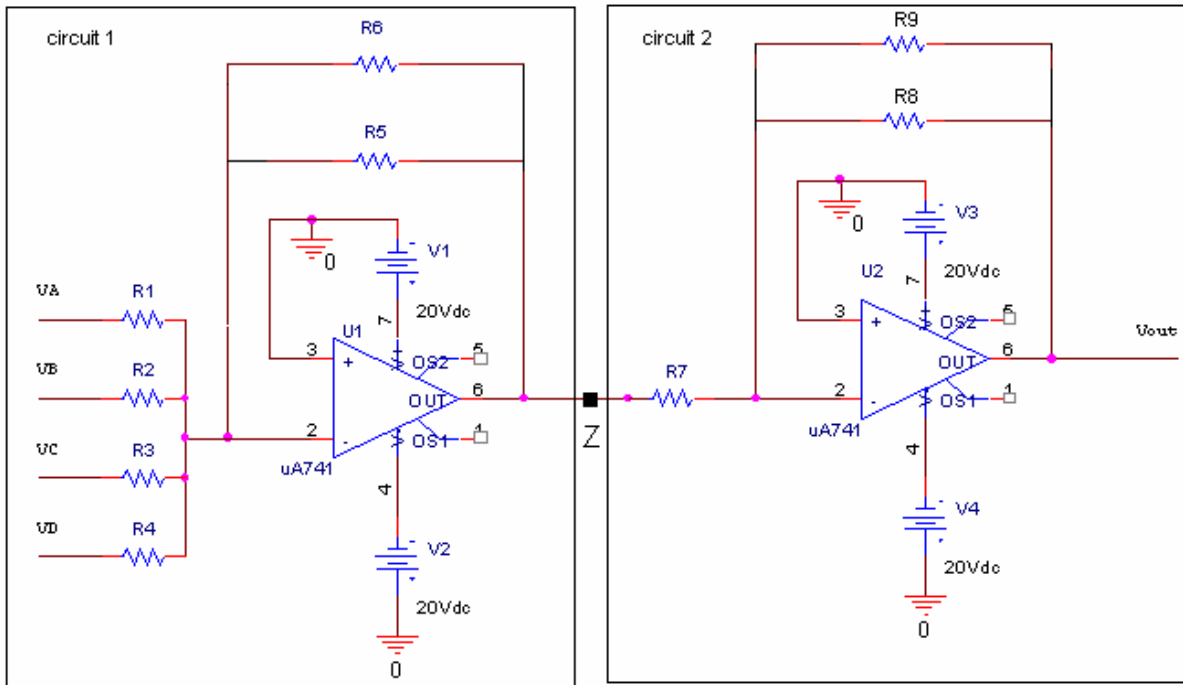


Questions about Digital to Analog Conversion Using Op-Amps

Fall 2004

Question 4 -- Digital-to-Analog Converter (20 points)

The circuit below converts digital signals into analog signals. This circuit produces an analog output voltage equal to the binary word DCBA in terms of the four inputs. Please assume that the input voltage levels for this circuit is 5 Volts for a logic “one” and 0 Volts for a logic “zero” and that $R5 = 6K\Omega$, $R6 = 6K\Omega$, $R7 = 2K\Omega$, $R8 = 20K\Omega$, and $R9 = 20K\Omega$.



a) What kind of circuit is circuit 1? Give an equation for the output of circuit 1 at point Z in terms of VA, VB, VC, and VD. Substitute numerical values for the known resistances, but leave in terms of the unknown resistances: R1, R2, R3 and R4. (4 points)

b) What kind of circuit is circuit B? Give an equation for the output of circuit B at Vout in terms of the input voltage at Z (Vz). Substitute numerical values for the known resistances. (3 points)

c) Find an equation relating V_{out} to the input voltages at V_A , V_B , V_C , and V_D . Substitute numerical values for the known resistances, but leave in terms of the unknown resistances: R_1 , R_2 , R_3 and R_4 . (2 points)

a) Select values for R_1 , R_2 , R_3 , and R_4 so that the output voltage will be the decimal equivalent of $DCBA$. For example, if $DCBA=1010$, or equivalently $V_D=V_B=5$ V, $V_A=V_C=0$ V, then $V_{out} = 10$ V. The circuit should work for **all** possible $DCBA$ combinations. (8 points)

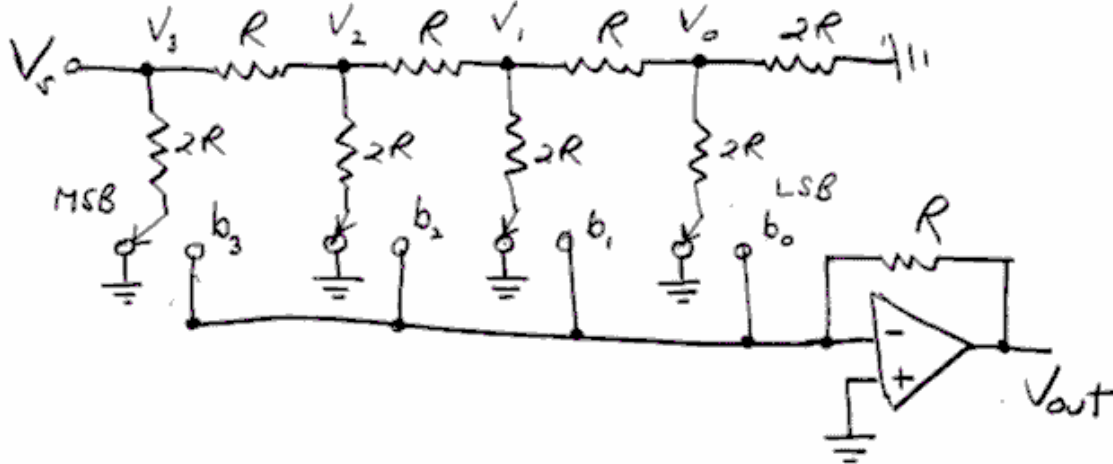
b) Show that the circuit correctly converts binary input 1001 to an AC voltage. What decimal number does 1001 represent? (3 points)

Fall 2004 Solution
(none available)

Spring 2004

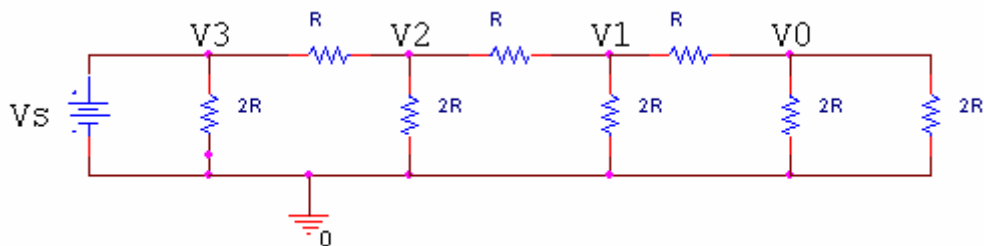
Question 4 -- Digital-to-Analog Converter (20 points)

For a computer or other digital device to interface with external analog circuits and devices, a digital-to-analog converter (DAC) is required. The most common DAC is a R-2R resistor ladder network, which requires only two precision resistor values R and 2R. A 4-bit R-2R resistor ladder network is shown below:



The digital input to the DAC is a 4-bit binary number represented by bits b_0 , b_1 , b_2 and b_3 , where b_0 is the least significant bit (LSB) and b_3 is the most significant bit (MSB). Each bit in the circuit controls a switch between ground and the *inverting* input op amp. When a bit is 1, the corresponding switch is connected to the op-amp; when a bit is zero, the corresponding switch is connected to ground.

a) If we assume this is an ideal op-amp, we can analyze the voltage levels in the circuit by removing the op amp. Below is a picture of the circuit when all bits are zero. Use the simplified circuit below to determine the voltage levels at V_0 , V_1 , V_2 and V_3 in terms of the source voltage, V_s . (8 points)



b) If we assume that closing switches has negligible effect on the voltage levels we found in part a), what will be the output of the DAC circuit, V_{out} (in terms of V_s), when the input ($b_3 b_2 b_1 b_0$) is: (10 points)

0001:

0010:

0100:

1000:

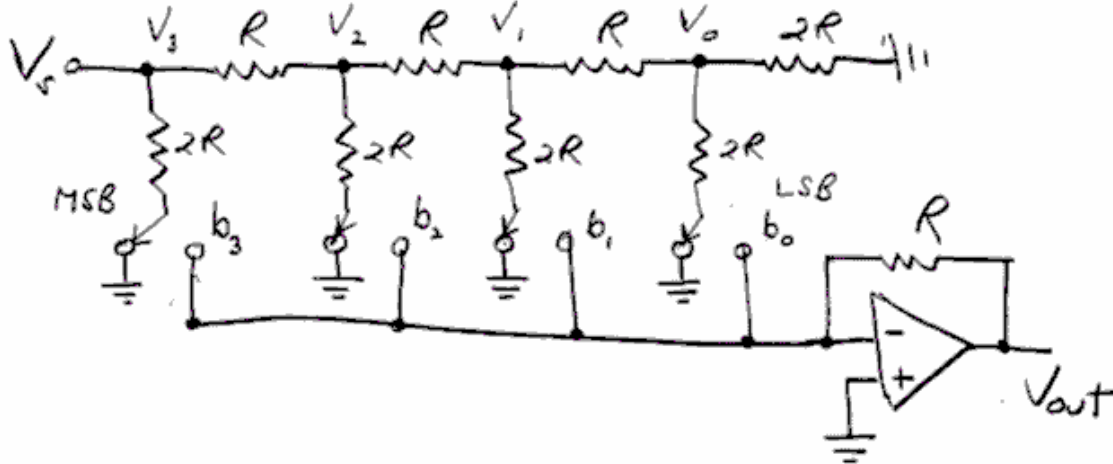
1111: [Hint: Use principal of superposition.]

c) In terms of V_s , what is the range of the analog output for a 4 bit binary input? (ie. If the input ranges from 0000 to 1111, what is the output range?) (2 points)

Spring 2004 solution

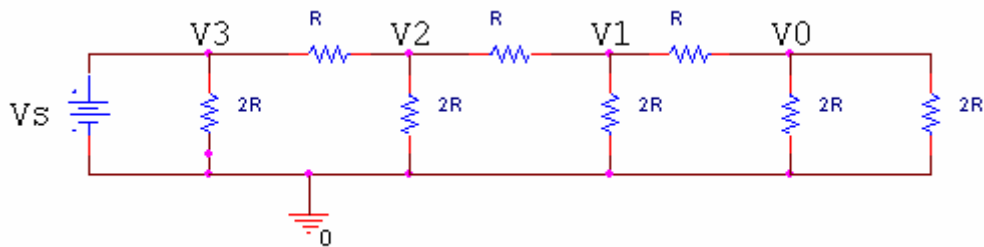
Question 4 -- Digital-to-Analog Converter (20 points)

For a computer or other digital device to interface with external analog circuits and devices, a digital-to-analog converter (DAC) is required. The most common DAC is a R-2R resistor ladder network, which requires only two precision resistor values R and 2R. A 4-bit R-2R resistor ladder network is shown below:

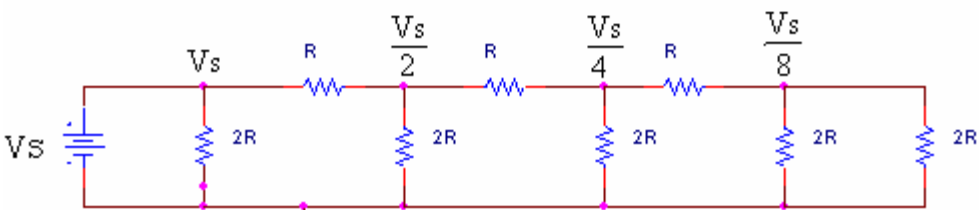
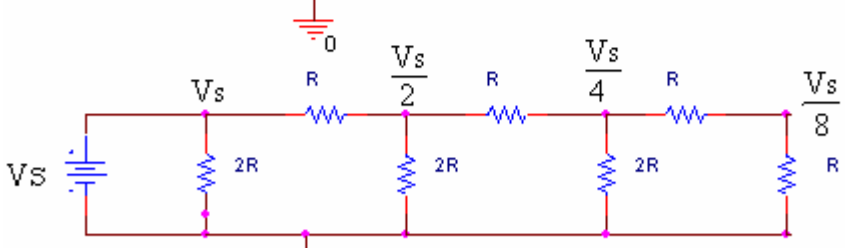
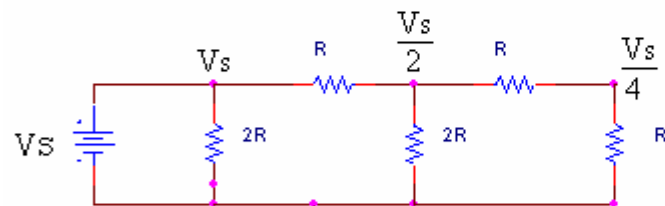
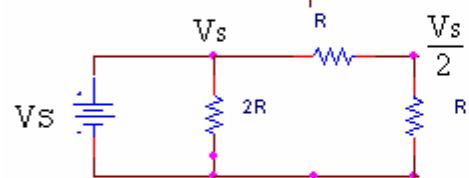
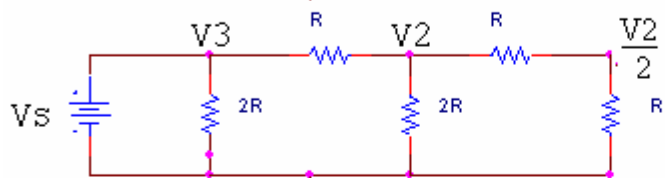
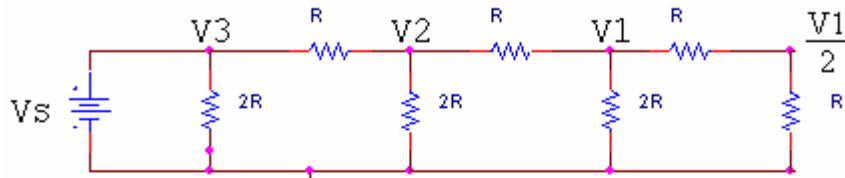
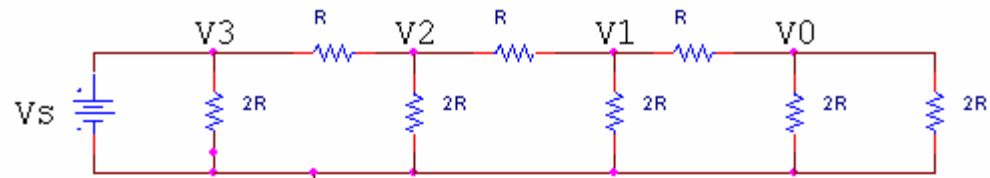


The digital input to the DAC is a 4-bit binary number represented by bits b_0 , b_1 , b_2 and b_3 , where b_0 is the least significant bit (LSB) and b_3 is the most significant bit (MSB). Each bit in the circuit controls a switch between ground and the *inverting* input op amp. When a bit is 1, the corresponding switch is connected to the op-amp; when a bit is zero, the corresponding switch is connected to ground.

a) If we assume this is an ideal op-amp, we can analyze the voltage levels in the circuit by removing the op amp. Below is a picture of the circuit when all bits are zero. Use the simplified circuit below to determine the voltage levels at V_0 , V_1 , V_2 and V_3 in terms of the source voltage, V_s . (8 points)



See following page for circuit analysis. When you combine 2 2R resistors in parallel, you get $(2R \cdot 2R)/(2R + 2R) = R$. If you do this to the two 2R resistors in parallel at V_0 , you get a voltage divider that divides V_1 in half. If you add the 2 R resistors and combine them in parallel with the 2R resistor at V_1 , you get another voltage divider that divides V_2 in half. You can continue this process until you get the voltage divider that divides V_3 into half to get V_2 . You know that V_3 is V_s . Therefore, V_2 is $V_s/2$. You can then apply the relationships in reverse to get all the voltages.



b) If we assume that closing switches has negligible effect on the voltage levels we found in part a), what will be the output of the DAC circuit, V_{out} (in terms of V_s), when the input (b3 b2 b1 b0) is: (10 points)

This assumption can be made because the op-amp is ideal and it tries to keep the voltages at the inputs the same. The positive input is grounded, therefore, the negative input is ground and the input circuit does not change regardless of the position of the switches. The value of V_{out} is determined by the inverting op-amp, which is acting on the input voltage of the corresponding bit with an input resistance of $2R$ and a feedback resistance of R .

$$0001: V_{out} = -(R/2R)V_0 = (-1/2)(V_s/8) \quad \mathbf{V_{out} = -(1/16)V_s}$$

$$0010: V_{out} = -(R/2R)V_1 = (-1/2)(V_s/4) \quad \mathbf{V_{out} = -(1/8)V_s}$$

$$0100: V_{out} = -(R/2R)V_2 = (-1/2)(V_s/2) \quad \mathbf{V_{out} = -(1/4)V_s}$$

$$1000: V_{out} = -(R/2R)V_3 = (-1/2)(V_s) \quad \mathbf{V_{out} = -(1/2)V_s}$$

1111: [Hint: Use principal of superposition.]

$$V_{out} = -(1/16)V_s + -(1/8)V_s + -(1/4)V_s + -(1/2)V_s$$

$$V_{out} = -(1/16+2/16+4/16+8/16)V_s$$

$$\mathbf{V_{out} = -(15/16)V_s}$$

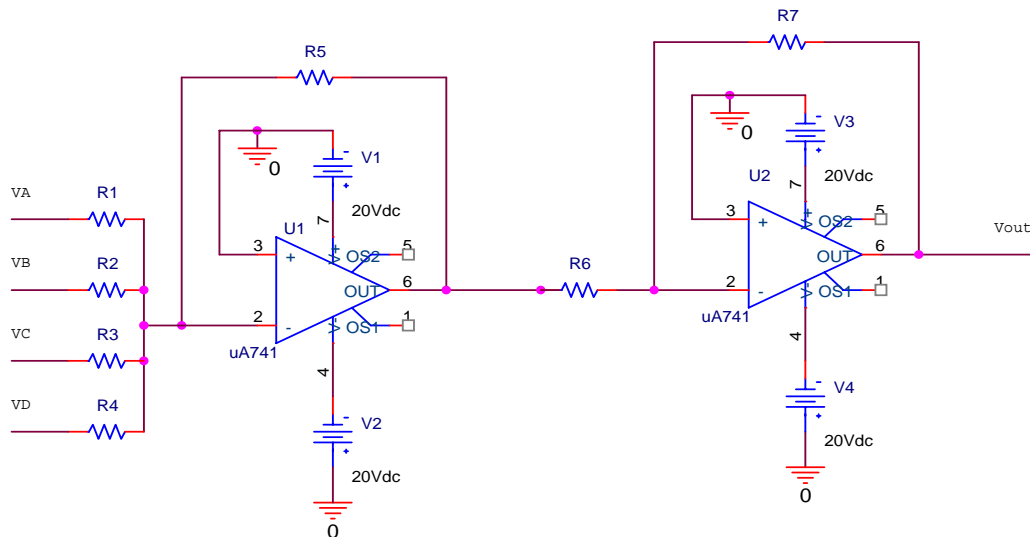
c) In terms of V_s , what is the range of the analog output for a 4 bit binary input? (ie. If the input ranges from 0000 to 1111, what is the output range?) (2 points)

The range of outputs is from 0 volts to $-(15/16)V_s$.

Fall 2003

Question 4 -- Digital-to-Analog Converter (20 points)

The circuit below converts digital signals into analog signals. This circuit produces an analog output voltage equal to the binary word DCBA in terms of the four inputs. Please assume that the input voltage levels for this circuit is 5 Volts for a logic of "one" and 0 Volts for a logic "zero" and that $R5 = 5K\Omega$, $R6 = 2K\Omega$ and $R7 = 30K\Omega$.



a) Select values for $R1$, $R2$, $R3$, and $R4$ so that the output voltage will be the decimal equivalent of DCBA. For example, if $DCBA=1010$, or equivalently $VD=VB=5$ V, $VA=VC=0$ V, then $V_{out} = 10$ V. The circuit should work for **all** possible DCBA combinations. (12 points)

$R1 =$

$R2 =$

$R3 =$

$R4 =$

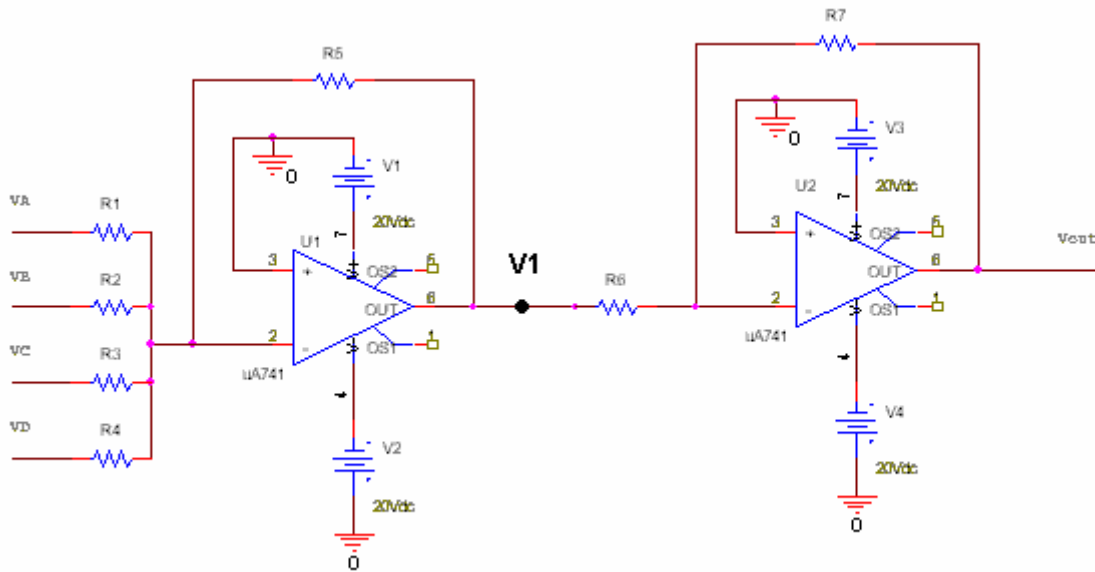
b) Show that the circuit correctly converts binary input 0111 to an AC voltage. What decimal number does 0111 represent? (4 points)

c) Explain one way you could modify the values of the resistors in this circuit so that the output voltage gives $4 \cdot N$ (rather than N), when N is the digital number at the input. For example, when the input is DCBA=1010, then $V_{out} = 4 \cdot 10 \text{ V}$ or 40V. You can modify any of the resistors R1-R7. (4 points)

Fall 2003 Solution

Question 4 -- Digital-to-Analog Converter (20 points)

The circuit below converts digital signals into analog signals. This circuit produces an analog output voltage equal to the binary word DCBA in terms of the four inputs. Please assume that the input voltage levels for this circuit is 5 Volts for a logic “one” and 0 Volts for a logic “zero” and that $R5 = 5K\Omega$, $R6 = 2K\Omega$ and $R7 = 30K\Omega$.



a) Select values for R1, R2, R3, and R4 so that the output voltage will be the decimal equivalent of DCBA. For example, if DCBA=1010, or equivalently $VD=VB=5$ V, $VA=VC=0$ V, then $Vout = 10$ V. The circuit should work for **all** possible DCBA combinations. (12 points)

$$V1 = (-R5)[(VA/R1) + (VB/R2) + (VC/R3) + (VD/R4)] \quad Vout = (-R7/R6)V1$$

$$Vout = (R5 * R7 / R6)[(VA/R1) + (VB/R2) + (VC/R3) + (VD/R4)]$$

$$(R5 * R7 / R6) = (5K * 30K) / 2K = 75K$$

$Vout$	VA	VB	VC	VD	plug in	solve
1	5V	0	0	0	$75K(5/R1)=1$	$R1=375K$
2	0	5V	0	0	$75K(5/R2)=2$	$R2=187.5K$
4	0	0	5V	0	$75K(5/R3)=4$	$R3=93.75K$
8	0	0	0	5V	$75K(5/R4)=8$	$R4=46.875K$

$R1 = 375K$

$R2 = 187.5K$

$R3 = 93.75K$

$R4 = 46.875K$

b) Show that the circuit correctly converts binary input 0111 to an AC voltage. What decimal number does 0111 represent? (4 points)

$$V_{out} = (R_5 * R_7 / R_6) [(V_A / R_1) + (V_B / R_2) + (V_C / R_3) + (V_D / R_4)]$$
$$V_{out} = (75K) [(5/375K) + (5/187.5K) + (5/93.75K) + (0/46.875K)]$$
$$V_{out} = 1 + 2 + 4 = 7 \text{ volts}$$

0111 = 7 decimal

c) Explain one way you could modify the values of the resistors in this circuit so that the output voltage gives 4*N (rather than N), when N is the digital number at the input. For example, when the input is DCBA=1010, then $V_{out} = 4 * 10 \text{ V}$ or 40V. You can modify any of the resistors R1-R7. (4 points)

The easiest way to do this is to modify one of the gain resistors:

$$R_5 = 4 * R_5 = 20K$$

or

$$R_7 = 4 * R_7 = 120K$$

or

$$R_6 = R_6 / 4 = 1.25K$$

You could also modify the four input resistors:

$$R_1 = R_1 / 4 = 93.75 \text{ and } R_2 = R_2 / 4 = 46.875K \text{ and}$$

$$R_3 = R_3 / 4 = 23.47 \text{ and } R_4 = R_4 / 4 = 11.73K$$

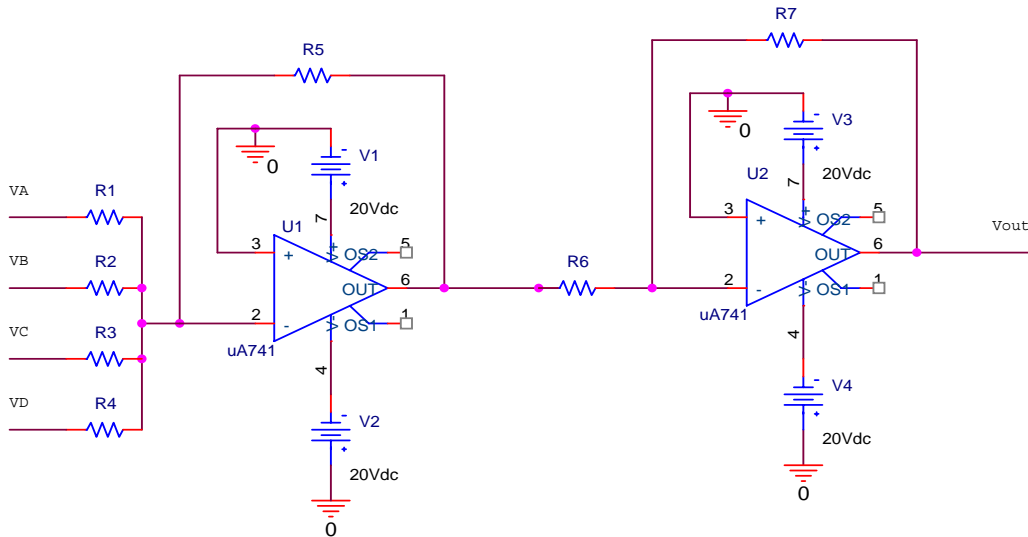
A combination which changes the gain would work also:

$$R_5 = R_5 * 2 = 10K \text{ and } R_6 = R_6 / 2 = 2.5K$$

Spring 2003

Question 4) Digital-to-Analog Converter (20 points)

The circuit below converts digital signals into analog signals. This circuit produces an analog output voltage equal to the binary word DCBA in terms of the four inputs. Please assume that the input voltage levels for this circuit is 5 Volts for a logic of “one” and 0 Volts for a logic “zero” and that $R5 = 10K\Omega$, $R6 = 1K\Omega$ and $R7 = 10K\Omega$.



a) Select values for $R1$, $R2$, $R3$, and $R4$ so that the output voltage will be the decimal equivalent of DCBA. For example, if $DCBA=1010$, or equivalently $VD=VB=5$ V, $VA=VC=0$ V, then $V_{out} = 10$ V. The circuit should work for **all** possible DCBA combinations. (12 points)

$R1 =$

$R2 =$

$R3 =$

$R4 =$

b) Show that the circuit correctly converts binary input 0110 to an AC voltage. What decimal number does 0110 represent? (4 pts)

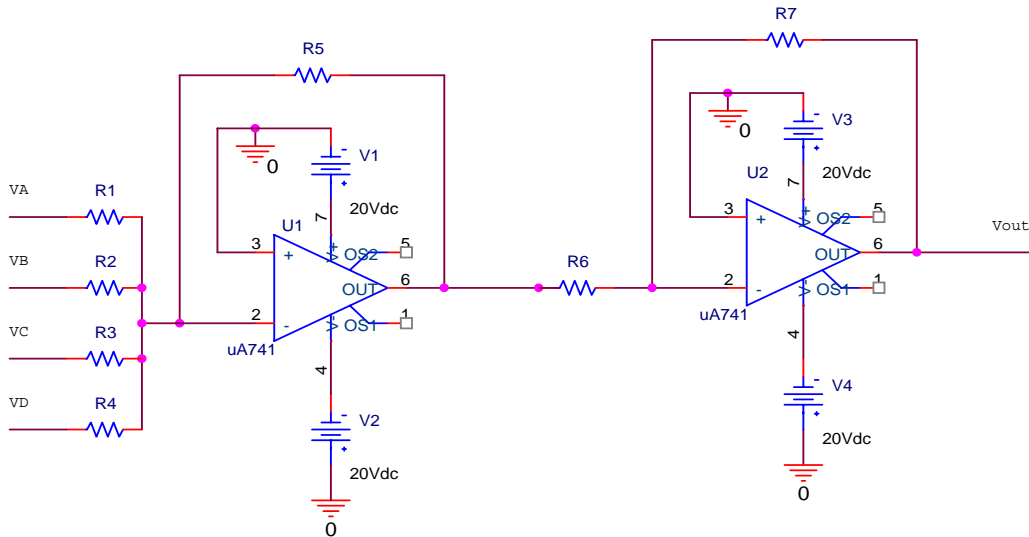
c) The D/A converter in this question is actually two op-amp configurations you have already seen combined together. Draw a box around each configuration on the previous page and identify which type of op-amp configuration it is. (4 pts)

Spring 2003 solution
(none available)

Fall 2002

Question 4) Digital-to-Analog Converter (20 points)

The circuit below is a “Voltage Adder” followed by an “Inverting Op-Amp”, that converts digital signals into analog signals. This circuit produces an analog output voltage equal to the binary word ABCD in terms of the four inputs. Please assume that the input voltage levels for this circuit is 5 Volts for a logic of “one” and 0 Volts for a logic “zero” and that $R5 = 15K\Omega$, $R6 = 2K\Omega$ and $R7 = 10K\Omega$.



a) Select values for $R1$, $R2$, $R3$, and $R4$ so that the output voltage will be the decimal equivalent of ABCD. For example, if $ABCD=1010$, or equivalently $VA=VC=5$ V, $VB=VD=0$ V, then $V_{out} = 10$ V. The circuit should work for all possible ABCD combinations. (12 points)

$R1 =$

$R2 =$

$R3 =$

$R4 =$

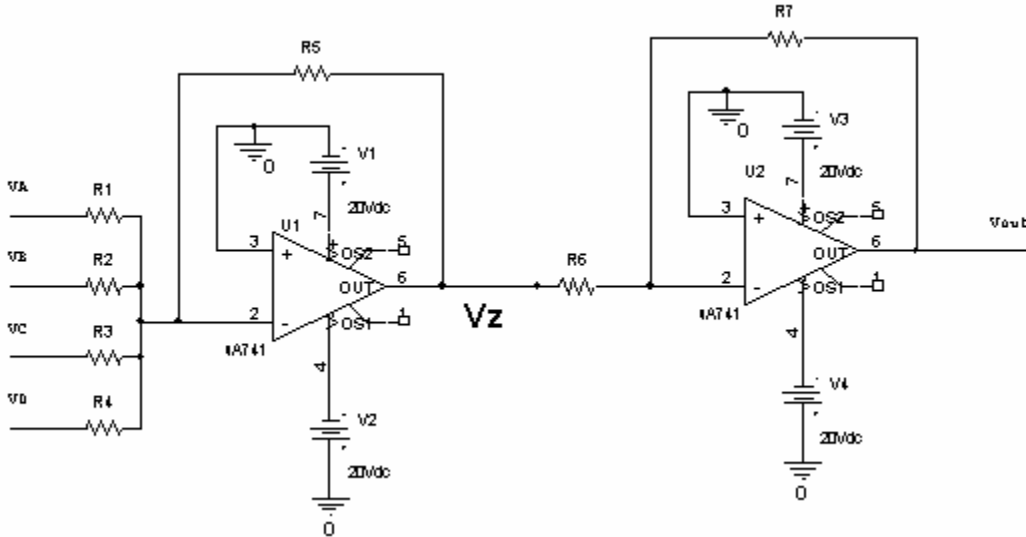
b) What is the maximum binary input (ABCD=?) to this circuit (4 points)?

c) Show that your selection of resistors above is correct by comparing the calculated V_{out} with the maximum digital input (4 points)

Fall 2002 Solution

Question 4) Digital-to-Analog Converter (20 points)

The circuit below is a “Voltage Adder” followed by an “Inverting Op-Amp”, that converts digital signals into analog signals. This circuit produces an analog output voltage equal to the binary word ABCD in terms of the four inputs. Please assume that the input voltage levels for this circuit is 5 Volts for a logic of “one” and 0 Volts for a logic “zero” and that $R5 = 15K\Omega$, $R6 = 2K\Omega$ and $R7 = 10K\Omega$.



a) Select values for R1, R2, R3, and R4 so that the output voltage will be the decimal equivalent of ABCD. For example, if ABCD=1010, or equivalently $VA=VC=5\text{ V}$, $VB=VD=0\text{ V}$, then $V_{out} = 10\text{ V}$. The circuit should work for all possible ABCD combinations. (12 points)

$$V_{out} = -(R7/R6)V_z \quad V_z = -(R5/R1)VA - (R5/R2)VB - (R5/R3)VC - (R5/R4)VD$$

$$V_{out} = (R7 \times R5 / R6) [(VA/R1) + (VB/R2) + (VC/R3) + (VD/R4)]$$

$$(R7 \times R5 / R6) = 10K \times 15K / 2K = 75K$$

Vout	VA	VB	VC	VD	Equation	Result
1V	0	0	0	5V	$1V = 75K(5V/R4)$	$R4 = 375K\text{ ohms}$
2V	0	0	5V	0	$2V = 75K(5V/R3)$	$R3 = 187.5K\text{ ohms}$
4V	0	5V	0	0	$4V = 75K(5V/R2)$	$R2 = 93.75K\text{ ohms}$
8V	5V	0	0	0	$8V = 75K(5V/R1)$	$R1 = 46.875K\text{ ohms}$

$$R1 = 46.875K\text{ ohms} \quad R2 = 93.75K\text{ ohms} \quad R3 = 187.5K\text{ ohms} \quad R4 = 375K\text{ ohms}$$

b) What is the maximum binary input (ABCD=?) to this circuit (4 points)?

$$ABCD = 1111 \text{ (15 in decimal)}$$

c) Show that your selection of resistors above is correct by comparing the calculated V_{out} with the maximum digital input (4 points)

$$V_{out} = (R7 \times R5 / R6) [(VA/R1) + (VB/R2) + (VC/R3) + (VD/R4)]$$

$$V_{out} = 15V = 75K [(5V/46.875K) + (5V/93.75K) + (5V/187.5K) + (5V/375K)] ?$$

$$15V = 75 [0.107 + 0.053 + 0.027 + 0.013] = 75K \times 0.2m = 15 ?$$

15=15 It checks.

Spring 2002

4) Digital-to-Analog Converter (20 points)

- a) In figure below, $R5 = 20K\Omega$, $R6 = 1K\Omega$ and $R7 = 5K\Omega$. This configuration of op-amps and resistors can produce an analog output voltage equal to the binary word ABCD input at the left. Assume that you are working with TTL devices, so the voltage levels for ones and zeros are TTL levels (0V and 5V). Select values for $R1$, $R2$, $R3$, and $R4$ so that the output voltage will be the decimal equivalent of ABCD. For example if $ABCD=1010$, or equivalently $V_A=V_C=5v$, $V_B=V_D=0v$, $V_{out} = 10v$. The circuit should work for all possible ABCD combinations. (12 points)

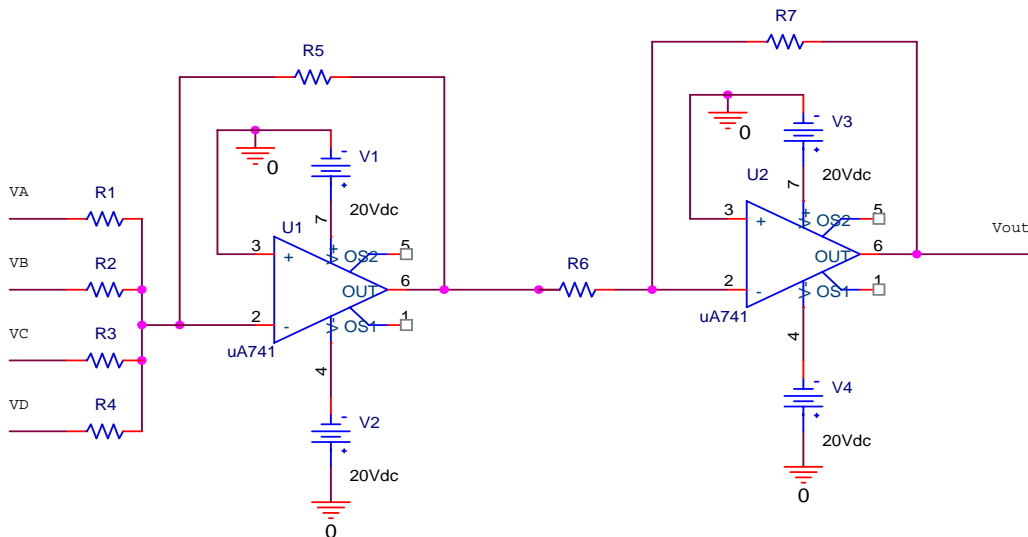
$R1 =$

$R2 =$

$R3 =$

$R4 =$

- b) Your choices of resistors should work for any number, but specifically show that your values work for the two binary numbers $ABCD = 0010$ and $ABCD = 1011$. (8 points)



(Show your work here for question 4.)

b) *Answer:*

$$ABCD = 0010 \text{ (binary)} = 2 \text{ (decimal)}$$

$$V_{out} = 2V = ? [VA/R1 + VB/R2 + VC/R3 + VD/R4] * (R6 * R5) / R7$$

$$2V = ? [5/250K] * 100K = 2V \text{ (checks)}$$

$$ABCD = 1101 \text{ (binary)} = 11 \text{ (decimal)}$$

$$V_{out} = 11V = ? [VA/R1 + VB/R2 + VC/R3 + VD/R4] * (R6 * R5) / R7$$

$$11V = ? [5/62.5K + 5/250K + 5/500K] * 100K$$

$$11V = ? [.08 + .02 + .01] * 100 = 11V \text{ (checks)}$$

Name _____

Section _____

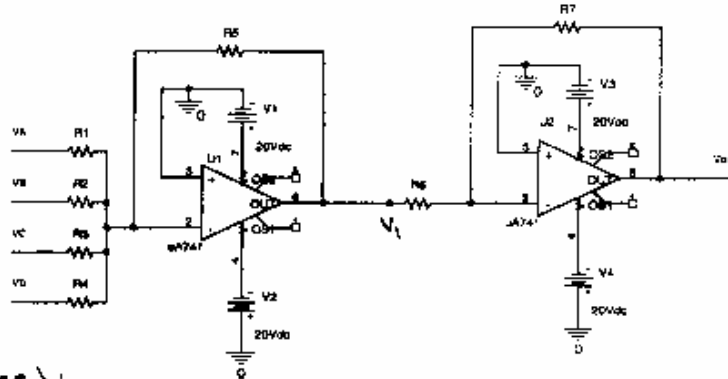
Please show all work on all questions for full credit, some explanation of your answer is required.

4. Digital-to-Analog Converter (20 points)

a) In figure below, $R_5 = 100K\Omega$, $R_6 = 1K\Omega$ and $R_7 = 2K\Omega$. This configuration of op-amps and resistors can produce an analog output voltage equal to the binary word ABCD input at the left. Assume that you are working with TTL devices, so the voltage levels for ones and zeros are TTL levels (0V and 5V). Select values for R_1 , R_2 , R_3 , and R_4 so that the output voltage will be the decimal equivalent of ABCD. For example if $ABCD=1010$, or equivalently $V_A=V_C=5V$, $V_B=V_D=0V$, $V_{out} = 10V$. The circuit should work for all possible ABCD combinations. (16 points)

$R_1 = 125K$ $R_2 = 250K$ $R_3 = 500K$ $R_4 = 1M$

b) Your choices of resistors should work for any number, but specifically show that your values work for the two binary numbers $ABCD = 0110$ and $ABCD = 1001$. (8 points)



What we need:

$ABCD_{\text{binary}} = 8A + 4B + 2C + D$

$1 \rightarrow 5V$
 $0 \rightarrow 0V$

$\Rightarrow V_{out} = \frac{8}{5} V_A + \frac{4}{5} V_B + \frac{2}{5} V_C + \frac{1}{5} V_D$

Next page →

Name _____

Section _____

Please show all work on all questions for full credit, some explanation of your answer is required.

4. (continued)

What circuit does:

$$V_{out} = -\frac{R_7}{R_6} V_1 = -\frac{2K}{1K} V_1 = -2V_1$$

$$V_1 = -\frac{R_5}{R_1} V_A - \frac{R_5}{R_2} V_B - \frac{R_5}{R_3} V_C - \frac{R_5}{R_4} V_D$$

$$\rightarrow V_{out} = 2\frac{R_5}{R_1} V_A + 2\frac{R_5}{R_2} V_B + 2\frac{R_5}{R_3} V_C + 2\frac{R_5}{R_4} V_D$$

For the circuit to be what we want:

$$2\frac{R_5}{R_1} = \frac{8}{5}, \quad 2\frac{R_5}{R_2} = \frac{4}{5}, \quad 2\frac{R_5}{R_3} = \frac{2}{5}, \quad 2\frac{R_5}{R_4} = \frac{1}{5} \Rightarrow R_1 = 105K, R_2 = 100K, R_3 = 50K, R_4 = 1M\Omega$$

b) ABCD = 0110 $\Rightarrow V_A = 0, V_B = 5V, V_C = 5V, V_D = 0$

$$\Rightarrow V_1 = -\frac{R_5}{R_1} 0 - \frac{R_5}{R_2} 5V - \frac{R_5}{R_3} 5V - \frac{R_5}{R_4} 0V = -\frac{100K}{250K} \cdot 5V - \frac{100K}{50K} 5V = -2V - 1V = -3V$$

$$V_{out} = -2V_1 \Rightarrow \boxed{V_{out} = 6V}$$

ABCD = 1001 $\Rightarrow V_A = 5V, V_B = 0V, V_C = 0V, V_D = 5V$

$$\Rightarrow V_1 = -\frac{R_5}{R_1} 5V - \frac{R_5}{R_2} 0 - \frac{R_5}{R_3} 0 - \frac{R_5}{R_4} 5V = -\frac{100K}{250K} \cdot 5V - \frac{100K}{1M} 5V = -4V - 0.5V = -4.5V$$

$$V_{out} = -2V_1 \Rightarrow \boxed{V_{out} = 9V}$$

Fall 2000

2. Digital-to-Analog Converter

The configuration of op-amps and resistors shown below can produce an analog output voltage (across the last 100kΩ resistor on the right) equal to the binary word ABCD input at the left. Assume that you are working with TTL devices, so the voltage levels for ones and zeros are TTL levels (0V and 5V). Select values for R1, R2, R3, and R4 so that the output voltage will be the decimal equivalent of ABCD, for any choice of ABCD.

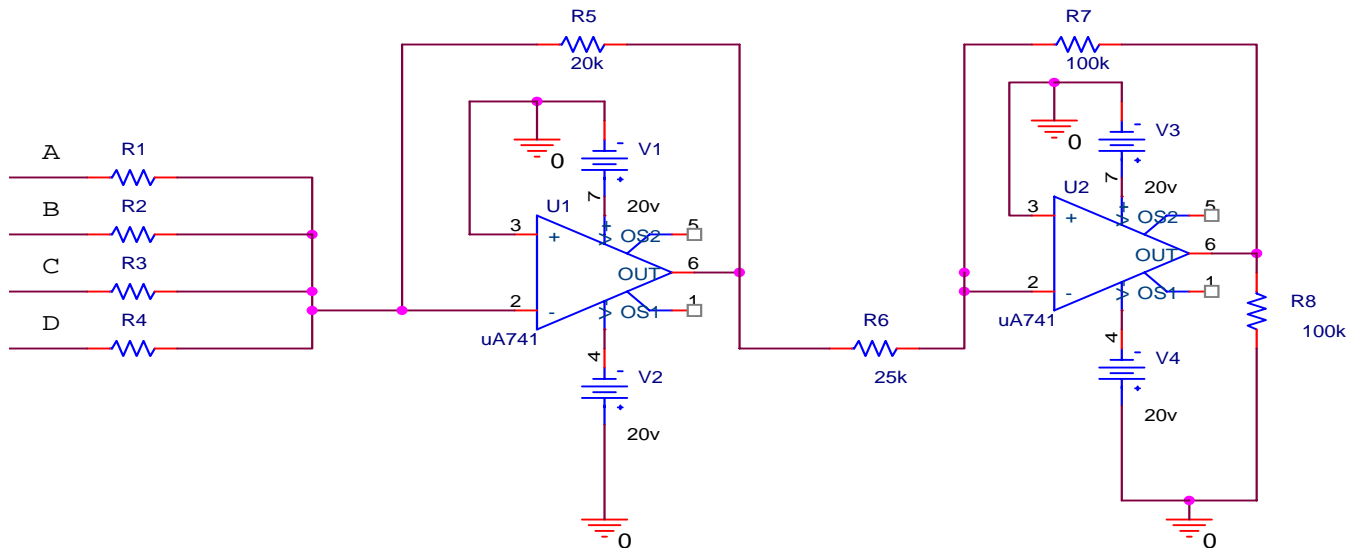
R1 =

R2 =

R3 =

R4 =

Your choices of resistors should work for any number, but specifically show that your values work for the two binary numbers ABCD = 0110 and ABCD = 1001.



$ABCD = 0110$ (binary) = 6 (decimal)

$V_{out} = 6V = ? [VA/R1 + VB/R2 + VC/R3 + VD/R4] * (R6 * R5) / R7$

$6V = ? [5/100K + 5/200K] * 80K = 6V$ (checks)

$ABCD = 1001$ (binary) = 9 (decimal)

$V_{out} = 9V = ? [VA/R1 + VB/R2 + VC/R3 + VD/R4] * (R6 * R5) / R7$

$9V = ? [5/50K + 5/400K] * 80K$

$9V = ? [.08 + .02 + .01] * 80 = 9V$ (checks)