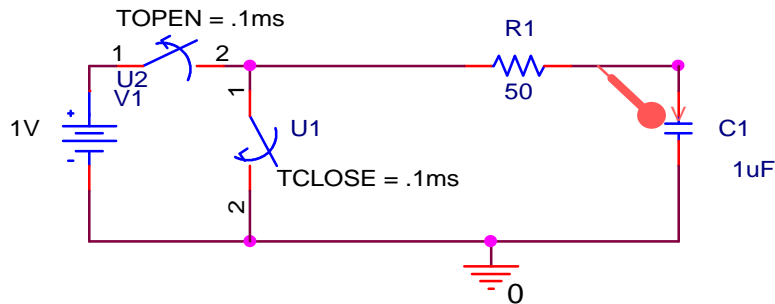Data Analysis Example

Assume that you have the following circuit:

That is, a 1 Volt DC source has been connected across a resistor-capacitor combination as shown. At time t=0.1ms, the source is disconnected from the circuit and the resistor-capacitor combination is shorted to ground. When this occurs, the voltage across the capacitor decays to zero. Assume that you have measured the following:

| t (us)	V	I (mA)
0	1	?
10	0.8	17
20	0.7	13.5
30	0.55	11
40	0.45	9
50	0.4	8
60	0.3	6.5
70	0.25	5
80	0.2	4
90	0.15	3
100	0.14	2.5
110	0.1	2
120	0.08	1.6
130	0.07	1.4
140	0.06	1.2
150	0.05	0.9

We would like to use this information to confirm that the current-voltage equation for a capacitor, $I = C \frac{dV}{dt}$, does indeed hold. For this expression, we can estimate the time derivative of the voltage by using the finite difference form $\frac{dV}{dt} = \frac{V_2 - V_1}{t_2 - t_1}$. For each of the times where data was taken, we can find the finite difference derivative. The capacitance can then be found by dividing the current by the time derivative of the voltage. Since the finite difference expression is only approximate, the best answer can be determined by taking the average of all of the values determined. When doing this, the average value turns out to be very close to the value for the capacitance. Thus, the current-voltage expression is shown to be correct, at least on the average.

dV/dt	C
20000	0.00000085
10000	0.00000135
15000	7.33333E-07
10000	0.0000009
5000	0.0000016
10000	0.00000065
5000	0.000001
5000	0.0000008
5000	0.0000006
1000	0.0000025
4000	0.0000005
2000	0.0000008
1000	0.0000014
1000	0.0000012
1000	0.0000009
Average C	1.05222E-06

Using this information, we can check to see if the current tracks the capacitance times the time derivative of the voltage. Because the derivative is so crudely taken, the two curves do not match point-by-point, but they do look the same on the average.
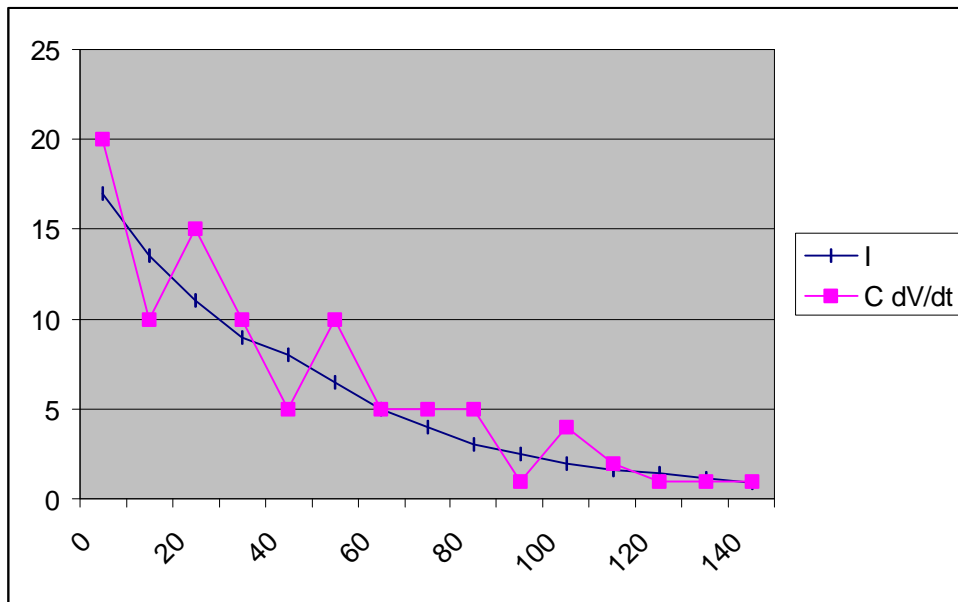

Excel provides a method for finding the slopes of functions that can be more accurate than the simple finite difference derivative used here. The slope function (more details are provided below) finds the best fit line (using linear regression) to any number of data points. If you chose two data points, slope will produce the same result as the finite

difference formula used above. However, by taking more points, the results become much smoother, primarily because the errors from point-to-point tend to be random. For the same data used above, the slope formula gives the following results:
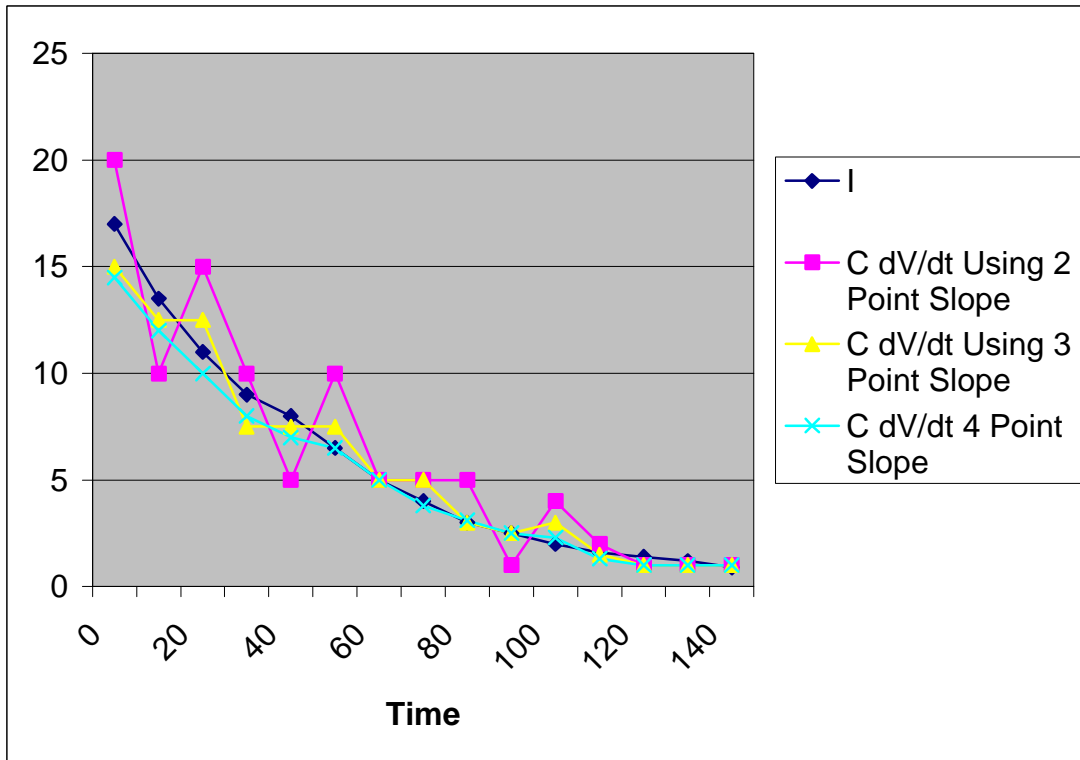

The results are not perfect, since the slope is an approximation and is particularly less accurate near the ends of the curve. Ideally the slope value should be assigned to the middle of the range it is evaluated over. However, Excel does not do this easily.

One can also fit the current and voltage expressions to exponential functions, then take the derivative of the voltage to check the capacitance current-voltage equation another way. The voltage measured as a function of time looks like
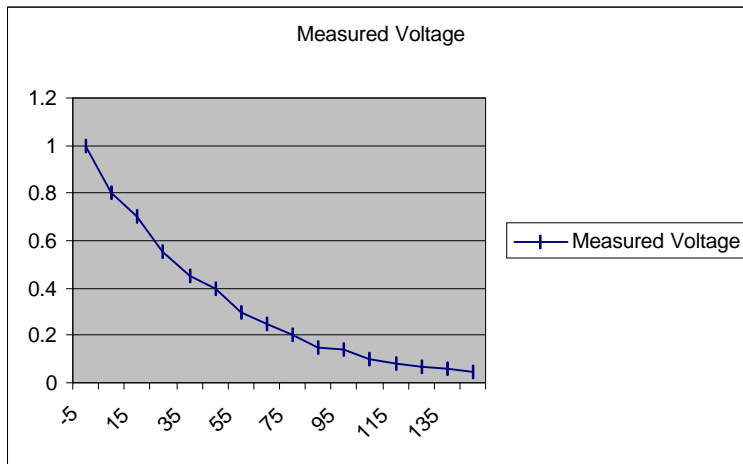
where the time scale is in microseconds. Note that the voltage drops from around 1Volt to a little less than 0.3Volt in about 50 microseconds. Thus, for exponential decay, the time

constant is about 0.00005 seconds. If we assume that the voltage starts at 1 Volt, we can write it as a function of time as $V = V_o e^{-t/t}$. Plotting this function on top of the measured voltage, we obtain
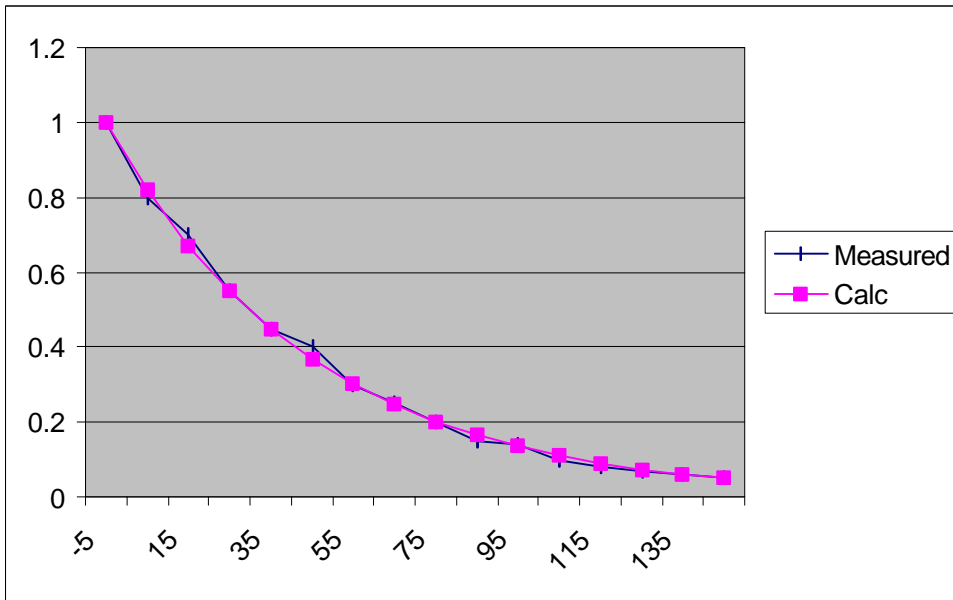
so the agreement is seen to be very good. Thus, the voltage does indeed decay exponentially.