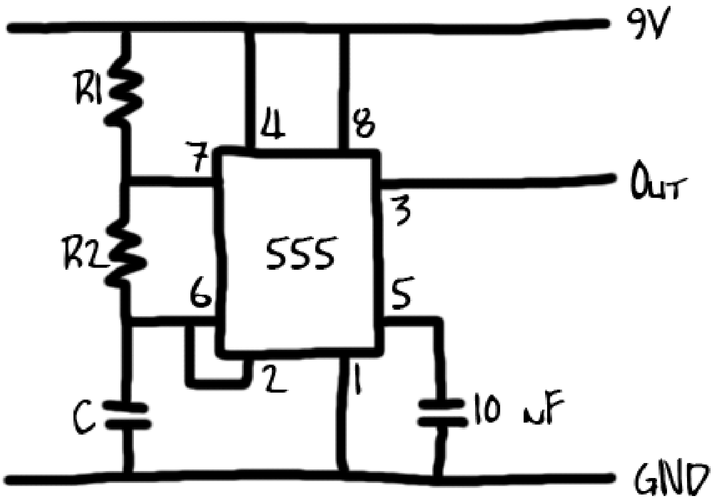
Question: Where does the 0.693 multiplier for the 555 timer time constant come from?
The generic 555 timer astable multivibrator circuit is shown below.



(Image from www.ohmslawcalculator.com)
For this circuit, in steady-state, the voltage on the capacitor oscillates between $\frac{1}{3}V_{CC}$ and $\frac{2}{3}V_{CC}$, where V_{CC}=9V. The capacitor charges through R_1 and R_2 and discharges through R_2. The two characteristic time constants are $\tau_{charge} = 0.693(R_1 + R_2)C$ and $\tau_{discharge} = 0.693R_2C$.
Qualitatively these time constants make sense because to charge from *0V* to $V_{CC}\left(1-e^{-1}\right) = V_{CC}(0.632)$ requires a time equal to the $\tau = RC$ time constant. To go from $\frac{1}{3}V_{CC}$ to $\frac{2}{3}V_{CC}$ is a smaller change (*0.333 being smaller than 0.632*). However, we do not have to guess at this number because we can calculate it exactly.
For the general case (either charge or discharge) characterized by $\tau = RC$, assume a general expression for the charging of the capacitor $V_C = V_{CC}\left(1-e^{-t/\tau}\right)$. Evaluate this expression at the two voltages to determine the time difference. $\frac{1}{3}V_{CC} = V_{CC}\left(1-e^{-t_1/\tau}\right)$ and $\frac{2}{3}V_{CC} = V_{CC}\left(1-e^{-t_2/\tau}\right)$. Cancel the V_{CC} from each expression and take the natural log (*ln*).

$$\frac{1}{3} = 1-e^{-t_1/\tau} \text{ and } \frac{2}{3} = 1-e^{-t_2/\tau}$$

$$\frac{2}{3} = e^{-t_1/\tau} \text{ and } \frac{1}{3} = e^{-t_2/\tau}$$

$$\ln 3 - \ln 2 = \frac{t_1}{\tau} \text{ and } \ln 3 - \ln 1 = \ln 3 - 0 = \ln 3 = \frac{t_2}{\tau}$$

Subtract the first expression from the second expression to obtain the time it takes to raise the voltage from $\frac{1}{3}V_{CC}$ to $\frac{2}{3}V_{CC}$.

$$\ln 2 = \frac{t_2}{\tau} - \frac{t_1}{\tau} \text{ or } t_2 - t_1 = (\ln 2)\tau = 0.693\tau$$

QED