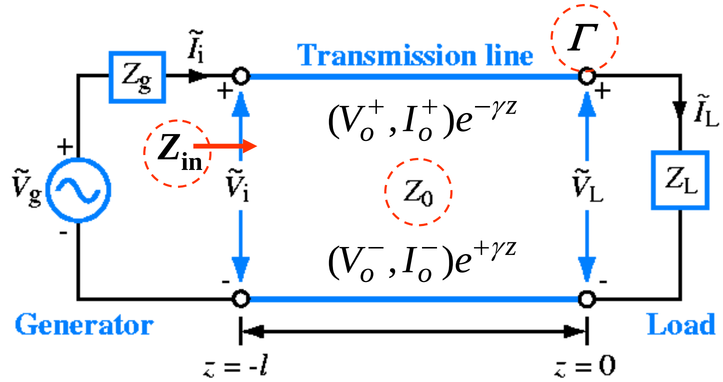
Transmission Lines



Steady State (Continuous Wave)

Phasor Notation

$$v(z,t) = \mathcal{Re}[\tilde{V}(z)e^{j\omega t}]$$

$$i(z,t) = \mathcal{Re}[\tilde{I}(z)e^{j\omega t}]$$

Voltage Wave

$$\tilde{V}(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z} = V_o^+(e^{-\gamma z} + \Gamma e^{\gamma z}) = V_o^+ e^{-\gamma z}(1+\Gamma(z)) \quad \text{(general form)}$$

$$\tilde{V}(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z} = V_o^+(e^{-j\beta z} + \Gamma e^{j\beta z}) = V_o^+ e^{-j\beta z}(1+\Gamma(z)) \quad \text{(for lossless lines)}$$

Current Wave

$$\tilde{I}(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z} = \frac{V_o^+}{Z_o}(e^{-\gamma z} - \Gamma e^{\gamma z}) = \frac{V_o^+}{Z_o} e^{-\gamma z}(1-\Gamma(z)) \quad \text{(general form)}$$

$$\tilde{I}(z) = I_o^+ e^{-j\beta z} + I_o^- e^{j\beta z} = \frac{V_o^+}{Z_o}(e^{-j\beta z} - \Gamma e^{j\beta z}) = \frac{V_o^+}{Z_o} e^{-j\beta z}(1-\Gamma(z)) \quad \text{(for loesslesse line)}$$

Wavelength

$\lambda = \frac{u_p}{f}$, where u_p is the propagation velocity; $\beta = \frac{2\pi}{\lambda}$ and $u_p = \frac{\omega}{\beta}$

Propagation Constant

$$\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')} \quad \text{(general form)}$$

$$\gamma = j\beta = j\omega\sqrt{L'C'}, \quad \alpha = 0 \quad \text{(for lossless lines)}$$

Characteristic Impedance

$$Z_o = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \qquad Z_o = \sqrt{\frac{L'}{C'}} = Z_o^o \quad \textbf{(for loesslesseline)}$$

Phase Velocity

$$u_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{(For lossless or low-loss lines)}$$

Total Wave Impedance at location *z*:

$$\tilde{Z}(z) = \frac{\tilde{V}(z)}{\tilde{I}(z)} = Z_o \frac{1+\Gamma(z)}{1-\Gamma(z)}$$

Reflection Coefficient

$$\Gamma(z) = \frac{V_o^-}{V_o^+} e^{2\gamma z} = \Gamma e^{2\gamma z} \quad \text{(General form)}$$

$$\Gamma(z) = \Gamma e^{j2\beta z} \quad \text{(For lossless lines)}$$

Normalized impedance

$$\left(z_L = \frac{Z_L}{Z_o} \right)$$

Reflection from load:

$$\Gamma = \Gamma_L = \frac{V_o^-}{V_o^+} = -\frac{I_o^-}{I_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta_\gamma}$$

Reflection from generator:

$$\Gamma_g = \frac{Z_g - Z_o}{Z_g + Z_o}$$

Input Impedance (*z* = - *l*)

$$Z_{in} = Z_o \frac{1+\Gamma_{in}}{1-\Gamma_{in}}, \qquad \Gamma_{in} = \Gamma e^{-2\gamma l} = |\Gamma| e^{j\theta_\Gamma - 2\gamma l}$$

For lossless lines: $\Gamma_{in} = \Gamma e^{-j2\beta l} = |\Gamma| e^{-j(2\beta l - \theta_\Gamma)}$

Input impedance of a **lossless transmission line** of length ***d*** with load Z_L,

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\beta d}{Z_o + jZ_L \tan\beta d} = Z_o \frac{z_L + j\tan\beta d}{1 + jz_L \tan\beta d}$$

Forward wave amplitude:

$$V_0^+ = \tilde{V}_g \frac{Z_{in}}{Z_g + Z_{in}} \frac{1}{e^{\gamma l} + \Gamma e^{-\gamma l}} = \tilde{V}_g \frac{Z_{in}}{Z_g + Z_{in}} \frac{e^{-\gamma l}}{1 + \Gamma_l} \quad \text{(General form)}$$

$$V_0^+ = \tilde{V}_g \frac{Z_{in}}{Z_g + Z_{in}} \frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} \quad \text{(For lossless lines)}$$

Magnitude of voltage: $\left|\tilde{V}(d)\right| = |V_o^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_\gamma)\right]^{1/2}$

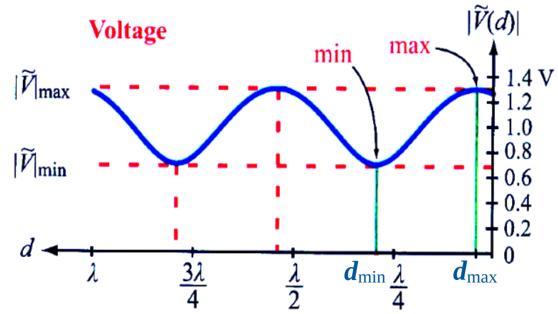
(*Lossless T-line*)

Standing Wave Ratio:

$$S = SWR = \frac{|V|_{\max}}{|V|_{\min}} = \frac{|I|_{\max}}{|I|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \geq 1$$

$$|V|_{\max} = |V_o^+|(1+|\Gamma|)$$

$$|V|_{\min} = |V_o^+|(1-|\Gamma|)$$



Average Power:

$$P_{av}(z) = \frac{1}{2}\Re e\{\tilde{V}(z)\tilde{I}^*(z)\} = \frac{|\tilde{V}(z)|^2}{2}\Re e\{\frac{1}{Z^*(z)}\};$$

For lossless line: $P_{av} = \frac{|V_o^+|^2}{2Z_o}(1 - |\Gamma|^2)$

<u>Low-Loss lines</u> (*G′*≈0 and *R′*<< ω*L′*):

$$Z_o \approx \sqrt{\frac{L'}{C'}}\left(1 - j\frac{R'}{2\omega L'}\right) = Z_o^o\left(1 - j\frac{R'}{2\omega L'}\right); \quad Z_o^o = \sqrt{\frac{L'}{C'}}$$

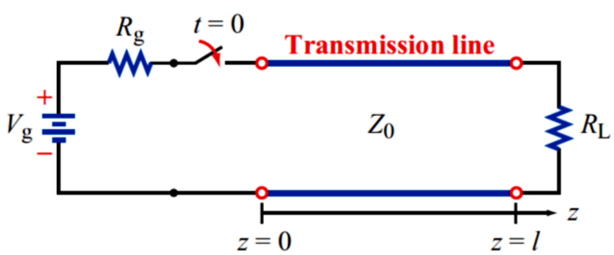
$$\gamma = \alpha + j\beta \qquad \alpha \approx \frac{R'}{2Z_o^o} \quad \text{and} \quad \beta \approx \omega\sqrt{L'C'}$$

<u>For distortionless line</u>: G′/C′ = R′/L′

$$Z_o = \sqrt{\frac{L'}{C'}} = Z_o^o$$

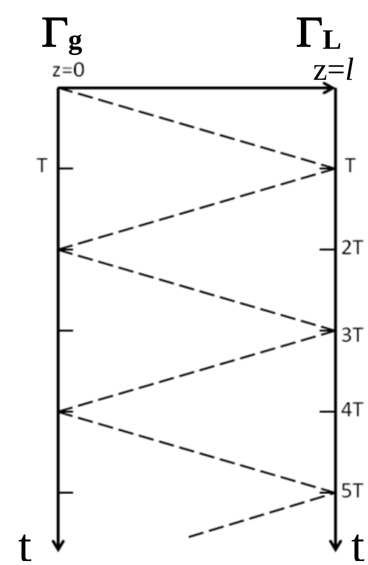
$$\gamma = \alpha + j\beta \quad \alpha = \frac{R'}{Z_o^o} \quad \text{and} \quad \beta = \omega\sqrt{L'C'}$$

<u>Transient Response:</u>



$$T = \frac{l}{u_p}$$

$$V_1^+ = \frac{V_g Z_o}{Z_g + Z_o}$$



<u>Impedance Matching</u> (Smith Chart):

$$z_l = \frac{1+\Gamma_l}{1-\Gamma_l} \qquad y_l = \frac{1}{z_l} = \frac{1-\Gamma_l}{1+\Gamma_l} \qquad \Gamma_l = \Gamma e^{-2\beta l}$$

<u>Goal:</u> $\Gamma_{in} = 0$, $z_{in} = 1$ ($y_{in} = 1$)

Short-circuited: $Z_{in} = jZ_o\tan\beta l$, or $z_{in} = j\tan\beta l$

Open-circuited: $Z_{in} = -jZ_o\cot\beta l$, or $z_{in} = -j\cot\beta l$