# Electric Fields

## MAXWELL'S EQUATIONS

**Differential Form**

$$\nabla \bullet \vec{D} = \rho$$

$$\nabla \times \vec{E} = 0$$

**Integral Form**

$$\oint_S \vec{D} \bullet d\vec{s} = Q_{encl.}$$

$$\oint_C \vec{E} \bullet d\vec{l} = 0$$

**Coulomb's Law**

$$d\vec{E} = \frac{1}{4\pi\varepsilon}\frac{dq}{R^2}\hat{R}$$

## ELECTRICAL POTENTIAL

$$Q_{encl.} = \int_V \rho_v dv \quad \text{or} \quad \int_S \rho_S ds \quad \text{or} \quad \int_l \rho_l dl$$

$$\vec{E} = -\nabla V$$

$$V = -\int_{\infty}^{P} \vec{E} \bullet d\vec{l} \qquad V_2 - V_1 = -\int_{P_1}^{P_2} \vec{E} \bullet d\vec{l}$$

$\nabla^2 V = -\dfrac{\rho}{\varepsilon}$, Poisson's Equation

$$dV = \frac{1}{4\pi\varepsilon}\frac{dq}{R}, \quad V = \frac{1}{4\pi\varepsilon}\int_{v'} \frac{\rho(\vec{R}')}{\left|\vec{R} - \vec{R}'\right|} dv'$$

**Finite Difference Method to Laplace's Equation**

$$V_{i,j} = \frac{V_{i-1,j} + V_{i+1,j} + V_{i,j-1} + V_{i,j+1}}{4}$$

## BOUNDARY CONDITIONS

**General**

$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \bullet (\vec{D}_1 - \vec{D}_2) = \rho_s$$

I.e., $E_{1t} = E_{2t} \qquad D_{1n} - D_{2n} = \rho_s$

**Dielectric-Dielectric**

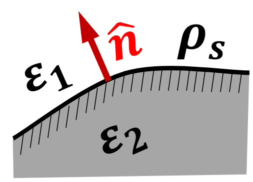
$$E_{1t} = E_{2t}$$

$$D_{1n} = D_{2n}$$

**Conductor-Dielectric**

$\vec{E} = 0$ in conductor

$D_n = \rho_s$ in dielectric




## FORCE

$$\vec{F} = q\vec{E}$$

$$\vec{F} = \hat{a}_{12}\frac{q_1 q_2}{4\pi\varepsilon_o R^2}$$

$$\vec{F} = -\nabla W_e$$

## ENERGY

$$W_e = \frac{1}{2}CV^2 = \frac{1}{2}\int_V \vec{D} \bullet \vec{E} dv = \int_V \frac{1}{2}\varepsilon E^2 dv$$

$$w_e = \frac{1}{2}\vec{D} \bullet \vec{E} = \frac{1}{2}\varepsilon E^2$$

## CAPACITANCE & RESISTANCE

$C = \dfrac{Q}{V}$, $Q$ : Charge on either conductor

V : Voltage difference between conductors

$C = \dfrac{\varepsilon S}{d}$ for parallel plates

$R = \dfrac{V}{I}$; $R = \dfrac{l}{\sigma S}$ for linear resistor

## ELECTRIC CIRCUITS

$V = RI, \quad \vec{J} = \sigma\vec{E}$

## MATERIALS: $\varepsilon$, $\sigma$

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_r\varepsilon_o\vec{E} = \varepsilon_o\vec{E} + \vec{P}$$

$$\vec{P} = \varepsilon_o\chi_e\vec{E}, \quad \varepsilon = \varepsilon_o(1 + \chi_e)$$

$$\varepsilon_o = 8.854 \cdot 10^{-12} = \frac{1}{36\pi} \cdot 10^{-9}\,\text{F/m}$$