Magnetic Fields

MAXWELL'S EQUATIONS

Differential Form

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

VECTOR POTENTIAL

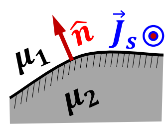
$$\vec{B} = \nabla \times \vec{A} \quad \text{or} \quad \Phi = \int_S \vec{B} \bullet d\vec{s} = \oint_C \vec{A} \bullet d\vec{l}$$

$$\nabla^2 \vec{A} = -\mu \vec{J}\,, \qquad \nabla \bullet \vec{A} = 0$$

$$\vec{A} = \frac{\mu}{4\pi} \int_{v'} \frac{\vec{J}(\vec{R}')}{\left|\vec{R} - \vec{R}'\right|} dv'$$

BOUNDARY CONDITIONS

General	Magnetic-Magnetic
$\hat{\mathbf{n}} \bullet (\vec{B}_1 - \vec{B}_2) = 0$	$B_{1n} = B_{2n}$
$\hat{\mathbf{n}} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$	$H_{1t} = H_{2t}$
$-H_{1t} + H_{2t} = J_s$	



FORCE

$$\vec{F} = q(\vec{u} \times \vec{B}) \quad \text{Newtons per particle}$$

$$d\vec{F} = I\, d\vec{l} \times \vec{B} \quad \text{Force on current wire}$$

$$\vec{F} = -\nabla W \quad \text{Energy gradient}$$

INDUCTANCE

$$L = \frac{\Lambda}{I}, \quad \Lambda = \sum_i \Phi_i = N\Phi, \qquad -V_{emf} = L\frac{dI}{dt}$$

$$L = \frac{N\Phi_m}{I} \quad \text{If all turns enclose the same flux}$$

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2 \Phi_{12}}{I}, \qquad \Phi_{12} = \int_S \vec{B}_1 \bullet d\vec{s}_2$$

$$L = \frac{N^2}{\Re} \quad \text{(magnetic circuit method)}$$

TRANSFORMER

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}, \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Integral Form

$$\oint_S \vec{B} \bullet d\vec{s} = 0$$

$$\oint_C \vec{H} \bullet d\vec{l} = \int_S \vec{J} \bullet d\vec{s} = I_{net}$$

$$\oint_C \vec{E} \bullet d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \bullet d\vec{s}$$

or $\oint_C \vec{E} \bullet d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \bullet d\vec{s} + \oint_C (\vec{u} \times \vec{B}) \bullet d\vec{l}$

Faraday's Law

$$V_{emf} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\int_S \vec{B} \bullet d\vec{s}$$

Biot-Savart Law

$$d\vec{H} = \frac{I}{4\pi} \frac{d\vec{l} \times \hat{R}}{R^2}$$

ENERGY & Force

$$W_m = \frac{1}{2} L I^2 = \frac{1}{2}\int_V \vec{B} \bullet \vec{H} dv = \int_V \frac{1}{2}\mu H^2 dv$$

$$w_m = \frac{1}{2}\vec{B} \bullet \vec{H} = \frac{1}{2}\mu H^2$$

Magnetic Force at Air Gap

$$F_m = \frac{1}{2}\mu_o H_g^2 S = \frac{B_g^2}{2\mu_o} S$$

MAGNETIC CIRCUITS

$$V_{mmf} = NI = \sum_i \Re_i \Phi_i$$

$$\Re_i = \frac{l_i}{\mu_i S_i}$$

MATERIALS

$$\vec{B} = \mu \vec{H} = \mu_r \mu_o \vec{H} = \mu_o (\vec{H} + \vec{M})$$

$$\vec{M} = \chi_m \vec{H}, \quad \mu = \mu_o (1 + \chi_m)$$

$\mu_o = 4\pi \times 10^{-7}$ H/m