Finding the Magnitude of a Transfer Function

- If we want to use the transfer function to calculate the output voltage of a system given an input voltage and H contains complex numbers, we need to find the magnitude of H: $|H(j\omega)|$

$$V_{out} = |H(j\omega)| V_{in}$$

- How do we find $|H(j\omega)|$?

$$|H(j\omega)| = \sqrt{H(-j\omega) H(j\omega)}$$

$|H(j\omega)|$ will always be real: it should never contain "j" and always be ≥ 0

- Example: Transfer function for an RC low-pass filter: $H(j\omega) = \dfrac{1/j\omega C}{R + 1/j\omega C}$

What is $|H(j\omega)|$?

1) $|H(j\omega)| = \sqrt{H(-j\omega)\,H(j\omega)}$

$$= \sqrt{\frac{-1/j\omega C}{R - 1/j\omega C} \times \frac{1/j\omega C}{R + 1/j\omega C}}$$

Properties of "j"

- $j \cdot j = -1$
- $-j \cdot j = 1$
- $(-j)\cdot(-j) = -1$

$$= \sqrt{\frac{-\frac{1}{j^2}\frac{1}{\omega^2 C^2}}{(R - 1/j\omega C)(R + 1/j\omega C)}} \quad (j^2 = -1)$$

$$= \sqrt{\frac{1/\omega^2 C^2}{R^2 - R/j\omega C + R/j\omega C - \frac{1}{j^2}\frac{1}{\omega^2 C^2}}} \quad (j^2 = -1)$$

$$= \sqrt{\frac{1/\omega^2 C^2}{R^2 + 1/\omega^2 C^2}} = \boxed{\frac{1}{\sqrt{\omega^2 C^2 R^2 + 1}} = |H(j\omega)|}$$

In terms of V_{in} and $|H|$:

$$\boxed{V_{out} = \frac{V_{in}}{\sqrt{(\omega RC)^2 + 1}}}$$