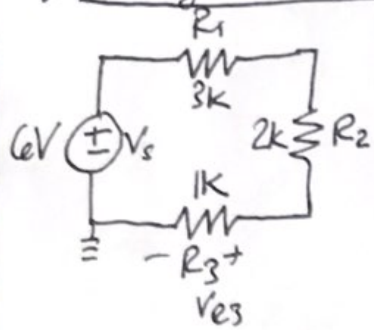
1) Voltage divider

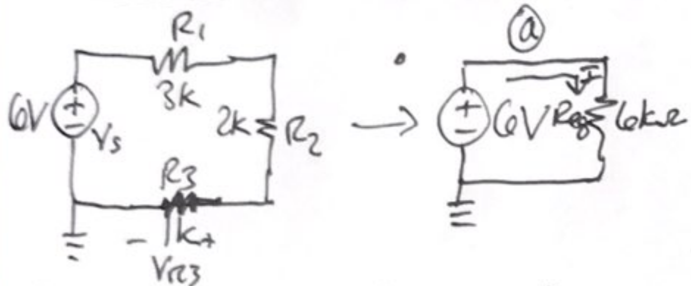


- What is the voltage across R_3 (i.e. V_{R3})? Use a voltage divider

$$\rightarrow V_{R3} = V_s \frac{R_3}{R_1+R_2+R_3}$$

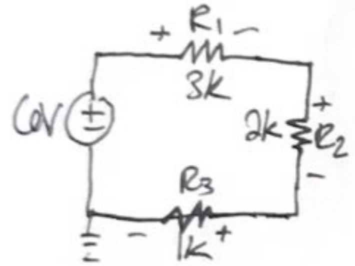
$$= 6V \frac{1k\Omega}{3k\Omega+2k\Omega+1k\Omega} = 6V \frac{1k\Omega}{6k\Omega} = \boxed{1V}$$

2) Circuit Reduction



- What is I? $I = \frac{6V}{6k\Omega} = 1mA$
- What is V_{R3}? $V_{R3} = I R_3 = 1mA \cdot 1k\Omega = \boxed{1V}$

3) KCL, KVL, and Ohm's Law

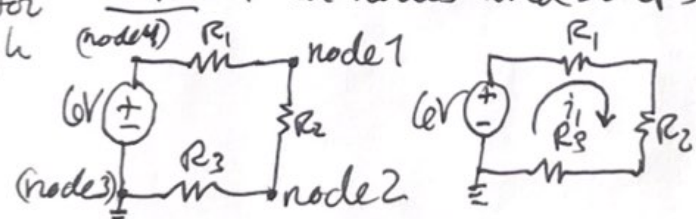


- Solving for voltage across each resistor or current through each resistor.

Step 1) Label components with reference marks "+" and "−" → determines direction of current flow through components

Step 2) Count # of unknowns = # of resistors = # linearly independent eqn's needed to solve the circuit

Step 3) Find nodes and loops



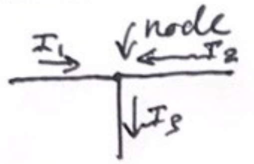
1) loop #1
2) node #1
3) node #2

Step 4) Pick the "best" nodes and loops
→ knowing which to pick will come with practice

Step 5) Use KVL and KCL (Ohm's Law) to write equations in terms of $V_{resistor}$ or $I_{resistor}$ variables.

(Doesn't matter which you choose - just be consistent!)

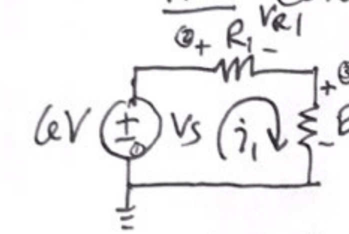
a) My conventions: KCL convention



$$0 = I_1 + I_2 - I_3$$

- Current flowing INTO a node (I_1, I_2) is positive ("+")
- Current flowing OUT of a node (I_3) is negative ("−")

KVL convention

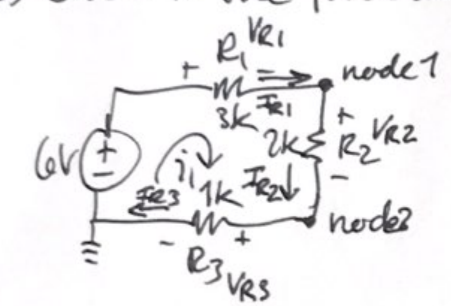


(1) (−) (2) (+) (3) (+)

$$0 = -V_s + V_{R1} + V_{R2}$$

- going in the direction of the loop, each component's voltage is assigned the sign (from reference marks) that is encountered first (i.e. we encounter the "−" on V_s first, so it gets a "−")

b) back to the problem



- loop #1 (KVL): $0 = V_{R1} + V_{R2} + V_{R3} - 6$ (1)

"Standard form" → constants on one side of "=", variables on the other: $6 = V_{R1} + V_{R2} + V_{R3}$

- node #1 (KCL): $0 = I_{R1} - I_{R2}$
- node #2 (KCL): $0 = I_{R2} - I_{R3}$

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* Want to solve for voltages, so must convert I to V/R via Ohm's Law
  - node #1 (KCL) in voltages: $0 = \frac{V_{R1}}{R_1} - \frac{V_{R2}}{R_2}$
  - node #2 (KCL) in voltages: $0 = \frac{V_{R2}}{R_2} - \frac{V_{R3}}{R_3}$

Put into matrix form: $b = Ax$ → $\underset{\text{sources}}{b} = \underset{\text{our case: coefficients + resistances}}{A}\; \underset{\text{(unknown) voltages}}{x = \text{unknowns}}$

Matrix multiplication: row1 × col1

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+by+cz \\ dx+ey+fz \\ gx+hy+iz \end{bmatrix}$$

(1)
$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ \frac{1}{R_1} & -\frac{1}{R_2} & 0 \\ 0 & \frac{1}{R_2} & -\frac{1}{R_3} \end{bmatrix}}\underset{x}{\begin{bmatrix} V_{R1} \\ V_{R2} \\ V_{R3} \end{bmatrix}} = \underset{b}{\begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}}$$

$V_{R1} + V_{R2} + V_{R3} = 6$

$0 = \frac{V_{R1}}{R_1} - \frac{V_{R2}}{R_2}$

$0 = \frac{V_{R2}}{R_2} - \frac{V_{R3}}{R_3}$

} matrix multiplication reproduces KCL & KVL equations

Plugging in numerical values:

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1/3000 & -1/2000 & 0 \\ 0 & 1/2000 & -1/1000 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_{R1} \\ V_{R2} \\ V_{R3} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}}_{b}$$

Solve with MATLAB or Calculator: $x = A^{-1}b$

$$x = \begin{bmatrix} V_{R1} \\ V_{R2} \\ V_{R3} \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} V$$