

# Intro to ECSE

## Quiz 2 Makeup

Fall 2022

<b>1.</b>	<b>/15</b>
<b>2.</b>	<b>/10</b>
<b>3.</b>	<b>/20</b>
<b>Total</b>	<b>/45</b>

Name \_\_\_\_\_

Notes:

**SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS.** No credit will be given for numbers that appear without justification. Use the backs of pages if there is not enough room on the front.

For partial credit on some questions, you may want to re-draw circuit diagrams as you simplify the circuits.

Many problems can be solved using more than one method. check your answers by using a second method.

At least skim through the entire quiz before you begin and then start with the problems you know best. The proctor will only answer clarification questions where wording is unclear or where there may be errors/typos. No other questions will be responded to.

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**Problem 1 (15 Points): Nodal Analysis**

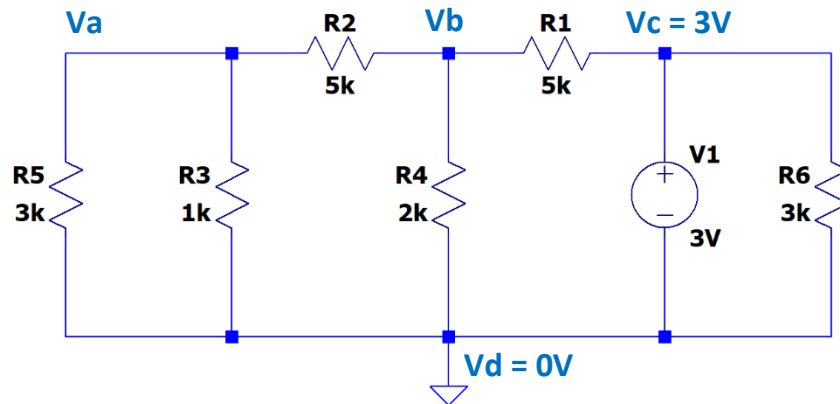


Figure 1

1.1) (2 pts) How many voltage nodes are in this circuit? Label them on the circuit schematic.

There are 4 nodes (1 pt)

Labeled correctly on circuit (1 pt)

1.2) (4 pts) Which of these node voltages are already known (if any)? Write their numerical values on the circuit schematic.

Vc = 3V (1 pt)

Vd = 0V (1 pt)

Labeled correctly on circuit (2 pts)

1.3) (2 pts) How many linearly-independent equations are needed to solve for all unknowns in this circuit? Why?

# of equations = # of nodes - # of voltage sources - 1 = 2 (1 pt)

Explanation (1 pt): There are 4 total nodes. After accounting for voltage sources (1), which each provide a known nodal voltage equal to the individual source voltage, and ground, which provides a known nodal voltage of 0V, only two unknowns are left to solve for, which requires two equations.

1.4) (4 pts) Write down the KCL equation for each unknown node in terms of the nodal voltages you labeled on the schematic in 1.1 and 1.2.

$$\text{At node a: } \frac{V_a}{3k} + \frac{V_a}{1k} + \frac{V_a - V_b}{5k} = 0 \quad (2 \text{ pts})$$

$$\text{At node b: } \frac{V_b - V_a}{5k} + \frac{V_b}{2k} + \frac{V_b - 3}{5k} = 0 \quad (2 \text{ pts})$$

1.5) (3 pts) Write the equations from 1.4 in matrix form.

In standard form:

$$\text{At node a: } V_a \left( \frac{1}{3k} + \frac{1}{1k} + \frac{1}{5k} \right) + V_b \left( -\frac{1}{5k} \right) = 0$$

$$\text{At node b: } V_a \left( -\frac{1}{5k} \right) + V_b \left( \frac{1}{5k} + \frac{1}{2k} + \frac{1}{5k} \right) = \frac{3}{5k}$$

In matrix form:

$$\begin{matrix} & (1 \text{ pt}) & & (1 \text{ pt}) & & (1 \text{ pt}) \\ \left[ \begin{array}{cc} \frac{1}{3k} + \frac{1}{1k} + \frac{1}{5k} & -\frac{1}{5k} \\ -\frac{1}{5k} & \frac{1}{5k} + \frac{1}{2k} + \frac{1}{5k} \end{array} \right] \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{3}{5k} \end{bmatrix} \end{matrix}$$

**Problem 2 (10 pts): Linear and Non-Linear Circuit Elements (Conceptual Questions)**

2.1) (2 pts) With regard to their IV characteristics, explain why a resistor is considered a linear circuit element, but a diode is considered a non-linear circuit element.

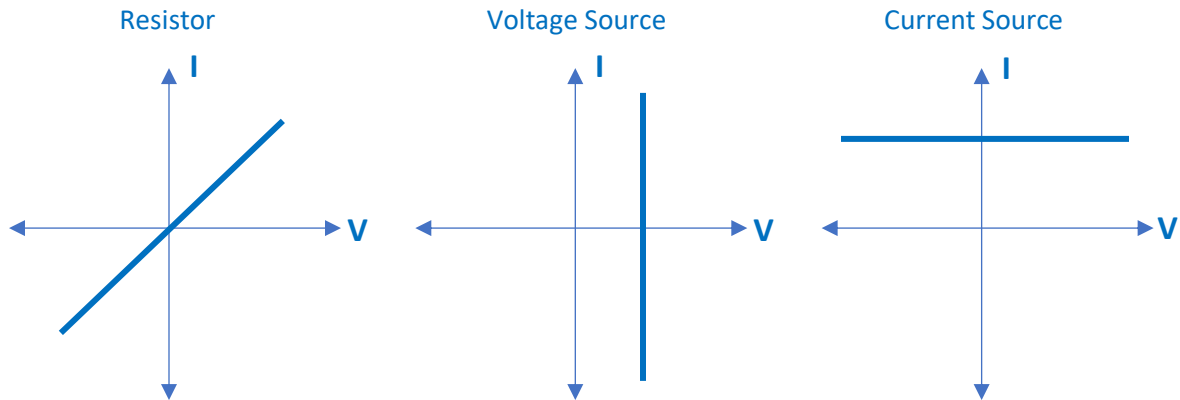
Explanation (2 pts)

A resistor's IV characteristic is a single, straight line with a constant slope (resistance). A diode, on the other hand, has an IV characteristic that is not a straight line and whose slope depends on the particular values of I and V it's evaluated at (changing resistance).

2.2) (4 pts) Identify two different linear circuit elements and draw their IV characteristics.

Choose 2 (1 pt each): resistor **OR** voltage source **OR** current source

Correct IV characteristic for each chosen circuit element above (1 pt each)



2.3) (2 pts) The transfer characteristic ( $V_{out}$  vs.  $V_{in}$ ) below belongs to an inverting amplifier with a gain of -1.5. What are the positive and negative supply voltages for this op-amp circuit?

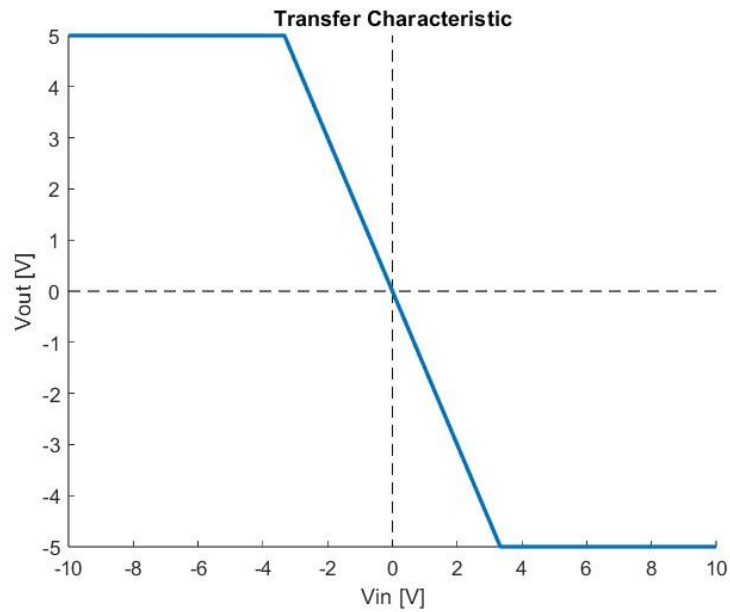


Figure 2

The positive and negative supply voltages are the voltages at which the transfer characteristic saturates.  $V_{s+} = +5$  V and  $V_{s-} = -5$  V (2 pts)

2.4) (2 pts) Under what operating conditions can an op-amp amplifier circuit (such as the one in Q2.3) be treated as a linear circuit element?

Explanation (2 pts)

An op-amp amplifier circuit can be considered to behave linearly when the output voltage does not exceed the supply voltages, so the output voltage does not saturate. This is known as the linear regime of operation for the op-amp circuit.

### Problem 3 (20 Pts): Operational Amplifiers

Shown below is a simple circuit for amplifying an audio signal. Assume that U1, U2, and U3 are ideal op-amps. For all parts of this problem, the positive ( $V_{s+}$ ) and negative ( $V_{s-}$ ) supply voltages are +5V and -5V. V1 is a sinusoidal wave with an amplitude of 1V.

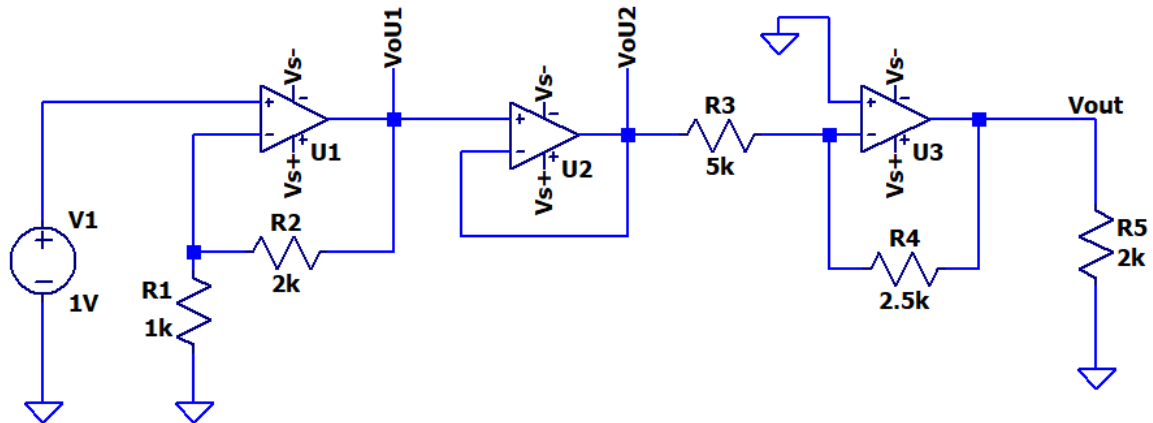


Figure 3

3.1) (3 Pts) What kind of op-amp circuit is stage U1? Write its transfer function both in terms of resistor names (if applicable) and numerically.

Stage U1 is a non-inverting amplifier (1 pt)

The transfer function for a non-inverting amplifier is  $H_1 = (1+R_2/R_1) = 3$  (2 pts)

3.2) (1 Pt) What is the output voltage after stage U1 ( $V_{oU1}$ )?

$V_{in} = 1V$  and the transfer function is  $H_1 = 3$ , so  $V_{oU1} = H_1 * V_{in} = 3V$ . (1 pt)

3.3) (3 Pts) What kind of op-amp circuit is stage U2? Write its transfer function both in terms of resistor names (if applicable) and numerically.

Stage U2 is a voltage follower (1 pt)

The transfer function for a voltage follower is  $H_2 = 1$  (2 pts)

3.4) (1 Pt) What is the output voltage after stage U2 ( $V_{oU2}$ )?

Since a voltage follower outputs the input voltage,  $V_{oU2} = V_{oU1} = 3V$  (1 pt)

3.5) (3 Pts) What kind of op-amp circuit is stage U3? Write its transfer function both in terms of resistor names (if applicable) and numerically.

Stage U3 is an inverting amplifier. (1 pt)

The transfer function of an inverting amplifier is  $H_3 = -R_4/R_3 = -0.5$  (2 pts)

3.6) (1 Pt) What is  $V_{out}$ ?

$V_{out} = V_{oU2} * (-R_4/R_3) = 3V * (-0.5) = -1.5V$  (1 pt)

3.7) (2 Pts) What is the overall transfer function  $H_{total}$  of the entire circuit in terms of resistor names?

$H_{total} = H_1 * H_2 * H_3$  (1 pt)

$H_{total} = (1+R_2/R_1) * (1) * (-R_4/R_3)$  (1 pt)

3.8) (3 Pts) What is the maximum amplitude that  $V_{in}$  can have and still not cause any of the op-amp circuit output voltages to saturate (i.e.  $V_{oU1}$ ,  $V_{oU2}$ , and  $V_{out}$  are not saturated)?

Gain of stage U1 = 3; Max.  $V_{oU1} = \text{Max. } V / 3 = 5V/3 = 1.66 \text{ V}$  at stage input (3 pts)

Gain of stage U2 = 1; Max  $V_{oU2} = \text{Max. } V / 1 = 5V/1 = 5 \text{ V}$  at stage input

Gain of stage U3 = -0.5; Max  $V_{out} = \text{Max. } V / (-0.5) = 5V/(-0.5) = 10 \text{ V}$  at stage input

$V_{in,max} = 1.66 \text{ V}$ , since it's the largest voltage that can be inputted and not saturate U1, which is the limiting stage due to it having the largest gain.

3.9) (3 Pts) Audio systems commonly have an LED indicator that lights up when the audio signal is nearing the maximum voltage that the circuit can output without distorting the signal.

Suppose you are given the following circuit schematic, which compares the input signal ( $V_{in}$ ) to a reference voltage ( $V_{ref}$ ) and are asked to choose resistor values such that the LED (D1) lights up when  $V_{in}$  is greater than  $V_{ref}$  and is off when  $V_{in}$  is less than  $V_{ref}$ .

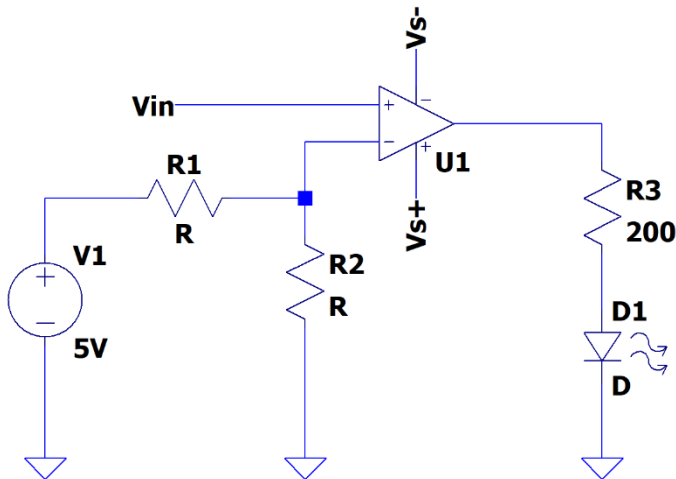


Figure 4

Available Resistors	
2kΩ	22kΩ
37kΩ	48kΩ

Which values for R1 and R2 from the available resistors above would provide a reference voltage of  $V_{ref} = 4.8V$  for the circuit? You may use each resistor value only once.

$$V_{ref} = V_1 \frac{R_2}{R_1 + R_2} \text{ (1 pt)}$$

$$4.8V = 5V \frac{R_2}{R_1 + R_2} \rightarrow 0.96 = \frac{R_2}{R_1 + R_2} \rightarrow 24R_1 = R_2$$



Of the possible resistor values, the only two that satisfy the above relationship are  $R_2 = 48k\Omega$  and  $R_1 = 2k\Omega$ . **(2 pts)**

Checking the result:  $V_{ref} = 5V \frac{48k\Omega}{2k\Omega + 48k\Omega} = (5V) \left( \frac{48k}{50k} \right) = 4.80V$