1) Voltage - unit: Volts (V)

EE language: { "voltage across" an element (i.e. resistor)
"Voltage at" a location (i.e. node)

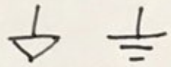
$$\text{Voltage} = \frac{\Delta \text{potential energy}}{\Delta \text{charge}} = \frac{dW}{dq}$$ ✓

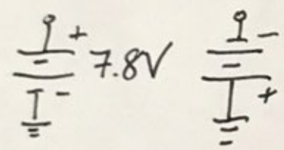
2) Current - unit: Amperes (A)

EE language: { "current through" an element (i.e. resistor)
"current to" } a location (a node)
"current from" }

$$\text{current} = \frac{\Delta \text{charge}}{\Delta \text{time}} = \frac{dq}{dt} = i$$

3) Ground : our reference point for voltage in circuits

Symbols: 



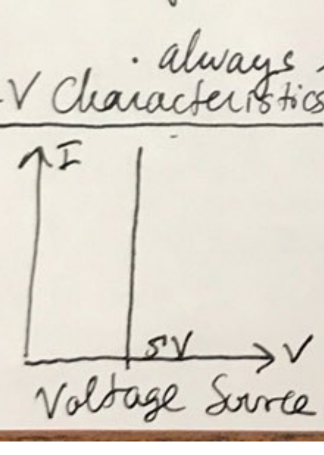
← voltage difference can be negative! $V<0$

4) Power : $P = \frac{\Delta \text{energy}}{\Delta \text{time}} = \frac{dW}{dt}$

EE language: "power applied" to an element/component
"power dissipated" in/by an element/component (i.e. resistor)

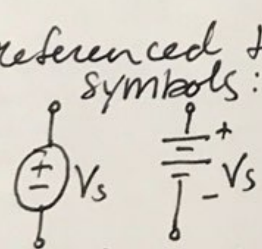
· always referenced to a component

5) I-V Characteristics

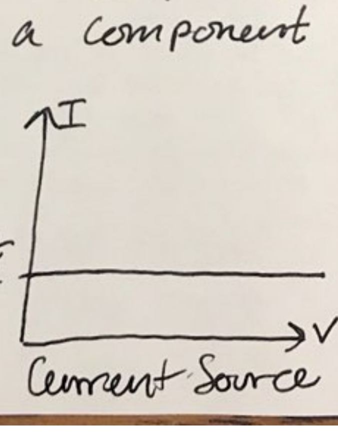


Voltage Source

symbols:

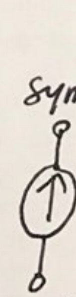


· supplies constant voltage across all currents

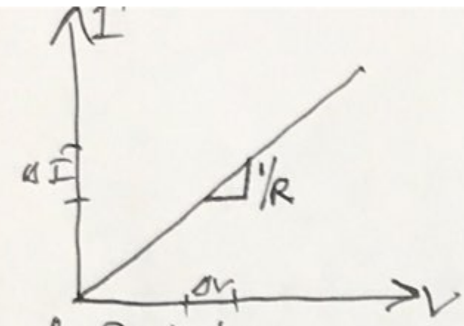


Current Source

symbol:



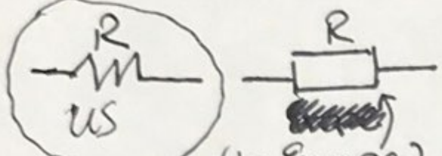
· supplies constant current across all voltages



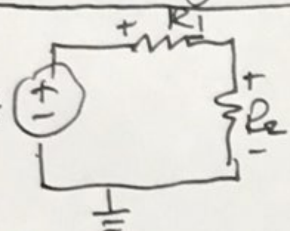
[redacted] $\Delta V = \Delta I R \rightarrow \frac{1}{R} = \frac{\Delta I}{\Delta V}$

slope: $\frac{dy}{dx} = \frac{\Delta I}{\Delta V}$

- Resistor



6) Passive Sign Convention



EE convention

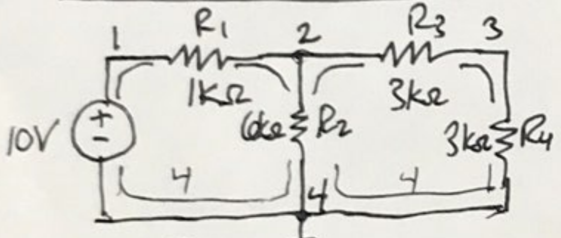
+ ⟶ −

↑ ↑

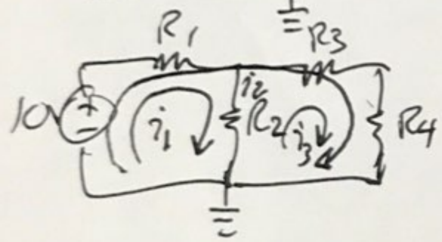
reference marks

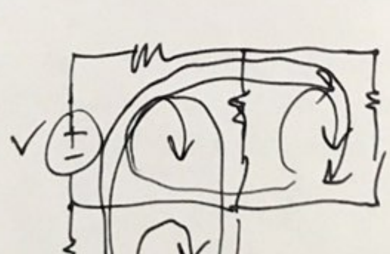
- For KVL & KCL, choose/place reference marks first
- For nodal analysis, choose current direction first

Example Problem



1) What is a node? Any connection between any number of components
2) What is a loop? Any enclosed circle in the circuit



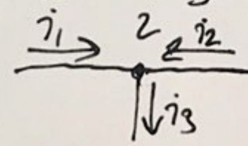


6 loops

7) Kirchoff's Current Law (KCL)

+ conservation of charge → all current entering a node must sum to zero

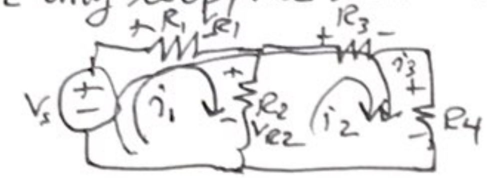
- current pointing away from the node gets a "−" sign, towards gets a "+" sign



$i_1 + i_2 - i_3 = 0$

8) **Kirchoff's Voltage Law (KVL)** – conservation of energy

- in any loop, the sum of the voltages must equal zero



$i_1: -V_S + V_{R1} + V_{R2} = 0$

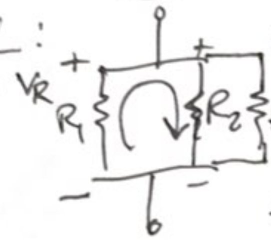
9) **Circuit Reduction**: combining elements to simplify a circuit

Resistors in parallel:

Symbol: $R_1 \| R_2$

$R_{eq}^{par} = R_1 \| R_2$

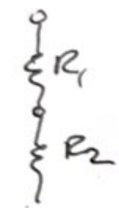
$= \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$



+ Resistors in parallel create a loop containing no other elements
+ Voltage across resistors in parallel is the same
+ $R_{eq}^{parallel} = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

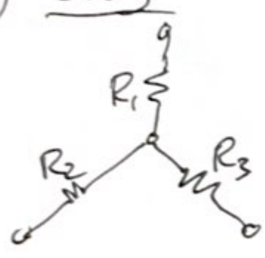
Resistors in Series:

Symbol: $R_1 + R_2$



+ have one common node to which no other elements are connected
+ $R_{eq}^{series} = R_1 + R_2$

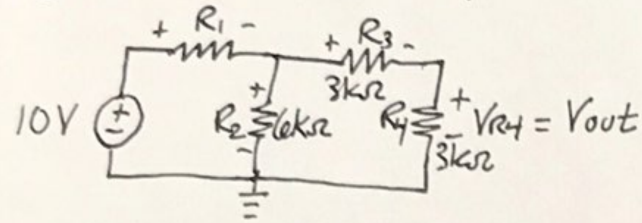
10) **"Quiz"**



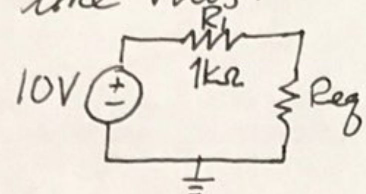
Series? Parallel? **[Neither?]**

- Not series because a 3rd element is connected in between the other 2
- Not parallel because none of the resistors form a loop (they're not connected at both ends)

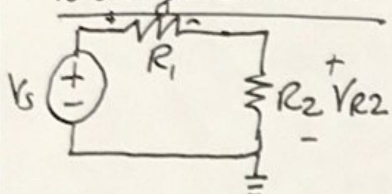
11) Task: Find $V_{out} = V_{R4}$



Goal is to solve this via the method of circuit reduction: We want a circuit that looks like this:



Voltage Divider

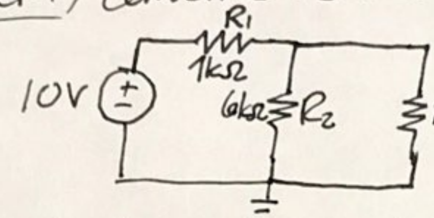


- In a series circuit with a voltage source, the source voltage is divided up between each of the resistors, proportional to their resistance values via the following formula:

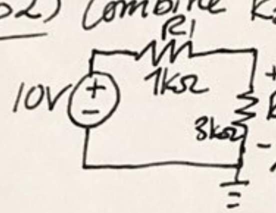
$$V_{out} = V_{in} \frac{R_{out}}{\sum_i R_i} \text{ where}$$

- R_{out} is the resistance over which the voltage of interest is dropped
- $\sum_i R_i$ is the sum of all resistances R_i in the series circuit.
- Example: $V_{R2} = \frac{R_2}{R_1 + R_2}$ in the circuit above

Step 1) Combine R_3 and R_4 (series)

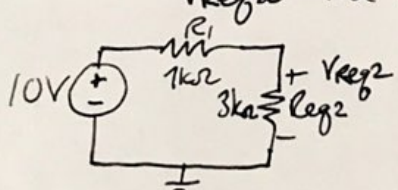


Step 2) Combine R_2 and R_{eq1}



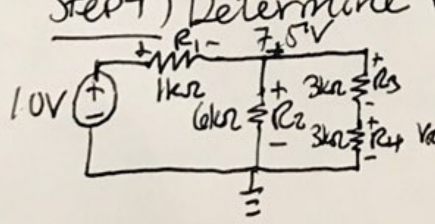
$$R_{eq2} = R_2 \| (R_3 + R_4)$$
$$= 6k\Omega \| (3k\Omega + 3k\Omega)$$
$$= 6k\Omega \| 6k\Omega$$
$$= \frac{1}{\frac{1}{6k\Omega} + \frac{1}{6k\Omega}} = \frac{1}{\left(\frac{2}{6k\Omega}\right)}$$
$$= 3k\Omega$$

Step 3) Determine V_{Req2} via a voltage divider



$$V_{Req2} = \frac{V_{in} R_{eq2}}{R_1 + R_{eq2}}$$
$$= 10V \frac{3k\Omega}{1k\Omega + 3k\Omega}$$
$$= 10V \times 3/4 = 7.5V$$

Step 4) Determine $V_{R4} = V_{out}$



- V_{R4} is given by the voltage divider: $V_{R4} = 7.5V \frac{3k\Omega}{3k\Omega + 3k\Omega}$

$$= 7.5V \times 1/2$$
$$= \underline{3.75V}$$

2) Voltage Divider Derivation

1) Find V_{R2} in terms of R_2 and I

- $V_{R2} = I R_2$

2) Find I in terms of V_S, R_1, and R_2

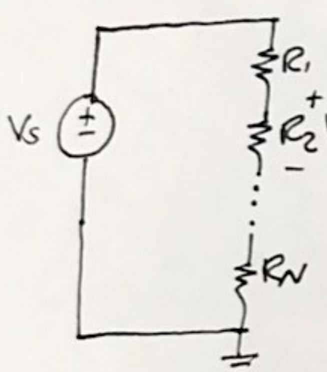
- $I = \dfrac{V_S}{R_1 + R_2}$

3) Substitute the expression for I from (2) into the expression from (1) for V_{R2}:

$$V_{R2} = \left(\frac{V_S}{R_1 + R_2}\right) R_2 = V_S \underbrace{\frac{R_2}{R_1 + R_2}}_{\text{voltage divider formula}}$$

Generalization:

- If we instead have N resistors in series in a circuit:



and we want to find the voltage drop across a particular resistor, say R_2, we use the following formula:

$$V_{R2} = V_S \frac{R_2}{R_1 + R_2 + \ldots + R_N} = V_S \frac{R_2}{\sum_{i=1}^{N} R_i}$$