4 / The Performance of Feedback Control Systems

4.1 INTRODUCTION

The ability to adjust the transient and steady-state performance is a distinct advantage of feedback control systems. In order to analyze and design control systems, we must define and measure the performance of a system. Then, based on the desired performance of a control system, the parameters of the system may be adjusted in order to provide the desired response. Since control systems are inherently dynamic systems, the performance is usually specified in terms of both the time response for a specific input signal and the resulting steady-state error.

The design *specifications* for control systems normally include several time-response indices for a specified input command as well as a desired steady-state accuracy. However, often in the course of any design, the specifications are revised in order to effect a compromise. Therefore, specifications are seldom a rigid set of requirements, but rather a first attempt at listing a desired performance. The effective compromise and adjustment of specifications can be graphically illustrated by examining Fig. 4.1. Clearly, the parameter p may minimize the performance measure M_2 by selecting p as a very small value. However, this results in large measure M_1, an undesirable situation. Obviously, if the performance measures are equally important, the crossover point at p_{min} provides the best compromise. This type of compromise is normally encountered in control system design. It is clear that if the original specifications called for both M_1 and M_2 to be zero, the specifications could not be simultaneously met and the specifications would have to be altered to allow for the compromise resulting with p_{min}.

The specifications stated in terms of the measures of performance indicate to the designer the quality of the system. In other words, the performance measures are an answer to the question: How well does the system perform the task it was designed for?

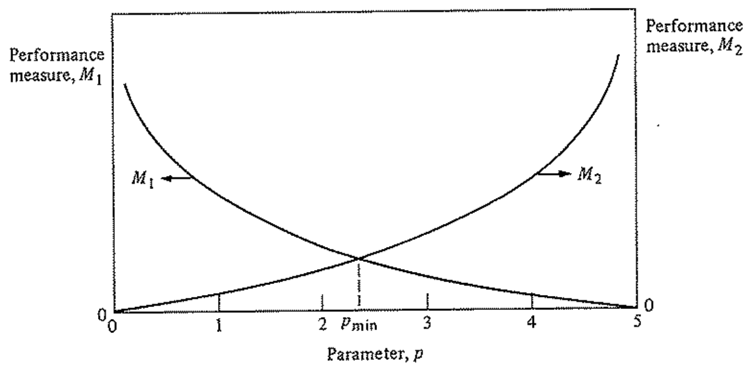

Fig. 4.1. Two performance measures vs. parameter p.

4.2 TIME-DOMAIN PERFORMANCE SPECIFICATIONS

The time-domain performance specifications are important indices since control systems are inherently time-domain systems. That is, the system transient or time performance is the response of prime interest for control systems. It is necessary to determine initially if the system is stable by utilizing the techniques of ensuing chapters. If the system is stable, then the response to a specific input signal will provide several measures of the performance. However, since the actual input signal of the system is usually unknown, a standard *test input signal* is normally chosen. This approach is quite useful since there is a reasonable correlation between the response of a system to a standard test input and the system's ability to perform under normal operating conditions. Furthermore, using a standard input allows the designer to compare several competing designs. Also, many control systems experience input signals very similar to the standard test signals.

The standard test input signals commonly used are (1) the step input, (2) the ramp input, and (3) the parabolic input. These inputs are shown in Fig. 4.2. The equations representing these test signals are given in Table 4.1, where the Laplace transform can be obtained by using Table 2.5. The ramp signal is the integral of the step input, and the parabola is simply the integral of the ramp input. A *unit impulse*

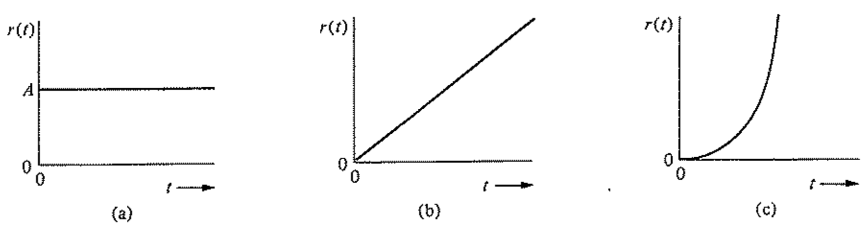

Fig. 4.2. Test input signals.

Table 4.1

Test signal	$r(t)$	$R(s)$
Step	$r(t) = A,\ t > 0$ $= 0,\ t < 0$	$R(s) = A/s$
Ramp	$r(t) = At,\ t > 0$ $= 0,\ t < 0$	$R(s) = A/s^2$
Parabolic	$r(t) = At^2,\ t > 0$ $= 0,\ t < 0$	$R(s) = 2A/s^3$

function is also useful for test signal purposes. The unit impulse is based on a rectangular function $f_\epsilon(t)$ such that

$$f_\epsilon(t) = \begin{cases} 1/\epsilon, & 0 \leq t \leq \epsilon, \\ 0, & t > \epsilon, \end{cases}$$

where $\epsilon > 0$. As ϵ approaches zero, the function $f_\epsilon(t)$ approaches the impulse function $\delta(t)$, which has the following properties:

$$\int_0^\infty \delta(t)\,dt = 1,$$

$$\int_0^\infty \delta(t-a)g(t) = g(a). \tag{4.1}$$

The impulse input is useful when one considers the convolution integral for an output $c(t)$ in terms of an input $r(t)$, which is written as

$$c(t) = \int_0^t g(t-\tau)r(\tau)\,d\tau$$
$$= \mathcal{L}^{-1}\{G(s)R(s)\}. \tag{4.2}$$

This relationship is shown in block diagram form in Fig. 4.3. Clearly, if the input is an impulse function of unit amplitude, we have

$$c(t) = \int_0^t g(t-\tau)\,\delta(0)\,d\tau. \tag{4.3}$$

The integral only has a value at $\tau = 0$, and therefore

$$c(t) = g(t),$$

the impulse response of the system $G(s)$. The impulse response test signal can often

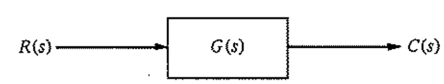

Fig. 4.3. Open-loop control system.

be used for a dynamic system by subjecting the system to a large amplitude, narrow width pulse of area A.

The standard test signals are of the general form

$$r(t) = t^n, \tag{4.4}$$

and the Laplace transform is

$$R(s) = \frac{n!}{s^{n+1}}. \tag{4.5}$$

Clearly, one may relate the response to one test signal to the response of another test signal of the form of Eq. (4.4). The step input signal is the easiest to generate and evaluate and is usually chosen for performance tests.

Initially, let us consider a single-loop second-order system and determine its response to a unit step input. A closed-loop feedback control system is shown in Fig. 4.4. The closed-loop output is

$$\begin{aligned} C(s) &= \frac{G(s)}{1 + G(s)} R(s) \\ &= \frac{K}{s^2 + ps + K} R(s). \end{aligned} \tag{4.6}$$

Utilizing the generalized notation of Section 2.4, we may rewrite Eq. (4.6) as

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s). \tag{4.7}$$

With a unit step input, we obtain

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}, \tag{4.8}$$

for which the transient output, as obtained from the Laplace transform table in Appendix A, is

$$c(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta t + \theta), \tag{4.9}$$

where $\beta = \sqrt{1 - \zeta^2}$ and $\theta = \tan^{-1}\beta/\zeta$. The transient response of this second-order system for various values of the damping ratio ζ is shown in Fig. 4.5. As ζ

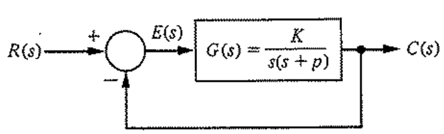

Fig. 4.4. Closed-loop control system.

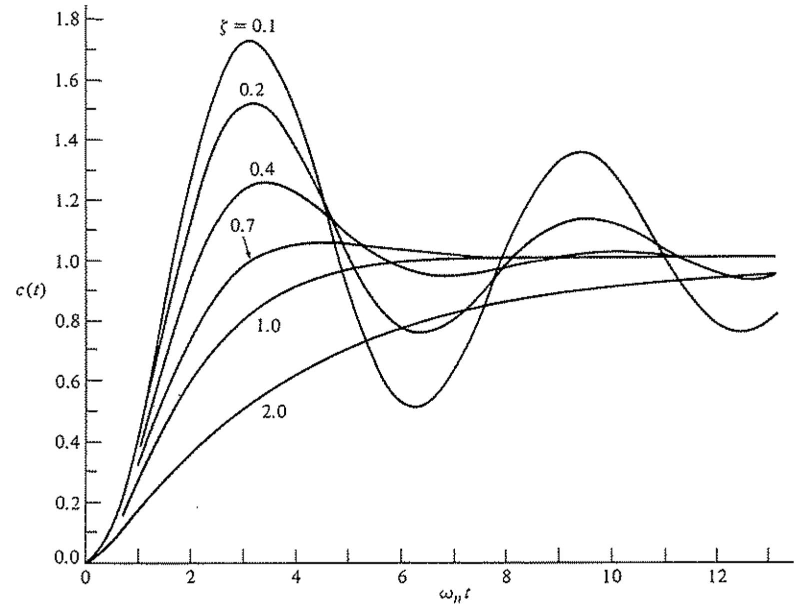

Fig. 4.5. Transient response of a second-order system (Eq. 4.9) for a step input.

decreases, the closed-loop roots approach the imaginary axis and the response becomes increasingly oscillatory.

The Laplace transform of the unit impulse is $R(s) = 1$, and therefore the output for an impulse is

$$C(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \tag{4.10}$$

which is $T(s) = C(s)/R(s)$, the transfer function of the closed-loop system. The transient response for an impulse function input is then

$$c(t) = \frac{\omega_n}{\beta} e^{-\zeta\omega_n t} \sin \omega_n \beta t, \tag{4.11}$$

which is simply the derivative of the response to a step input. The impulse response of the second-order system is shown in Fig. 4.6 for several values of the damping ratio, ζ. Clearly, one is able to select several alternative performance measures from the transient response of the system for either a step or impulse input.

Standard performance measures are usually defined in terms of the step response of a system as shown in Fig. 4.7. The swiftness of the response is measured by the rise time T_r and the peak time. For underdamped systems with an overshoot, the 0–100% rise time is a useful index. If the system is overdamped, then the peak time is not defined and the 10–90% rise time, T_{r_1}, is normally used.

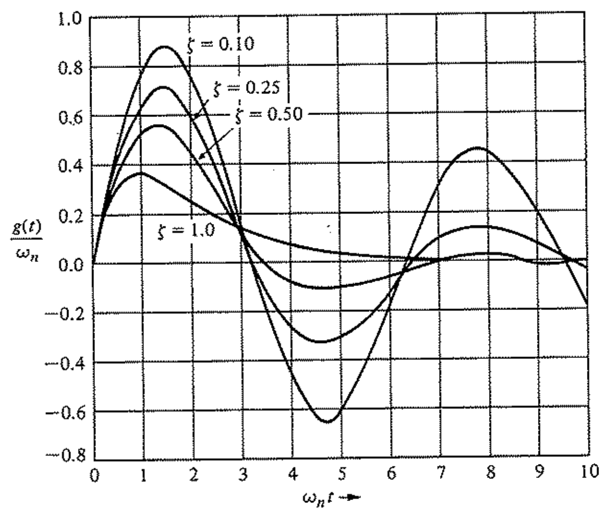

Fig. 4.6. Response of a second-order system for an impulse function input.

The similarity with which the actual response matches the step input is measured by the percent overshoot and settling time T_s. The *percent overshoot*, P.O., is defined as

$$\text{P.O.} = \frac{M_{p_t} - 1}{1} \times 100\% \tag{4.12}$$

for a unit step input, where M_{p_t} is the peak value of the time response. The *settling time*, T_s, is defined as the time required for the system to settle within a certain

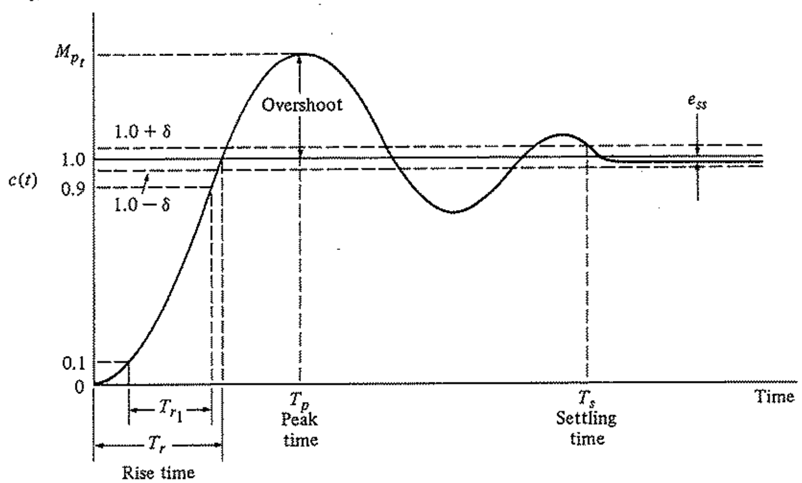

Fig. 4.7. Step response of a control system (Eq. 4.9).

percentage δ of the input amplitude. This band of $\pm\delta$ is shown in Fig. 4.7. For the second-order system with closed-loop damping constant $\zeta\omega_n$, the response remains within 2% after four time constants, or

$$T_s = 4\tau = \frac{4}{\zeta\omega_n}. \tag{4.13}$$

Therefore, we will define the settling time as four time constants of the dominant response. Finally, the steady-state error of the system may be measured on the step response of the system as shown in Fig. 4.7.

Therefore, the transient response of the system may be described in terms of
(1) the swiftness of response, T_r and T_p;
(2) the closeness of the response to the desired M_{p_t} and T_s.

As nature would have it, these are contradictory requirements and a compromise must be obtained. In order to obtain an explicit relation for M_{p_t} and T_p as a function of ζ, one can differentiate Eq. (4.9) and set it equal to zero. Alternatively, one may utilize the differentiation property of the Laplace transform which may be written as

$$\mathcal{L}\left\{\frac{dc(t)}{dt}\right\} = sC(s)$$

when the initial value of $c(t)$ is zero. Therefore, we may acquire the derivative of $c(t)$ by multiplying Eq. (4.8) by s and thus obtaining the right side of Eq. (4.10). Taking the inverse transform of the right side of Eq. (4.10) we obtain Eq. (4.11), which is equal to zero when $\omega_n\beta t = \pi$. Therefore we find that the peak time relationship for this second-order system is

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}, \tag{4.14}$$

and the peak response as

$$M_{p_t} = 1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}. \tag{4.15}$$

Therefore, the percent overshoot is

$$\text{P.O.} = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}. \tag{4.16}$$

The percent overshoot vs. the damping ratio ζ is shown in Fig. 4.8. Also, the normalized peak time, $\omega_n T_p$, is shown vs. the damping ratio ζ in Fig. 4.8. Again, we are confronted with a necessary compromise between the swiftness of response and the allowable overshoot.

The curves presented in Fig. 4.8 are only exact for the second-order system of Eq. (4.8). However, they provide a remarkably good source of data, since many systems possess a dominant pair of roots and the step response can be estimated by utilizing Fig. 4.8. This approach, while an approximation, avoids the evaluation

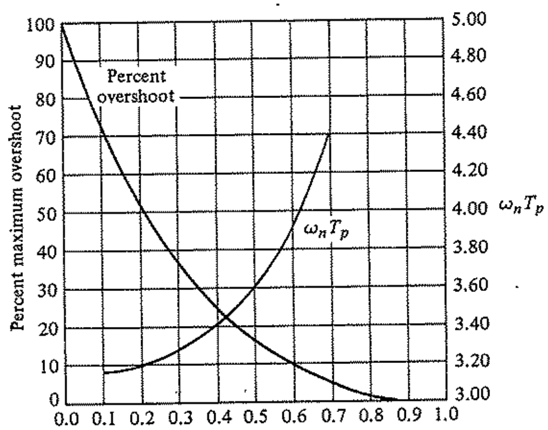

Fig. 4.8. Percent overshoot and peak time versus damping ratio ζ for a second-order system (Eq. 4.8).

of the inverse Laplace transformation in order to determine the percent overshoot and other performance measures. For example, for a third-order system with a closed-loop transfer function

$$T(s) = \frac{1}{(s^2 + 2\zeta s + 1)(\gamma s + 1)}, \tag{4.17}$$

the s-plane diagram is shown in Fig. 4.9. This third-order system is normalized with $\omega_n = 1$. It was ascertained experimentally that the performance as indicated by the percent overshoot, M_{p_t}, and the settling time, T_s, was represented by the second-order system curves when [4]

$$|1/\gamma| \geq 10|\zeta\omega_n|.$$

In other words, the response of a third-order system can be approximated by the

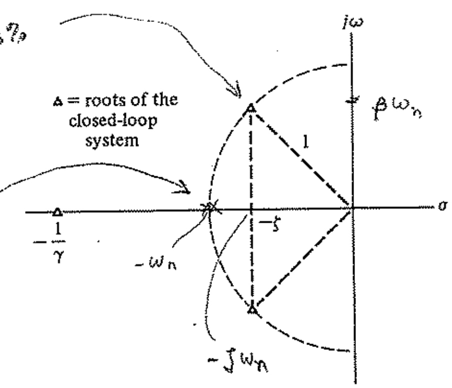

Fig. 4.9. An s-plane diagram of a third-order system.

dominant roots of the second-order system as long as the real part of the dominant roots is less than 1/10 of the real part of the third root.

Using a computer simulation, when $\zeta = .45$ one can determine the response of a system to a unit step input. When $\gamma = 2.25$ we find that the response is overdamped since the real part of the complex poles is $-.45$, while the real pole is equal to $-.444$. The settling time is found via the simulation to be 12.8 seconds. If $\gamma = .90$ or $1/\gamma = 1.11$ is compared to $\zeta\omega_n = .45$ of the complex poles we find that the overshoot is 12% and the settling time is 6.4 seconds. If the complex roots were entirely dominant we would expect the overshoot to be 20% and the settling time to be $4/\zeta\omega_n = 4.4$ seconds.

Also, we must note that the performance measures of Fig. 4.8 are only correct for a transfer function without finite zeros. If the transfer function of a system possesses finite zeros and they are located relatively near the dominant poles, then the zeros will materially affect the transient response of the system [5].

The transient response of a system with one zero and two poles may be affected by the location of the zero [5]. The percent overshoot for a step input as a function of $a/\zeta\omega_n$ is given in Fig. 4.10 for the system transfer function

$$T(s) = \frac{(\omega_n^2/a)(s + a)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The correlation of the time-domain response of a system with the s-plane location of the poles of the closed-loop transfer function is very useful for selecting the specifications of a system. In order to clearly illustrate the utility of the s-plane, let us consider a simple example.

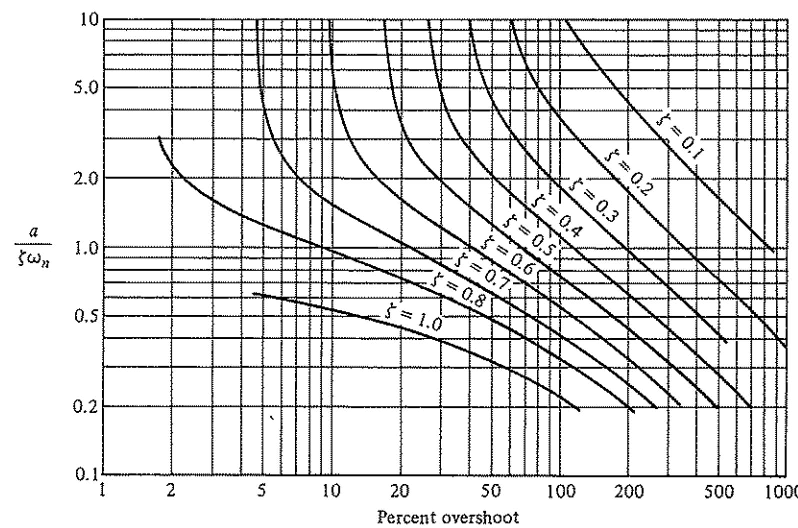

Fig. 4.10. Percent overshoot as a function of ζ and ω_n when a second-order transfer function contains a zero.

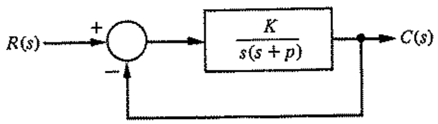

Fig. 4.11. Single-loop feedback control system.

Example 4.1. A single-loop feedback control system is shown in Fig. 4.11. It is desired to select the gain K and the parameter p so that the time-domain specifications will be satisfied. The transient response to a step should be as fast in responding as reasonable and with an overshoot of less than 5%. Furthermore, the settling time should be less than four seconds. The minimum damping ratio ζ for an overshoot of 4.3% is 0.707. This damping ratio is shown graphically in Fig. 4.12. Since the settling time is

$$T_s = \frac{4}{\zeta\omega_n} \leqslant 4 \text{ sec}, \tag{4.18}$$

we require that the real part of the complex poles of $T(s)$ is

$$\zeta\omega_n \geqslant 1.$$

This region is also shown in Fig. 4.12. The region that will satisfy both time-domain requirements is shown cross-hatched on the s-plane of Fig. 4.12. If the closed-loop roots are chosen as the limiting point, in order to provide the fastest response, as r_1 and $\hat{r}_1$, then $r_1 = -1 + j1$ and $\hat{r}_1 = -1 - j1$. Therefore, $\zeta = 1/\sqrt{2}$ and $\omega_n = 1/\zeta = \sqrt{2}$. The closed-loop transfer function is

$$\begin{aligned} T(s) &= \frac{G(s)}{1 + G(s)} = \frac{K}{s^2 + ps + K} \\ &= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \end{aligned} \tag{4.19}$$

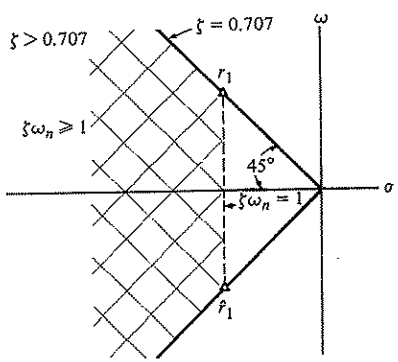

Fig. 4.12. Specifications and root locations on the s-plane.

Therefore, we require that $K = \omega_n^2 = 2$ and $p = 3\zeta\omega_n = 2$. A full comprehension of the correlation between the closed-loop root location and the system transient response is important to the system analyst and designer. Therefore, we shall consider the matter more fully in the following section.

4.3 THE S-PLANE ROOT LOCATION AND THE TRANSIENT RESPONSE

The transient response of a closed-loop feedback control system can be described in terms of the location of the poles of the transfer function. The closed-loop transfer function is written in general as

$$T(s) = \frac{C(s)}{R(s)} = \frac{\Sigma P_i(s)\, \Delta_i(s)}{\Delta(s)}, \tag{4.20}$$

where $\Delta(s) = 0$ is the characteristic equation of the system. For the single-loop system of Fig. 4.11, the characteristic equation reduces to $1 + G(s) = 0$. It is the poles and zeros of $T(s)$ that determine the transient response. However, for a closed-loop system, the poles of $T(s)$ are the roots of the characteristic $\Delta(s) = 0$ and the poles of $\Sigma P_i(s)\, \Delta_i(s)$. The output of a system without repeated roots and a unit step input can be formulated as a partial fraction expansion as

$$C(s) = \frac{1}{s} + \sum_{i=1}^{M} \frac{A_i}{s + \sigma_i} + \sum_{k=1}^{N} \frac{B_k}{s^2 + 2\alpha_k s + (\alpha_k^2 + \omega_k^2)}, \tag{4.21}$$

where the A_i and B_k are the residues. The roots of the system must be either $s = -\sigma_i$ or complex conjugate pairs as $s = -\alpha_k \pm j\omega_k$. Then the inverse transform results in the transient response as a sum of terms as follows:

$$c(t) = 1 + \sum_{i=1}^{M} A_i e^{-\sigma_i t} + \sum_{k=1}^{N} B_k \left(\frac{1}{\omega_k}\right) e^{-\alpha_k t} \sin \omega_k t. \tag{4.22}$$

The transient response is composed of the steady-state output, exponential terms, and damped sinusoidal terms. Obviously, in order for the response to be stable, that is, bounded for a step input, one must require that the real part of the roots, σ_i or α_k, be in the left-hand portion of the s-plane. The impulse response for various root locations is shown in Fig. 4.13. The information imparted by the location of the roots is graphic indeed and usually well worth the effort of determining the location of the roots in the s-plane.

4.4 THE STEADY-STATE ERROR OF FEEDBACK CONTROL SYSTEMS

One of the fundamental reasons for using feedback, despite its cost and increased complexity, is the attendant improvement in the reduction of the steady-state error of the system. As was illustrated in Section 3.5, the steady-state error of a stable closed-loop system is usually several orders of magnitude smaller than the error of

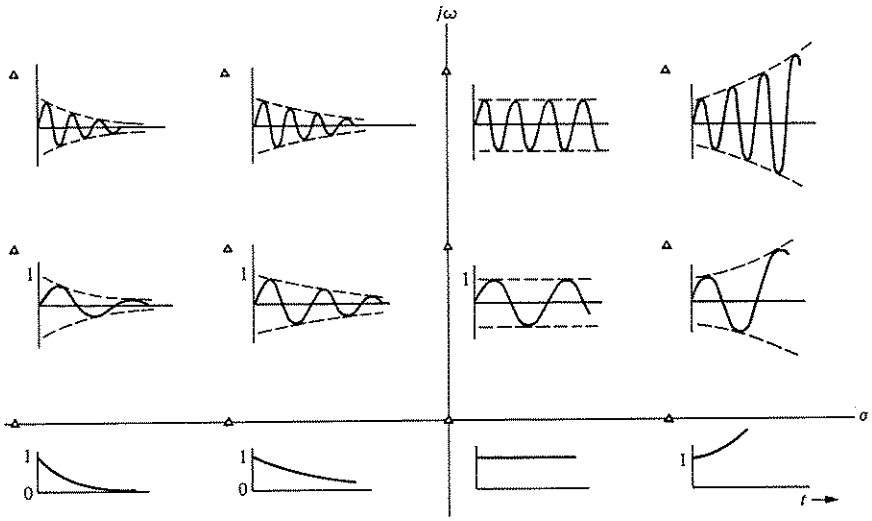

Fig. 4.13. Impulse response for various root locations in the s-plane. (The conjugate root is not shown.)

the open-loop system. The system actuating signal, which is a measure of the system error, is denoted as $E_a(s)$. However, the actual system error is $E(s) = R(s) - C(s)$. Considering the closed-loop feedback system of Fig. 4.14, we have

$$E(s) = R(s) - \frac{G(s)}{1 + GH(s)} R(s) = \frac{[1 + GH(s) - G(s)]}{1 + GH(s)} R(s). \tag{4.23}$$

The system error is equal to the actuating signal when $H(s) = 1$, which is a common situation, and then

$$E(s) = \frac{1}{1 + G(s)} R(s).$$

The steady-state error, when $H(s) = 1$, is then

$$\lim_{t\to\infty} e(t) = e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)}. \tag{4.24}$$

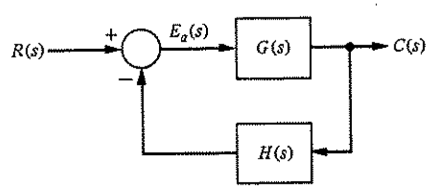

Fig. 4.14. Closed-loop control system.

It is useful to determine the steady-state error of the system for the three standard test inputs for a unity feedback system ($H(s) = 1$.).

Step Input

The steady-state error for a step input is therefore

$$e_{ss} = \lim_{s\to 0} \frac{s(A/s)}{1 + G(s)} = \frac{A}{1 + G(0)}. \tag{4.25}$$

Clearly, it is the form of the loop transfer function $GH(s)$ that determines the steady-state error. The loop transfer function is written in general form as

$$G(s) = \frac{K\prod_{i=1}^{M} (s + z_i)}{s^N \prod_{k=1}^{Q} (s + p_k)}, \tag{4.26}$$

where $\prod$ denotes the product of the factors. Therefore, the loop transfer function as s approaches zero depends upon the number of integrations N. If N is greater than zero, then $G(0)$ approaches infinity and the steady-state error approaches zero. The number of integrations is often indicated by labeling a system with a *type number* which is simply equal to N.

Therefore, for a type zero system, $N = 0$, the steady-state error is

$$\begin{aligned} e_{ss} &= \frac{A}{1 + G(0)} \\ &= \frac{A}{1 + (K\prod_{i=1}^{M} z_i/\prod_{k=1}^{Q} p_k)}. \end{aligned} \tag{4.27}$$

The constant $G(0)$ is denoted by K_p, the position error constant, so that

$$e_{ss} = \frac{A}{1 + K_p}. \tag{4.28}$$

Clearly, the steady-state error for a unit step input with one integration or more, $N \geqslant 1$, is zero since

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} \frac{A}{1 + (K\prod z_i/s^N \prod p_k)} \\ &= \lim_{s\to 0} \frac{As^N}{s^N + (K\prod z_i/\prod p_k)} = 0. \end{aligned} \tag{4.29}$$

Ramp Input

The steady-state error for a ramp (velocity) input is

$$e_{ss} = \lim_{s\to 0} \frac{s(A/s^2)}{1 + G(s)} = \lim_{s\to 0} \frac{A}{s + sG(s)} = \lim_{s\to 0} \frac{A}{sG(s)}. \tag{4.30}$$

Again, the steady-state error depends upon the number of integrations N. For a type zero system, $N = 0$, the steady-state error is infinite. For a type one system, $N = 1$, the error is

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} \frac{A}{s\{[K\prod(s + z_i)]/[s\prod(s + p_k)]\}} \\ &= \frac{A}{(K\prod z_i/\prod p_k)} = \frac{A}{K_v}, \end{aligned} \tag{4.31}$$

where K_v is designated the velocity error constant. When the transfer function possesses two or more integrations, $N \geq 2$, we obtain a steady-state error of zero.

Acceleration Input

When the system input is $r(t) = At^2/2$, the steady-state error is then

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} \frac{s(A/s^3)}{1 + G(s)} \\ &= \lim_{s\to 0} \frac{A}{s^2 G(s)}. \end{aligned} \tag{4.32}$$

The steady-state error is infinite for one integration; and for two integrations, $N = 2$, we obtain

$$e_{ss} = \frac{A}{K\prod z_i/\prod p_k} = \frac{A}{K_a}, \tag{4.33}$$

where K_a is designated the acceleration constant. When the number of integrations equals or exceeds three, then the steady-state error of the system is zero.

Table 4.2 Summary of Steady-State Errors

Number of integrations in G(s), type number	Input: Step, $r(t) = A$, $R(s) = A/s$	Ramp, At, A/s^2	Parabola, $At^2/2$, A/s^3
0	$e_{ss} = \frac{A}{1 + K_p}$	Infinite	Infinite
1	$e_{ss} = 0$	$\frac{A}{K_v}$	Infinite
2	$e_{ss} = 0$	0	$\frac{A}{K_a}$

Control systems are often described in terms of their type number and the error constants, K_p, K_v, and K_a. Definitions for the error constants and the steady-state error for the three inputs are summarized in Table 4.2. The usefulness of the error constants can be illustrated by considering a simple example.

Example 4.2. An automatic speed control system for an automobile was outlined in Problem 3.6. This system is commonly called cruise control. The block diagram of a specific speed control system is shown in Fig. 4.15. The throttle controller, $G_1(s)$, is

$$G_1(s) = K_1 + K_2/s. \tag{4.34}$$

The steady-state error of the system for a step input when $K_2 = 0$ and $G(s) = K_1$ is therefore

$$e_{ss} = \frac{A}{1 + K_p}, \tag{4.35}$$

where $K_p = K_e K_1$. When K_2 is greater than zero, we have a type one system,

$$G_1(s) = \frac{K_1 s + K_2}{s},$$

and the steady-state error is zero for a step input.

If the speed command was a ramp input, the steady-state error is then

$$e_{ss} = \frac{A}{K_v}, \tag{4.36}$$

where

$$K_v = \lim_{s\to 0} sG_1(s)G(s) = K_2 K_e.$$

The transient response of the automobile to a triangular wave input when $G_1(s) = (K_1 s + K_2)/s$ is shown in Fig. 4.16. The transient response clearly shows the effect of the steady-state error, which may not be objectionable if K_v is sufficiently large.

The error constants, K_p, K_v, and K_a, of a control system describe the ability of a system to reduce or eliminate the steady-state error. Therefore, they are utilized as numerical measures of the steady-state performance. The designer determines the error constants for a given system and attempts to determine methods of

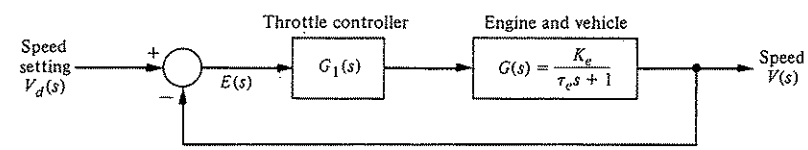

Fig. 4.15. An automobile speed control system.

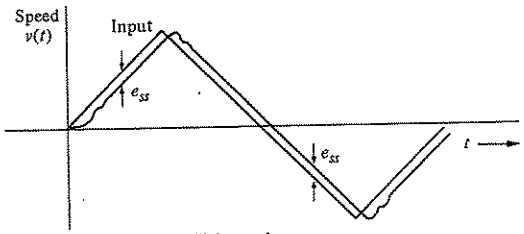

Fig. 4.16. Triangular wave response.

increasing the error constants while maintaining an acceptable transient response. In the case of the automobile speed control system, it is desirable to increase the gain factor K_eK_2 in order to increase K_v and reduce the steady-state error. How- ever, an increase in K_eK_2 results in an attendant decrease in the damping ratio, ζ, of the system and therefore a more oscillatory response to a step input. Again, a compromise would be determined which would provide the largest K_v based on the smallest ζ allowable.

4.5 PERFORMANCE INDICES

An increased amount of emphasis on the mathematical formulation and measurement of control system performance can be found in the recent literature on automatic control. A *performance index* is a quantitative measure of the performance of a system and is chosen so that emphasis is given to the important system speci- fications. Modern control theory assumes that the systems engineer can specify quantitatively the required system performance. Then a performance index can be calculated or measured and used to evaluate the system's performance. A quanti- tative measure of the performance of a system is necessary for the operation of modern adaptive control systems, for automatic parameter optimization of a control system, and for the design of optimum systems.

Whether the aim is to improve the design of a system or to design an adaptive control system, a performance index must be chosen and measured. Then the system is considered an *optimum control system* when the system parameters are adjusted so that the index reaches an extremum value, commonly a minimum value. A performance index, in order to be useful, must be a number that is always positive or zero. Then the best system is defined as the system which minimizes this index.

A suitable performance index is the integral of the square of the error, ISE, which is defined as

$$I_1 = \int_0^T e^2(t)\, dt. \tag{4.37}$$

The upper limit T is a finite time chosen somewhat arbitrarily so that the integral

approaches a steady-state value. It is usually convenient to choose T as the settling time, T_s. The step response for a specific feedback control system is shown in Fig. 4.17(b); and the error, in Fig. 4.17(c). The error squared is shown in Fig. 4.17(d); and the integral of the error squared, in Fig. 4.17(e). This criterion will discriminate between excessively overdamped systems and excessively underdamped systems. The minimum value of the integral occurs for a compromise value of the damping. The performance index of Eq. (4.37) is easily adapted for practical measurements, since a squaring circuit is readily obtained. Furthermore, the squared error is mathematically convenient for analytical and computational purposes.

Another readily instrumented performance criterion is the integral of the absolute magnitude of the error, IAE, which is written as

$$I_2 = \int_0^T |e(t)|\, dt. \tag{4.38}$$

This index is particularly useful for analog computer simulation studies. In order to

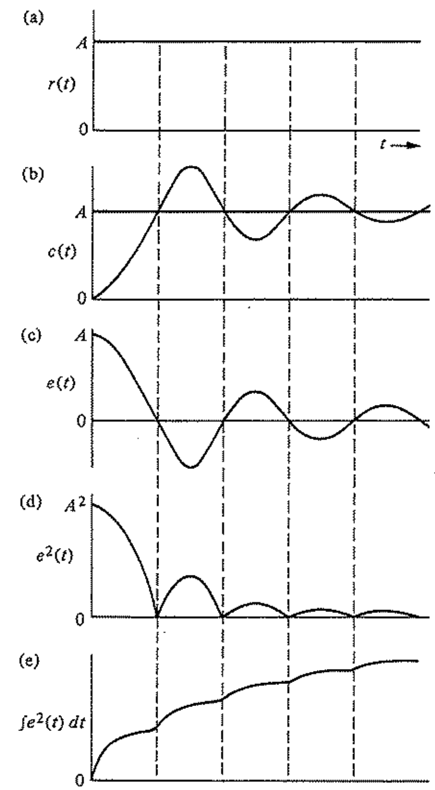

Fig. 4.17. The calculation of the integral squared error.