Intro to ECSE Lecture Notes 4/18/23

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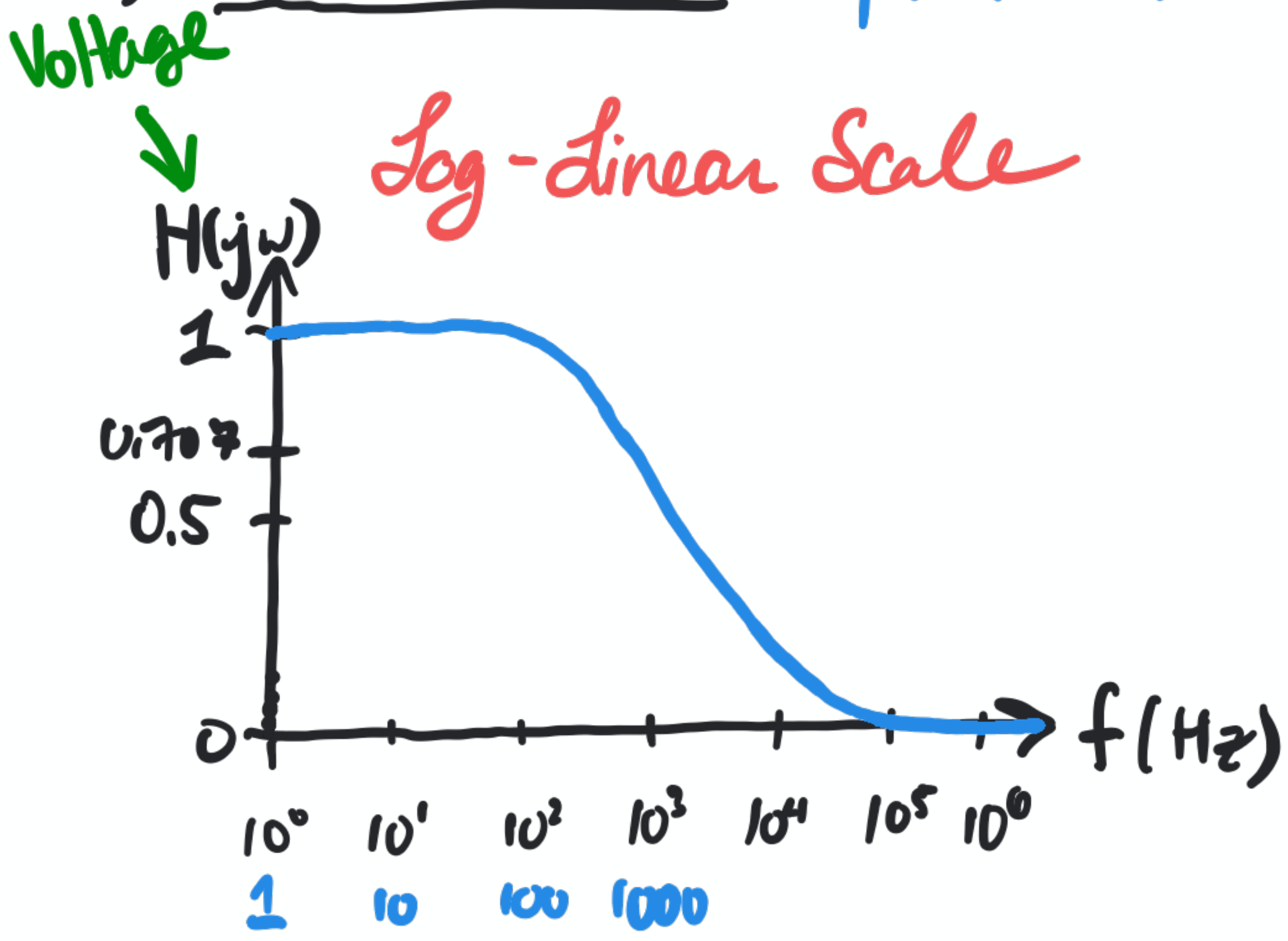
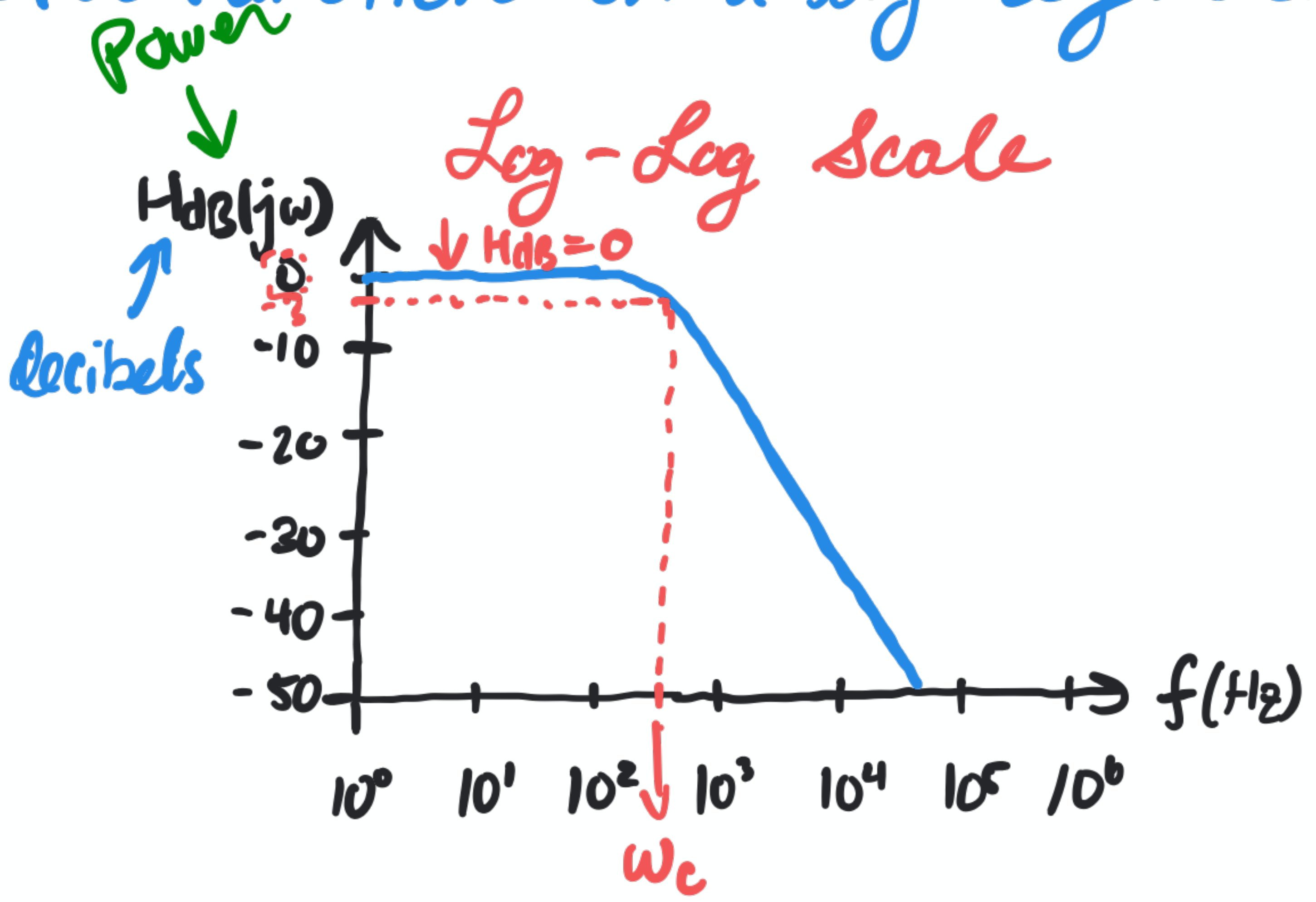
1) Bode Plots : plot of transfer function on a log-log scale





- The y-axis of a Bode plot is given in decibels
- A decibel is a logarithmic power ratio

$$\text{Power ratio} = \frac{P_{out}}{P_{in}}$$

output power

input power

2) Decibels: $\log_{10}$ based representation of a power ratio

$$\text{Power Ratio} = \frac{P_{out}}{P_{in}}$$

- In many cases, these power ratios are very small (10^{-6}) or very large (10^{6}), so it is more convenient to express them in terms of a logarithm:

$$\log_{10}(\text{Power Ratio}) = \log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 1 \text{ bel}$$

Alexander Graham Bell

$1 \text{bel} = 10$ decibels (metric prefix deci $= 0.1$)

$\updownarrow$ (?)

$$(?)\ 0.1\,dB = \log_{10}\left(\frac{P_{out}}{P_{in}}\right) \rightarrow 1\,dB = \underline{10\log_{10}\left(\frac{P_{out}}{P_{in}}\right)} = \text{Power}$$

- What is the power ratio 1/2 in dB?

$$(1/2)_{dB} = 10\log_{10}(1/2) = \underline{-3.01\,dB}$$

on a Bode Plot, this is the location of w_c (1/2 on a linear scale)

- If we want H in Volts/Volt expressed logarithmically

$$H_{V,dB} = 20\log_{10}(H)$$

↑ volts

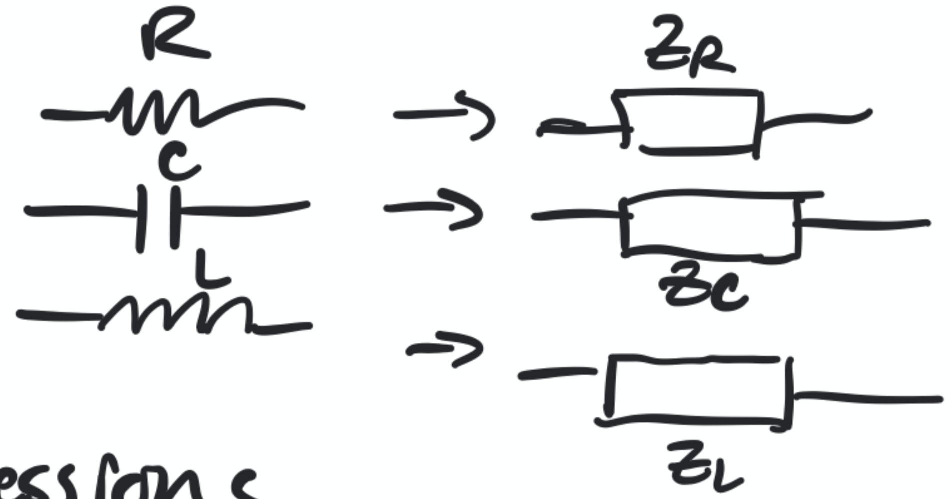
- What is HdB when $\frac{V_{out}}{V_{in}} = 1$?

$$H_{dB} = 10\log_{10}(1) = 0$$

$1 = 10^0 \rightarrow \log_{10}(10^a) = a$

Filter Circuit Steps:

1) Convert circuit elements to impedances



2) Find output voltage using a voltage divider using Z

3) Replace the Z variables with their expressions in terms of their component values ($Z_C = 1/j\omega C$)

4) Find $H = \frac{V_{out}}{V_{in}}$

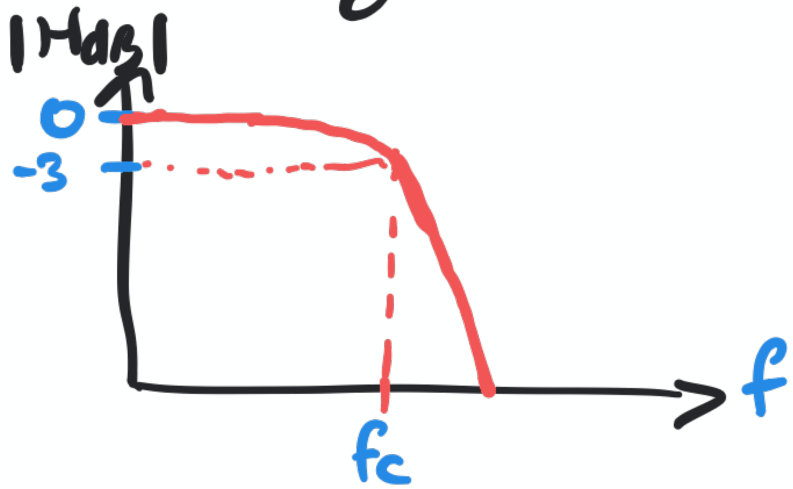
5) Use algebra to get $H(s) = \frac{N(s)}{s + a}$

numerator (may have s)

constant $= \omega_c$

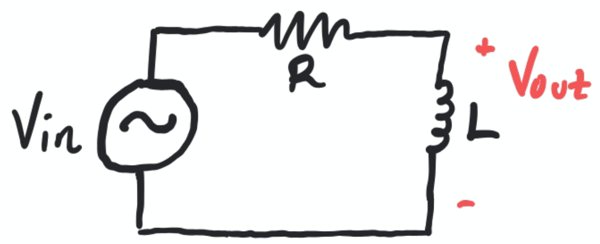
1st order circuits

6) Sketch a Bode Plot by evaluating $H(s \to 0)$ and $H(s \to \infty)$.

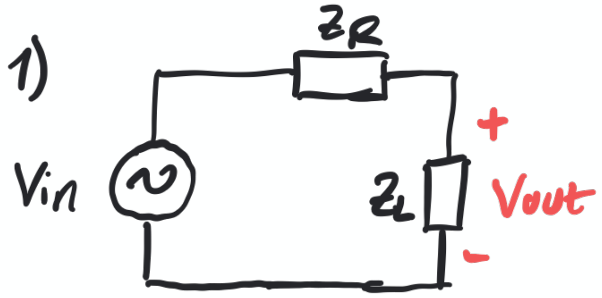


4*) Find $|H| = \sqrt{HH^*}$ and evaluate at different frequencies

Example Problem #1:



1)



2) $V_{out} = V_{in} \dfrac{Z_L}{Z_L + Z_R}$ ✓

$Z_L = sL$
$Z_R = R$

3) $V_{out} = V_{in} \dfrac{sL}{sL + R}$ ✓

4) $H(s) = \dfrac{sL}{sL + R}$ ✓

5) $H(s) = \dfrac{s}{s + R/L}$ ✓ $R/L = \omega_c$ corner frequency

4*) $|H(j\omega)| = \sqrt{H(j\omega) \cdot H(-j\omega)}$

$H(j\omega) = \dfrac{j\omega}{j\omega + R/L}$ $\quad j \cdot j = -1$, $j(-j) = 1$

$|H(j\omega)| = \sqrt{\left(\dfrac{j\omega}{j\omega + R/L}\right) \cdot \left(\dfrac{-j\omega}{-j\omega + R/L}\right)}$

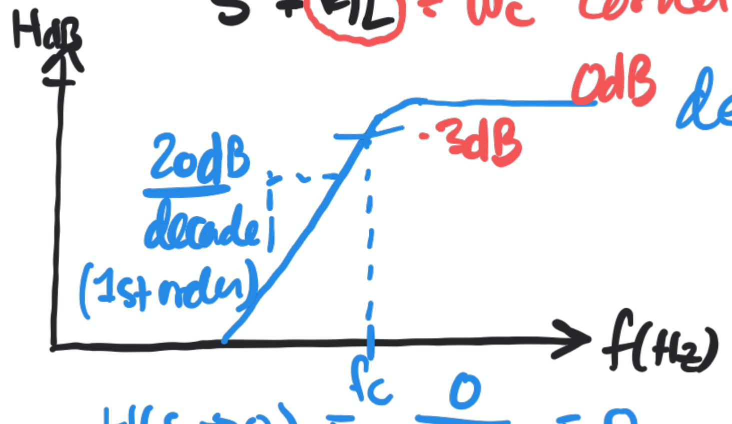
$= \sqrt{\dfrac{\omega^2}{\omega^2 + \cancel{j\omega R/L} - \cancel{j\omega R/L} + (R/L)^2}}$

$= \dfrac{\omega}{\sqrt{\omega^2 + (R/L)^2}} = |H(j\omega)|$

$V_{out} = |H(j\omega)| V_{in}$

Voltage amplitude that would be measured

6)



decade = factor of 10

for example (10 → 100)

inf

$H(s \to 0) = \dfrac{0}{0 + R/L} = 0$

$H(s \to \infty) = \dfrac{\infty}{\infty + R/L} \sim \dfrac{\infty}{\infty} \sim 1$

$f_c = \dfrac{R}{2\pi L}$

LT Spice: $R = 100\Omega$

$L = 5mH$

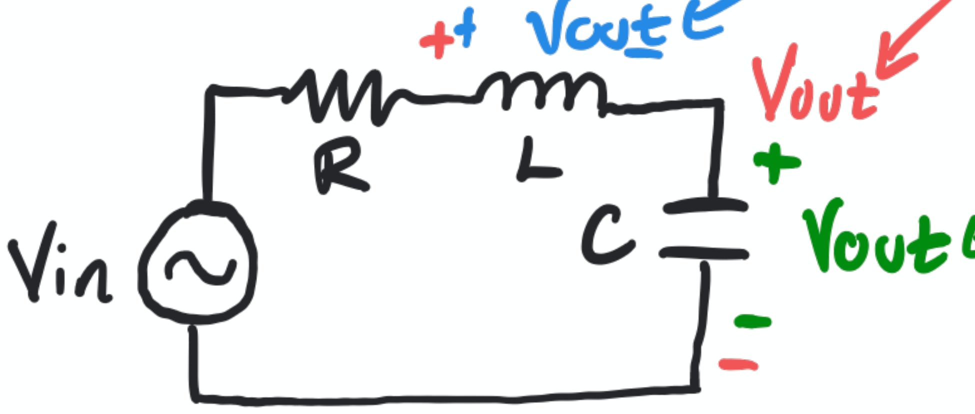
$f_c = 3183\ Hz$

Example Problem #2

option #3 ($V_{out} = V_L$)

option #1 ($V_{out} = V_L + V_C$)

option #2 ($V_{out} = V_C$)



#1) $$H(s) = \frac{sL + 1/sC}{R + sL + 1/sC} = \frac{s^2 + 1/LC}{s^2 + s\,R/L + 1/LC}$$

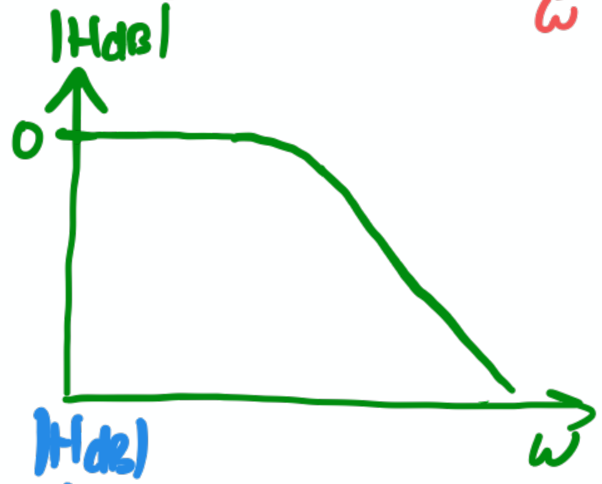
$\nearrow H(s \to 0) = 1$

$\searrow H(s \to \infty) = 1$

#2) $$H(s) = \frac{1/sC}{R + sL + 1/sC} = \frac{1/LC}{s^2 + s\frac{R}{L} + 1/LC}$$

$\nearrow H(s \to 0) : 1$

$\searrow H(s \to \infty) : 0$



#3) $$H(s) = \frac{sL}{R + sL + 1/sC} = \frac{s^2}{s^2 + s\frac{R}{L} + 1/LC}$$

$\nearrow H(s \to 0) : 0$

$\searrow H(s \to \infty) : 1$