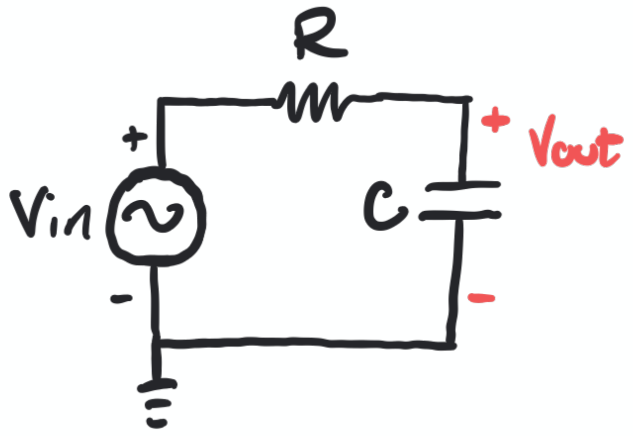
Intro to ECSE Lecture Notes 4/14

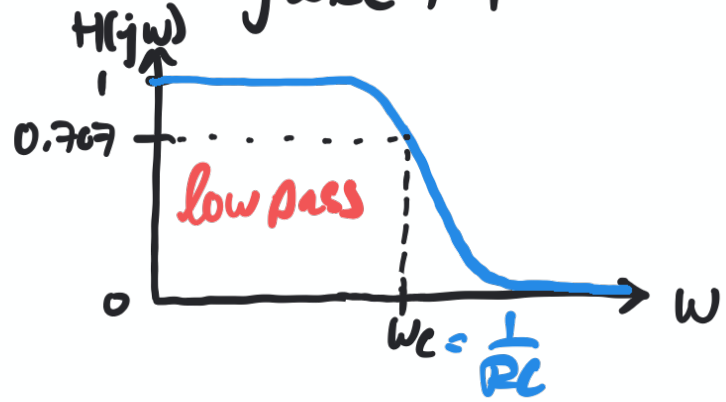
Filters... Part 3 : Review : first order RC filter



· Transfer function: $H(j\omega) = \dfrac{1}{j\omega RC + 1}$

· $H(\omega \to 0) \to 1$

· $H(\omega \to \infty) \to 0$



· What about the corner frequency? $|H(\omega_c)| = 1/\sqrt{2}$

· Why $1/\sqrt{2}$? $P = IV = \frac{V}{R} \cdot V = \frac{V^2}{R}$ $\quad H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{V}{V}$

Power transfer function: $|H(j\omega)|^2 \propto V^2 \to$ when power $\to 1/2$ is where we define ω_c

· $\omega_c = ?$ $\underbrace{|H(j\omega)|^2}_{\text{power transfer function}} = \frac{1}{2} \to |H(j\omega)|^2 = H(j\omega)\underbrace{H(-j\omega)}_{H^*(j\omega)} = \left(\frac{1}{j\omega RC + 1}\right) \cdot \left(\frac{1}{-j\omega RC + 1}\right)$

$= \dfrac{1}{(\omega RC)^2 + 1} = \dfrac{1}{2}$

$(j\omega RC) \cdot (-j\omega RC) = (\omega RC)^2 \cdot \underbrace{(j)(-j)}_{1}$

$j \cdot j = -1$

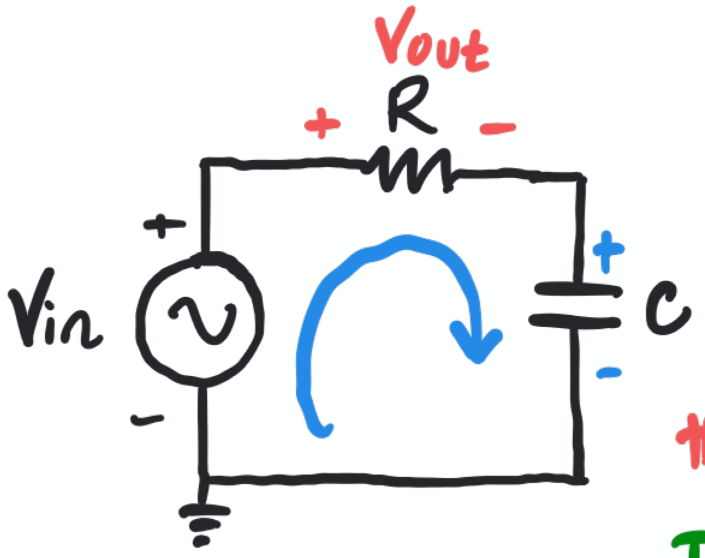
$j = \sqrt{-1}$

$(\omega RC)^2 + 1 = 2$

$$\boxed{\omega_c = \frac{1}{RC} \text{ corner frequency}}$$

$$\boxed{f_c = \frac{1}{2\pi RC} \text{ [Hz]}}$$

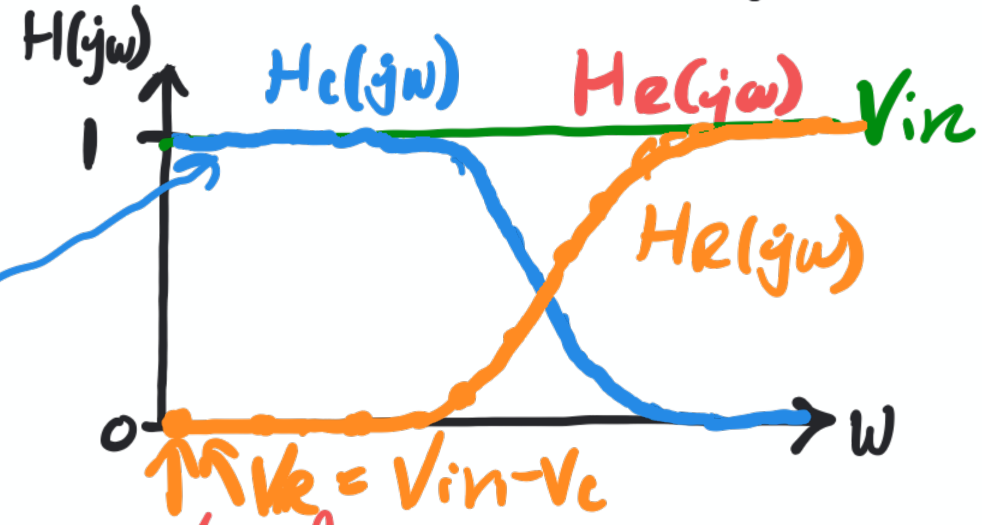
- What would we get if we measured across R instead?



- KVL : $V_{in} + V_R + V_C = 0$

$V_{in} = V_R + V_C$

$\#1, C: \; V_{in} = V_R + \underbrace{V_{out}}_{V_C}$



If $V_{in}(\omega)$ is 1 : what do we get when we measure across R?

- we get a high pass filter

$V_R = \underbrace{V_{in}}_{\text{constant}} - V_C$ ← curve defined by $H(j\omega)$

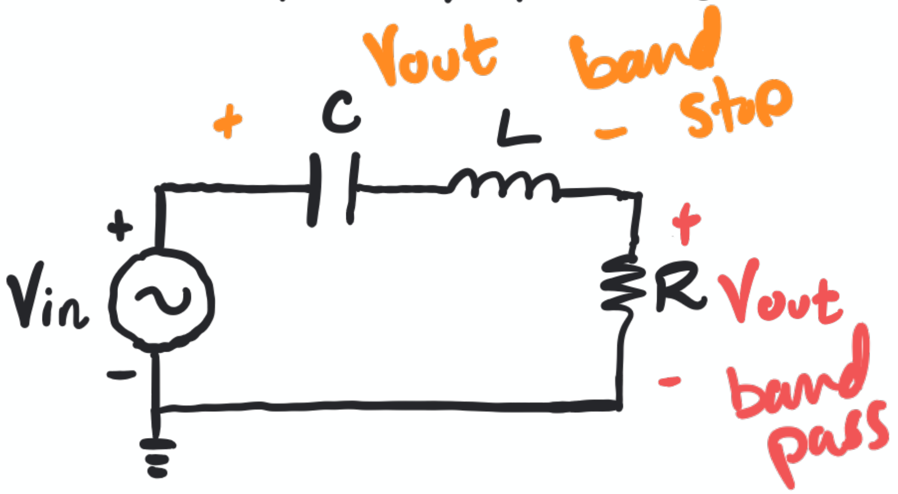
- Transfer function $H(j\omega) = \dfrac{Z_R}{Z_R + Z_C} = \dfrac{R}{R + 1/j\omega C} \cdot \dfrac{j\omega C}{j\omega C} = \dfrac{j\omega RC}{j\omega RC + 1}$

$H(\omega \to 0) = \dfrac{0}{0+1} = 0$

$H(\omega \to \infty) = \dfrac{\infty}{\infty + 1} \approx \dfrac{\infty}{\infty} = 1$

} high-pass filter

- Band-Pass Filter (2nd order) [Example using s]



a) What is Vout?

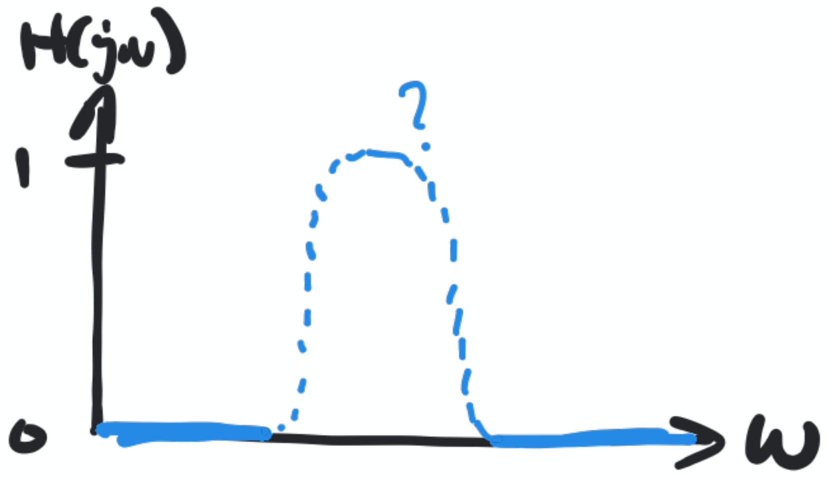
want $\dfrac{sc}{s^2+as+b}$ (c constant; a, b const?)

$$V_{out} = V_{in}\frac{Z_R}{Z_R+Z_C+Z_L} = \frac{R}{R+\frac{1}{sC}+\underbrace{sL}_{s^2}}V_{in}$$

$$= \frac{(s/L)}{(s/L)}\,\frac{R}{R+\frac{1}{sC}+sL}V_{in} = \frac{s\frac{R}{L}}{s^2+s\frac{R}{L}+\frac{1}{LC}}V_{in}$$

b) What is the transfer function?

$$H(s) = \frac{s\frac{R}{L}}{s^2+s\frac{R}{L}+\frac{1}{LC}}$$



c) What is $H(s\to 0)$? $H \to \dfrac{0}{0+0+\frac{1}{LC}} = 0$

d) What is $H(s\to\infty)$? $H \to \dfrac{s\,R/L}{s(s+R/L)+1/LC} \overset{\text{for } s \text{ very large}}{=} \dfrac{s\,R/L}{s(s+\cancel{R/L})+1/LC} = \dfrac{s\,R/L}{s^2+\cancel{1/LC}}$

$s \gg R/L$ $\quad s^2 \gg 1/LC$

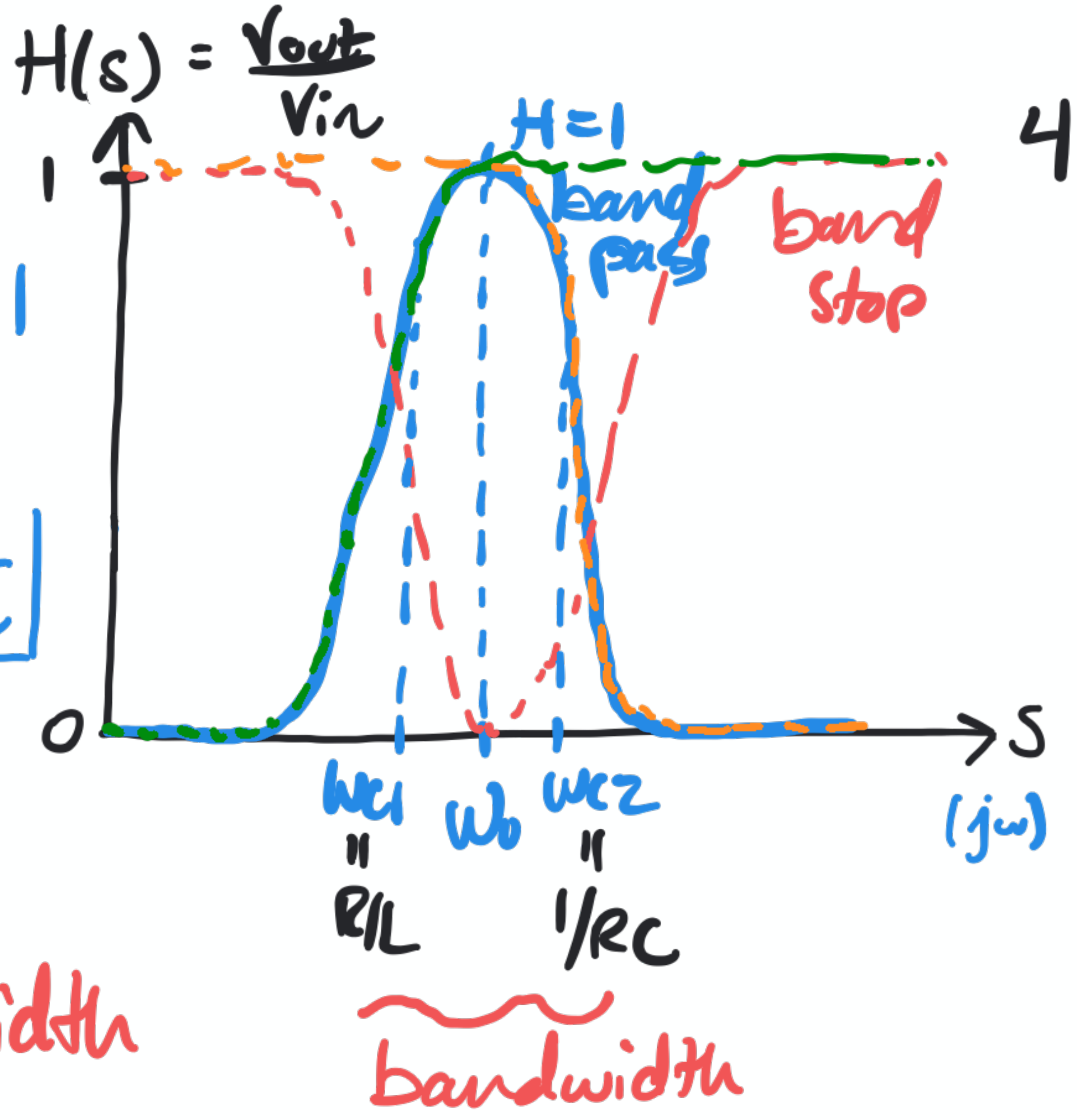
$$= \frac{s\,R/L}{s^2} = \frac{R/L}{s} \to \text{if } s\to\infty,\ H \to \frac{R/L}{\infty} \to 0$$

e) Where does $|H(j\omega)| = 1$?

$$H(s) = \frac{s\frac{R}{L}}{s^2 + s\frac{R}{L} + \frac{1}{LC}} \cdot \frac{\left(\frac{L}{sR}\right)}{\left(\frac{L}{sR}\right)} = \frac{1}{s\frac{L}{R} + 1 + \frac{1}{sRC}} = 1$$

$$s\frac{L}{R} + 1 + \frac{1}{sRC} = 1 \rightarrow s^2 = -\frac{1}{LC} = (j\omega)^2 = -\omega^2 \rightarrow \boxed{\omega = \frac{1}{\sqrt{LC}}}$$

center frequency → $\underline{\omega_0} = \frac{1}{\sqrt{LC}} \rightarrow H = 1$



f) What about the corner frequencies?

$$\left.\begin{aligned} \underline{\omega_{c1}} &= \frac{R}{L} \\ \underline{\omega_{c2}} &= \frac{1}{RC} \end{aligned}\right\} \quad \omega_{c2} - \omega_{c1} = \text{bandwidth of filter}$$