Intro to ECSE Lecture Notes

Frequency Response (Filters) Pt. 2

1) Filters as Transfer Functions

- Filters are defined by their transfer functions H(s)

$s = \underbrace{j}_{\text{imaginary \#}}\underbrace{\omega}_{\text{angular frequency}}$ $\omega = 2\pi \underbrace{f}_{\text{Hz}}$

2) s for different sinusoids (AC Steady State Signals)

sinusoid #1:  $f_1 = 60\,\text{Hz}$

sinusoid #2:  $f_2 = 120\,\text{Hz}$

Which one has the higher frequency?

1 or [✓ 2]

- What is s for #1? $s_1 = j\omega_1 = j2\pi f_1 = j376.99$ [rad/s]

  #2? $s_2 = j\omega_2 = j2\pi f_2 = j753.98$ [rad/s]

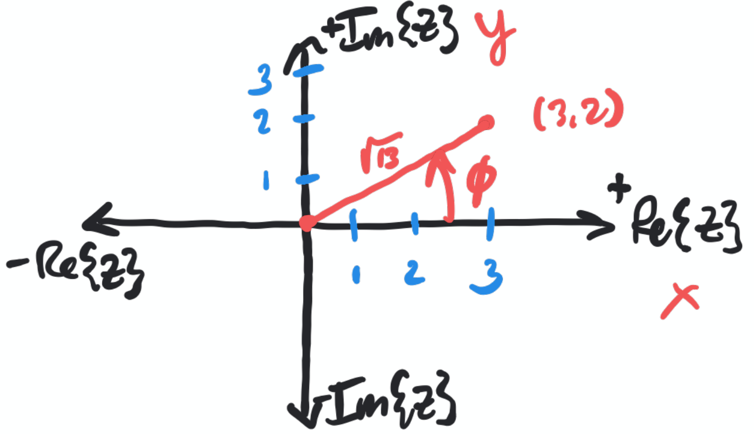
Complex Numbers and the Complex Plane

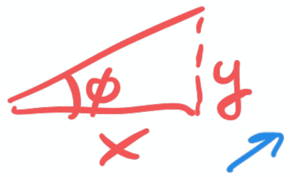
$$\underset{\text{complex number}}{z} = \overset{x}{\underset{\text{real part}}{a}} + j\overset{y}{\underset{\text{imaginary part}}{b}}$$

complex conjugate (*)

$$z^* = a - jb$$

- replace j with $-j$





y → Im{z}

$\tan^{-1}(y/x) = \phi$

x → Re{z}

- Plot the number: $3 + j2$

$\mathrm{Re}\{z\} = 3$

$\mathrm{Im}(z\} = 2$

- Find the magnitude $|z|$

$j \cdot j = -1$

$+j \cdot (-j) = 1$

$$\text{magnitude } |z| = \sqrt{zz^*} = \sqrt{(3+j2)(3-j2)}$$

$$= \sqrt{3^2 + j6 - j6 + 2^2}$$

$$= \sqrt{13}$$

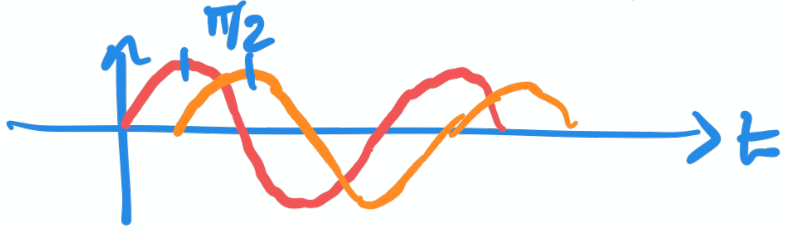
$$V_{out} = H(j\omega) \cdot V_{in}$$

$|V_{out}|$ is what we measure (real)

- Find the phase $\angle z$

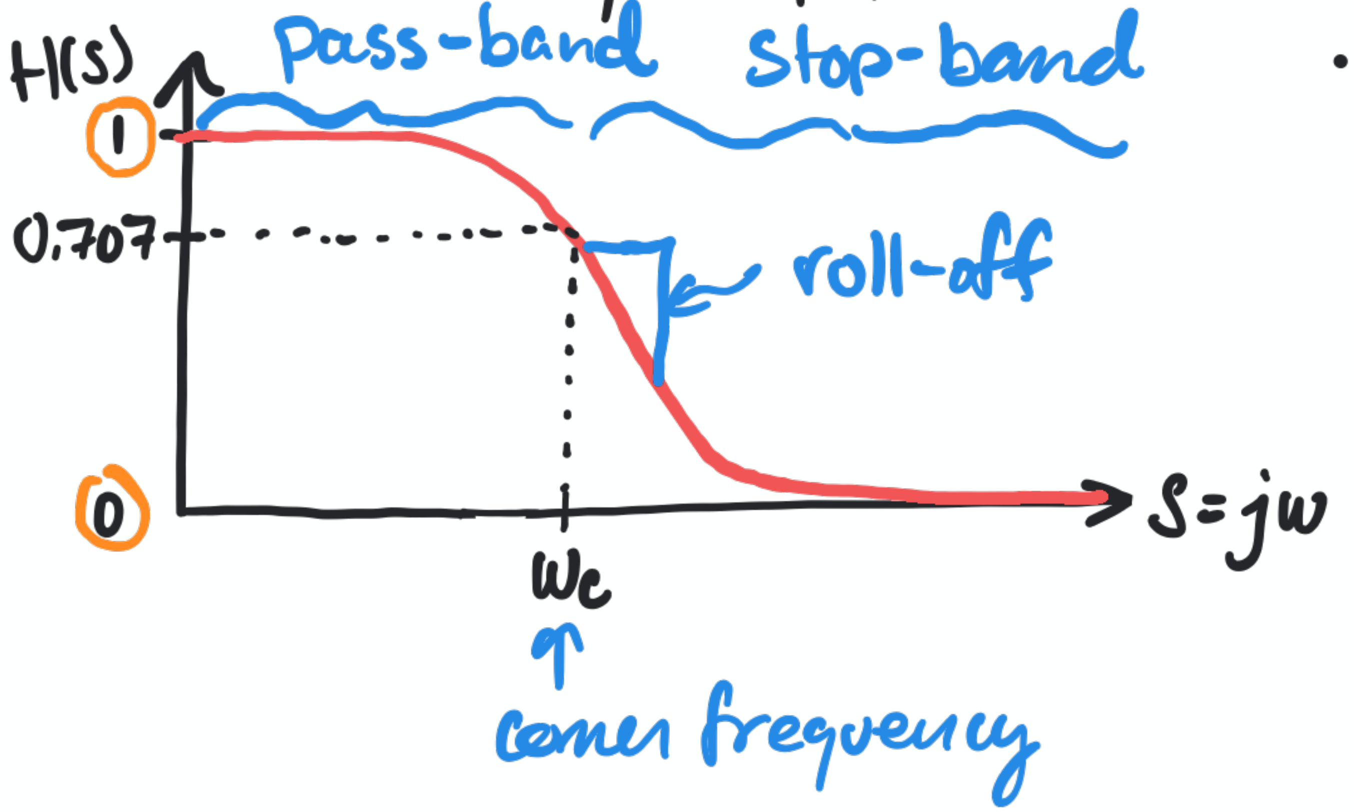
$$\angle z = \tan^{-1}(\mathrm{Im}(z)/\mathrm{Re}(z)) = \tan^{-1}(2/3) \cong \underline{33.7^\circ}$$

- phase is how much the output is shifted relative to the input



3) Anatomy of a Filter Transfer Function

low-pass filter



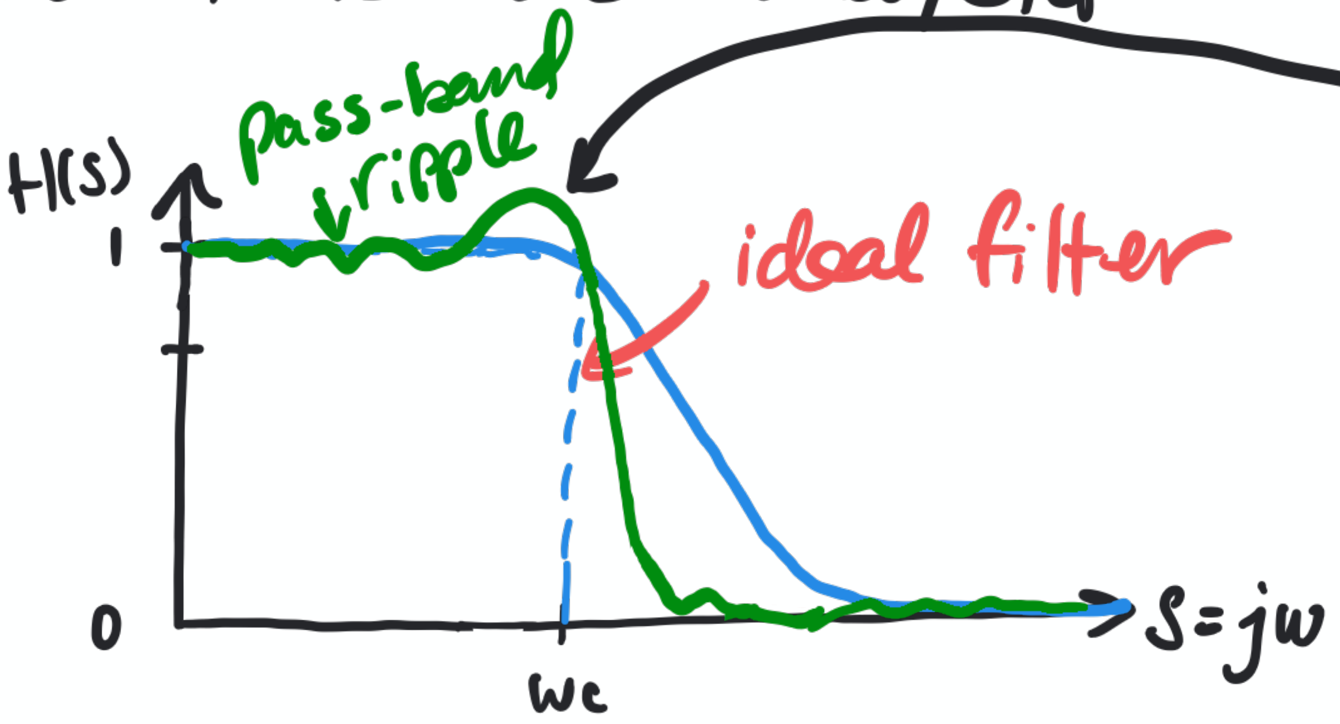
- corner frequency is where $|H(jw)|=1/\sqrt{2}$
  → tells us where the filter begins to change the output signal
- pass-band: filter allows frequency components to go through
- stop-band: filter blocks these frequencies
- roll-off: steepness of the transfer function of the filter → determines how quickly signal is attenuated vs. frequency

when $H=1 \rightarrow V_{out} = V_{in}$

$H=0 \rightarrow V_{out} = 0$

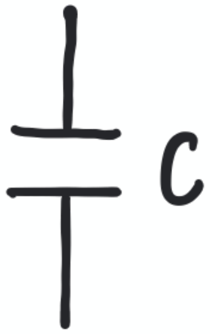
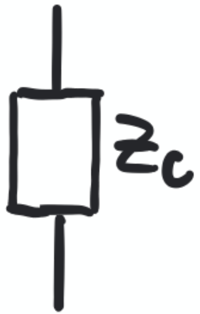
attenuation: opposite of amplification

4) Roll-off vs. Pass-band/Stop-band Flatness (vs. complexity)

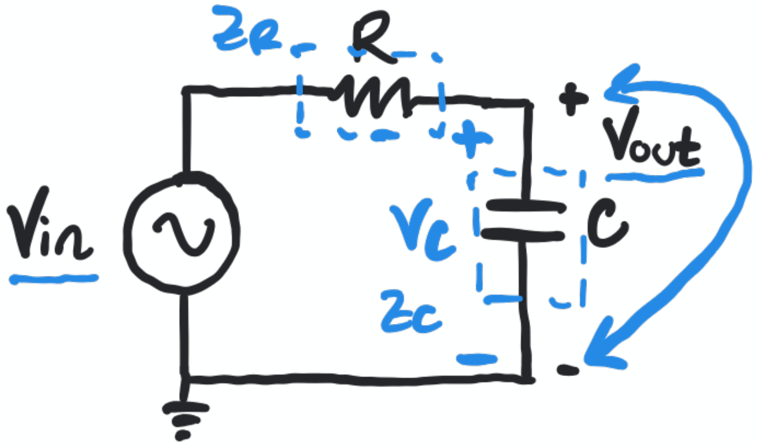


- stop-band/pass-band ripple can distort the signal because one component gets amplified more than others.

5) Review of RLC and Impedance

Element			
Impedance			
jw :	$Z_R = R$	$Z_C = 1/jwC$	$Z_L = jwL$
	Resistor	Capacitor	Inductor
s :	$Z_R = R$	$Z_C = 1/sC$	$Z_L = sL$
Circuit equivalent $S \rightarrow 0$		$Z_C \rightarrow \infty$  open circuit	$Z_L \rightarrow 0$  short circuit
$S \rightarrow \infty$		$Z_C \rightarrow 0$  short circuit	$Z_L \rightarrow \infty$  open circuit

6) 1st Order RC Circuit (Example using s)



a) What is V_{out}? $V_{out} = V_C$

Voltage divider

$$V_{out} = V_{in}\frac{Z_C}{Z_R + Z_C} = V_{in}\frac{1/j\omega C}{R + 1/j\omega C}\cdot\frac{\overset{1}{\widetilde{j\omega C}}}{j\omega C}$$

$$= V_{in}\frac{1}{j\omega RC + 1} \rightarrow \boxed{H(j\omega) = \frac{1}{j\omega RC + 1}}$$

b) What is $H(\omega \to 0)$?

$$H(\underline{\omega \to 0}) = \frac{1}{0 + 1} = \underline{1}$$

c) What is $H(\omega \to \infty)$?

$$H(\underline{\omega \to \infty}) = \frac{1}{\infty + 1} = \frac{1}{\infty} \rightarrow \underline{0}$$

d) What kind of filter is this?

low-pass filter