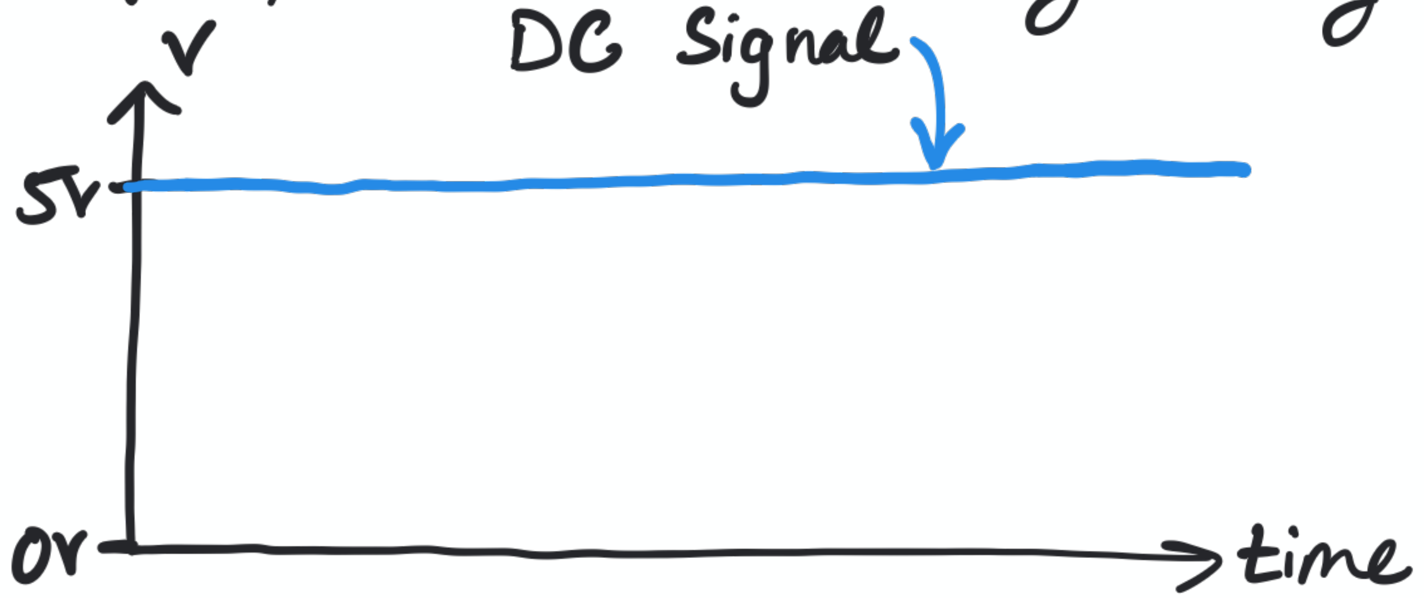
Intro to ECSE Lecture Notes: Time vs. Frequency Domain 4/4/23 1

1) Signals in the Time Domain

direct current

+ So far, we've been using mostly DC signals ($f = 0\,\text{Hz}$)

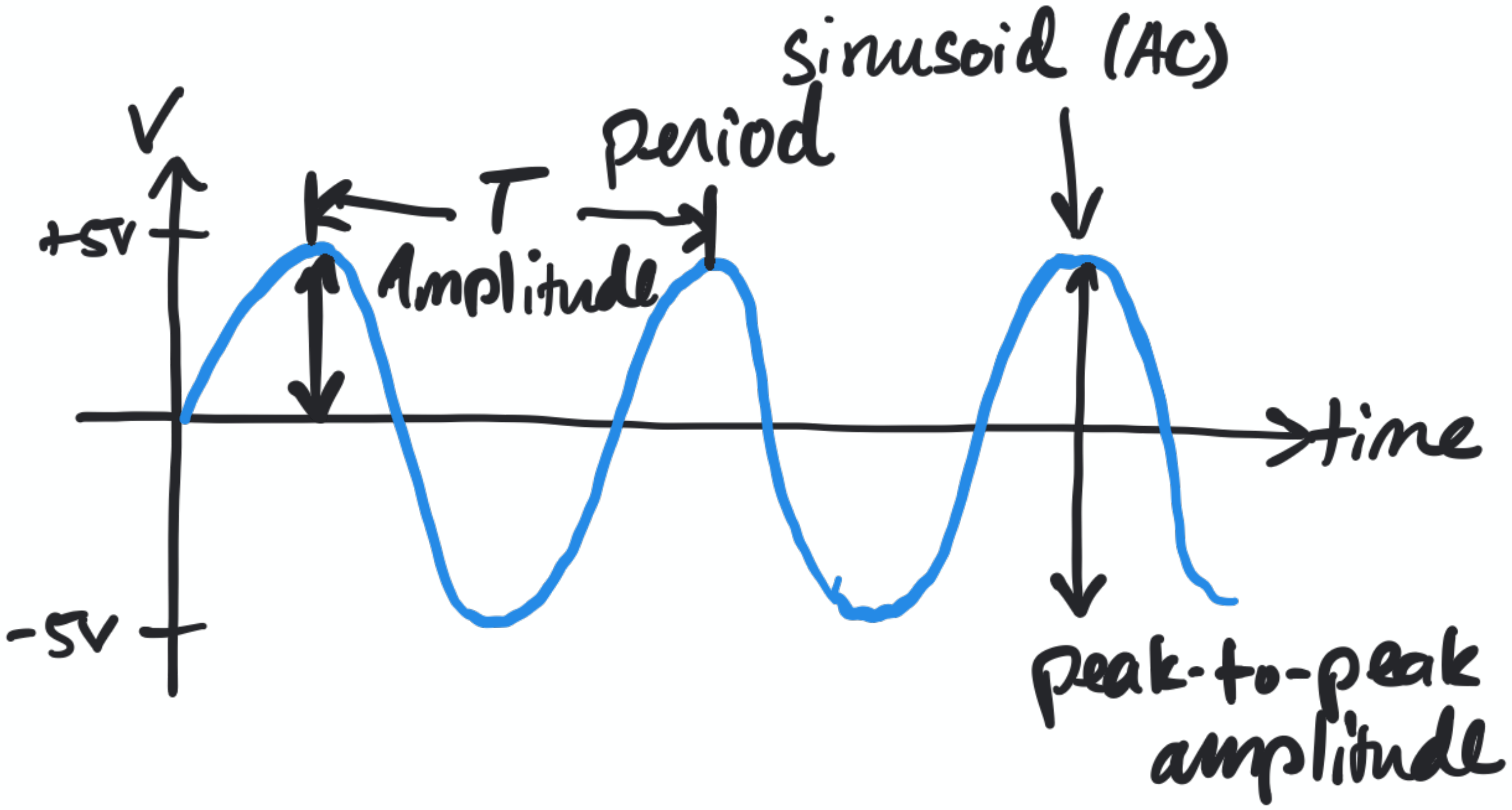


- Useful for power supplies, but not interesting for signal processing

typically AC = sinusoidal (periodic signal)

+ The "other" type of signal is AC (or alternating current)

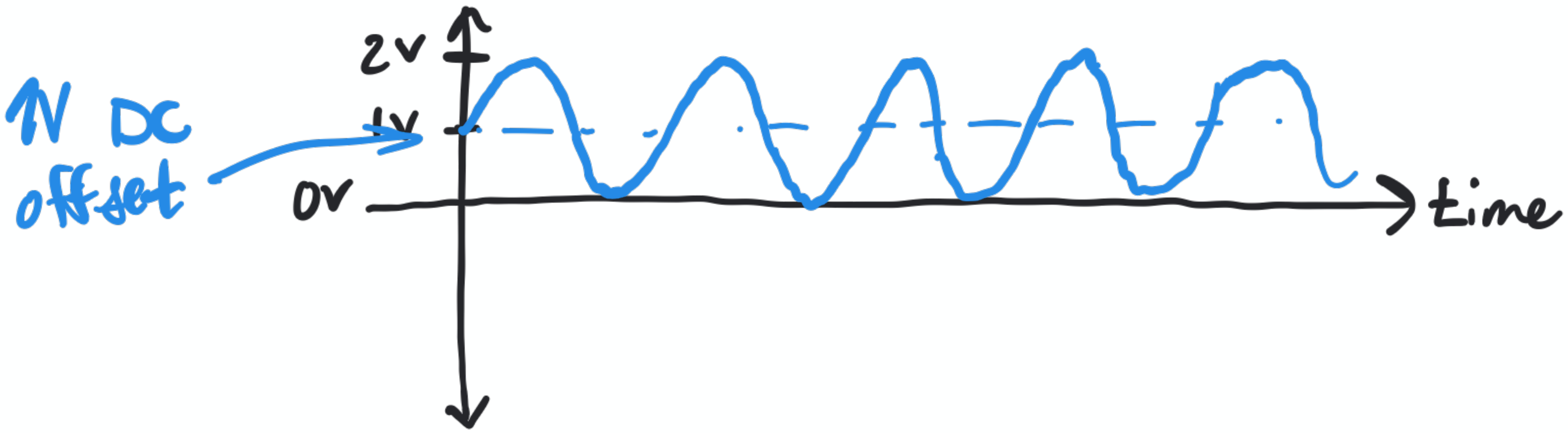
→ signal amplitude changes in time: $f > 0\,\text{Hz}$



frequency = $1/T$

To summarize: $\begin{cases} \text{DC} : f = 0 \rightarrow \text{does not change over time} \\ \text{AC} : f > 0 \rightarrow \text{Changes over time} \end{cases}$

+ In general, signals can have both a DC and an AC component



DC offset

sinusoidal

$V(t) = V_{DC} + V_{AC}(t)$

2) Capacitors and Inductors

- Resistors: behave according to Ohm's Law: $V = IR$ (resistance ↓ R)
  - resistors don't care about the signal's frequency

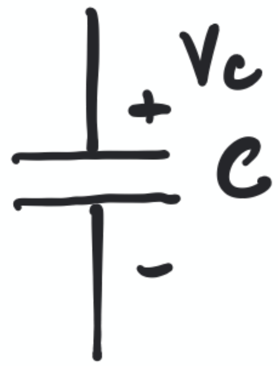
- The two remaining fundamental circuit elements:
  capacitors and inductors DO care about the signal frequency

  C and L are still linear circuit elements (in the frequency domain)
  not like Ohm's Law (time domain)

- Capacitors and Inductors in the time domain

Capacitor (C)

not like Ohm's law



capacitance C (Farads)

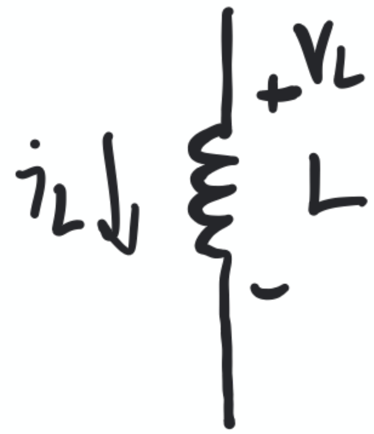
fF − μF (typical range)

$$i_c = C\frac{dv_c}{dt}$$ ← time domain

open circuit at DC

no voltage change = no current flow

Inductor (L)



inductance L (Henries)

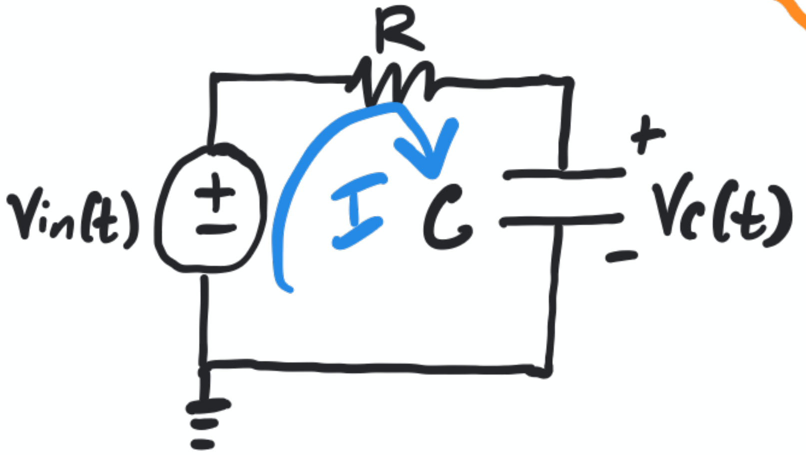
μH − H (typical range)

$$v_L = L\frac{di_L}{dt}$$

- no current change → no voltage → short circuit @ DC

3) Time Domain Math

a) 1st Order Circuits



1st order ODE

KVL

- Find $V_c(t)$: $-V_{in}(t) = V_R(t) + V_c(t)$

$I = i_c$

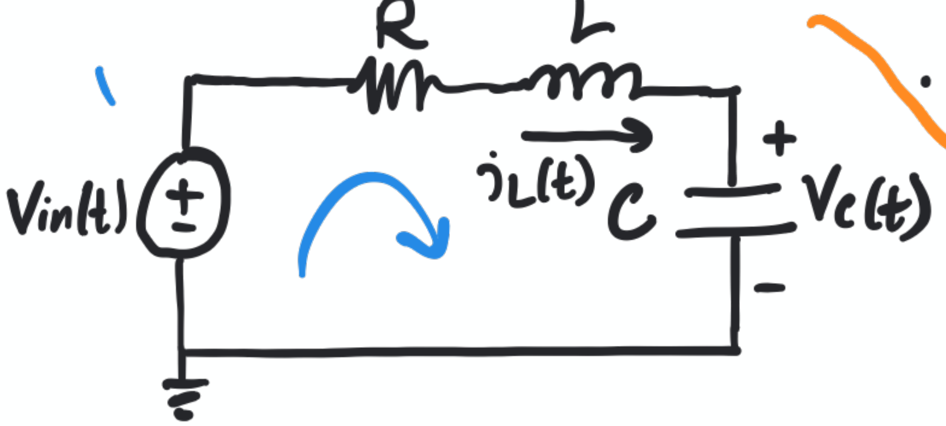
$$i_c = C\frac{dV_c(t)}{dt}$$

$$V_{in}(t) = IR + V_c(t)$$

$$V_{in}(t) = RC\frac{dV_c(t)}{dt} + V_c(t)$$

$$0 = \frac{dV_c(t)}{dt} + \frac{1}{RC}V_c(t) - \frac{V_{in}(t)}{RC}$$

b) 2nd Order Circuits



2nd order ODE

- Find $V_c(t)$: $$\frac{d^2V_c(t)}{dt^2} + \frac{R}{L}\frac{dV_c(t)}{dt} + \frac{1}{LC}V_c(t) = \frac{1}{LC}V_{in}(t)$$

$$i_c = C\frac{dV_c}{dt}$$

$$V_L = L\frac{di_c}{dt}$$

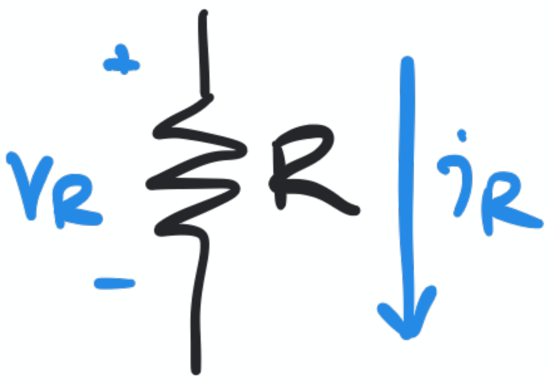
- often there's an easier way → frequency domain

4) Impedances

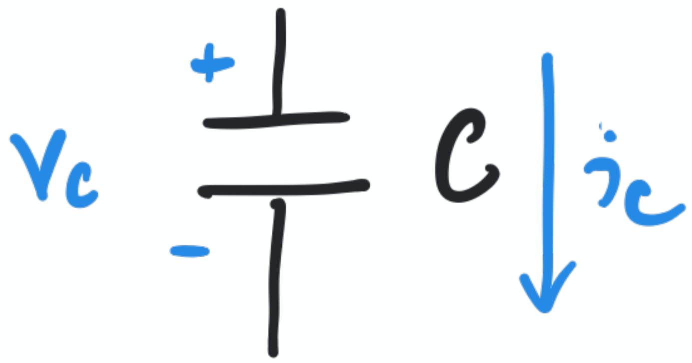
- In the frequency domain, there is an Ohm's-Law-like relationship between V and I for R, L, and C:

$$V = I\boxed{Z} \longleftarrow \text{Impedance } (Z)$$

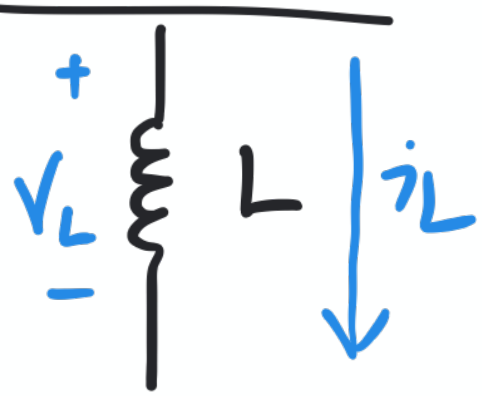
Resistor



Capacitor



Inductor



Impedance: $Z_R = R$

$Z_C = 1/sC$

$= 1/j\omega C$

imaginary # $= \sqrt{-1}$

freq. in Hz $= 1/s$ $\downarrow$ $\omega = 2\pi f$ = radial frequency [rad/s]

here $s = j\omega$

$Z_L = sL$

$= j\omega L$

• What is s? Via the LaPlace Transform:

$$s = \sigma + j\omega$$

σ: real number → decay or growth

ω: real number → oscillation → AC signals → $s = j\omega$

$$x(t) = e^{st} = e^{\sigma t} e^{j\omega t}$$

• What does this mean? If $x(t) = e^{st} = e^{\sigma t} e^{j\omega t}$ is the input

$e^{+\sigma t}$ → growth

$e^{-\sigma t}$ → decay

$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$

oscillation

input

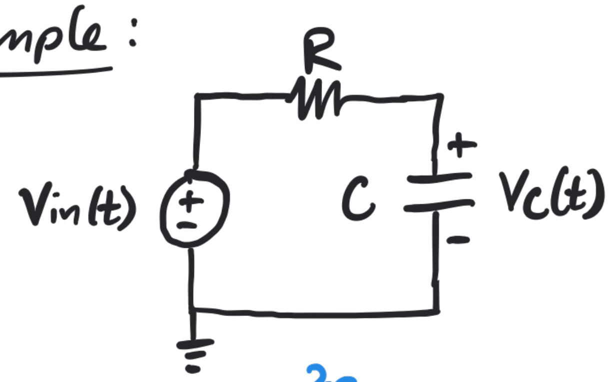
$$x(t) = e^{st} \longrightarrow \boxed{\begin{matrix}\text{Linear} \\ \text{System} \\ H(s)\end{matrix}} \longrightarrow y(t) = H(s)e^{st}$$

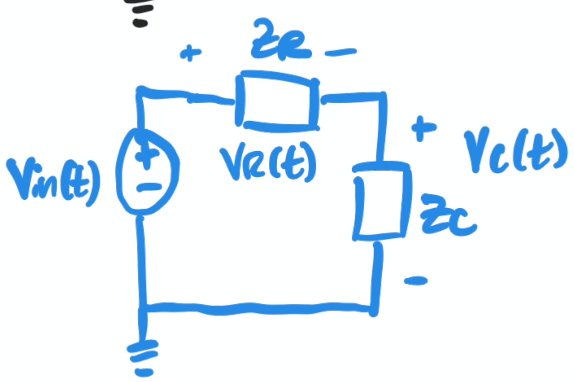
output

if we do a Laplace transform and then set $s = j\omega$ → Fourier Transform

• We will only be dealing with $s = j\omega$ → purely oscillatory signal

Example:





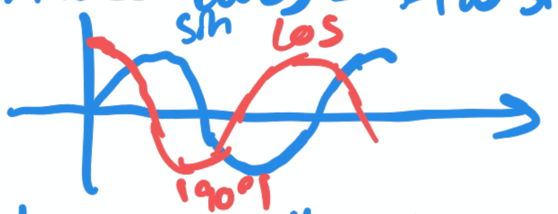
- If $V_{in}(t) = A\cos(\omega t)$, find $V_R(t)$ and $V_c(t)$.

$$V = IZ \quad Z_R = R, \quad Z_c = \frac{1}{j\omega C}$$

$$V_c = V_{in}\frac{Z_c}{Z_R + Z_c} = V_{in}\frac{1/j\omega C}{R + 1/j\omega C}\cdot\frac{j\omega C}{j\omega C} = \boxed{\frac{1}{j\omega RC + 1}V_{in}}$$

$$V_R = V_{in}\frac{Z_R}{Z_R + Z_c} = \frac{R}{R + 1/j\omega C}V_{in} = \boxed{\frac{j\omega RC}{j\omega RC + 1}V_{in}}$$

- $V_{in} = A\sin(\omega t)$
- $i_c \propto \frac{dV_c}{dt} \rightarrow \frac{d}{dt}\left(A\sin(\omega t)\right) = A\omega\cos(\omega t) = A\omega\sin(\omega t + \pi/2)$



- $|V_c|$ = what we measure (only real voltages)
- $\angle V_c$ = phase shift (imaginary number)