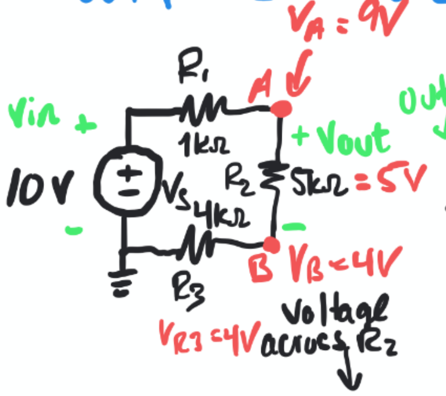
Intro to ECSE Lecture 8 2/3/23

Math as a Component

1) Voltage Divider: component in a circuit

- For an input voltage V_s, a voltage divider outputs a fraction of that voltage (V_{out})



$$V_{out} = V_s \frac{R_{out}}{\sum_{i=1}^{N} R_i}$$

component whose voltage is the output

sum of all resistances in series

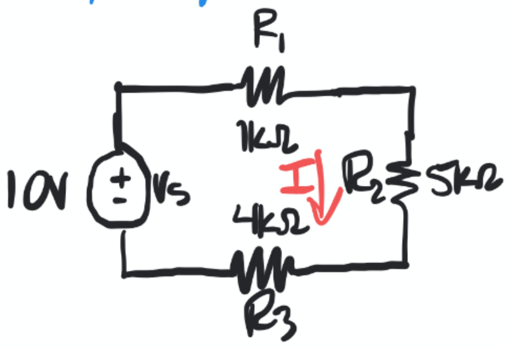
source voltage

Example: what is $V_{R2} = V_{out}$?

$$V_{out} = \frac{R_2}{R_1 + R_2 + R_3} V_{in} = \frac{5k\Omega}{1k\Omega + 5k\Omega + 4k\Omega} \cdot 10V = \frac{5}{10} \cdot 10 = \boxed{5V}$$

2) Circuit Analysis Method #1: Circuit Reduction

- combine R in series & parallel to get a simpler circuit we can solve with a voltage divider or Ohm's Law, then back calculate quantity of interest



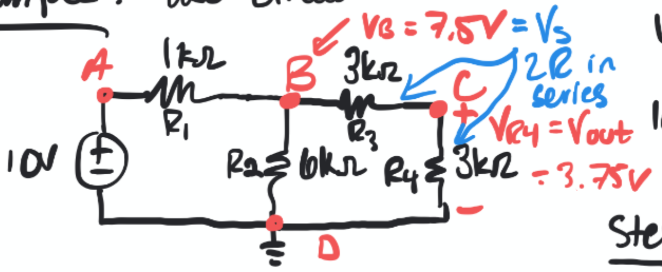
Ex: Find I:

a) combine $R_1, R_2, R_3 \to R_{eq}$

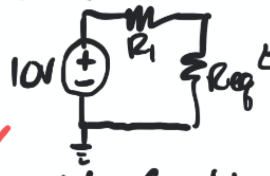
$$R_{eq} = R_1 + R_2 + R_3 \text{ (series)}$$
$$= 1k\Omega + 5k\Omega + 4k\Omega = 10k\Omega$$

b) $V_s = I R_{eq} \to I = \frac{V_s}{R_{eq}} = \frac{10V}{10k\Omega} = \underline{1mA}$

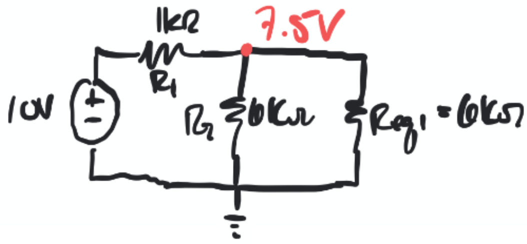
Example: Use circuit reduction to find V_{R4}



We want a circuit of the form



use a voltage divider to find V_{Req}

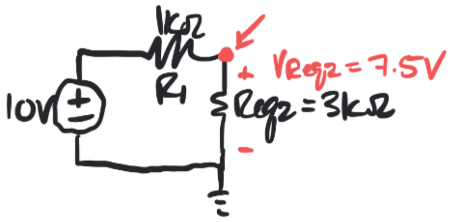


Step #1: Combine R_3 and R_4 (series)

$$R_{eq1} = R_3 + R_4 = 6k\Omega$$

Step #2: Combine R_2 and R_{eq1} (parallel)

$$R_{eq2} = R_{eq1} \| R_2 = \frac{1}{\frac{1}{R_{eq1}} + \frac{1}{R_2}}$$



$$= \frac{1}{\frac{1}{6k\Omega} + \frac{1}{6k\Omega}} = \frac{1}{2/6k\Omega} = \frac{6k\Omega}{2} = 3k\Omega$$

Step#3: $V_{Req2} = \underbrace{10V}_{V_s} \cdot \frac{\overbrace{3k\Omega}^{Req2}}{\underbrace{1k\Omega}_{R_1} + \underbrace{3k\Omega}_{Req2}} = 7.5V$

Step#4: (see 1st circuit)

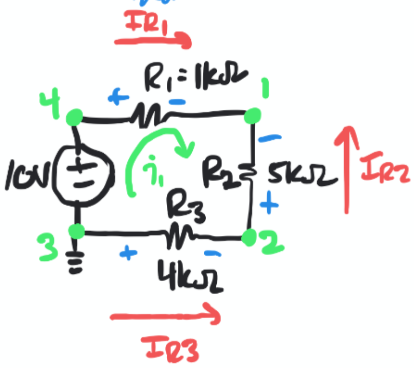
$V_B = 7.5V$, we want to find V_{R4}

$$V_{R4} = V_B \frac{R_4}{R_3 + R_4}$$

$$= 7.5 \frac{3k\Omega}{3k\Omega + 3k\Omega} = \boxed{3.75V}$$

3) Circuit Analysis Method #2: KVL/KCL/Ohm's Law

- Use KVL and KCL to derive equations for the voltage across R or current through R, then put them into a matrix form to solve them



Procedure: 1) Place reference marks across all resistors in the circuit: "+" and "-" signs. Current flows from "+" to "-". Defines direction of current flow through each resistor.

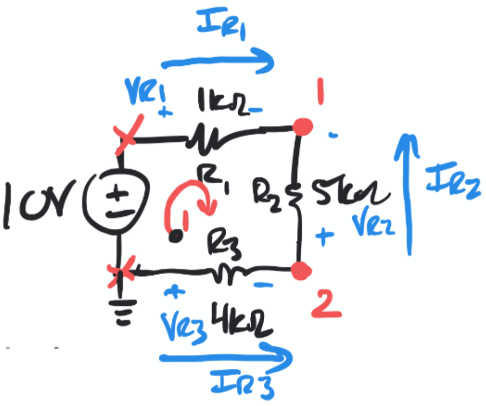
2) Count the number of unknowns in the circuit. Equal to the number of resistors: V_R or I_R are the unknowns.
In this case: V_{R1}, V_{R2}, V_{R3}
3 unknowns

3) Find all nodes and loops in the circuit: 4 nodes, 1 loop

4a) Choose best nodes and loops and write KCL and KVL equations

Hint1: don't choose nodes connected to ground or a voltage source

Hint2: don't include loops that include other loops

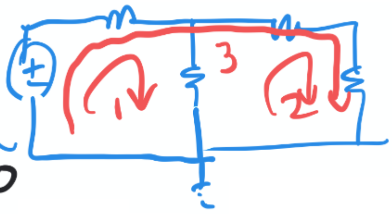


KCL:

node 1: $I_{R1} + I_{R2} = 0$

node 2: $-I_{R2} + I_{R3} = 0$

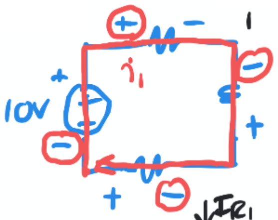
3 eq. 3 unknowns



Choose 1 and 2, don't choose 3

KVL:

✓ loop 1: $-10V + V_{R1} - V_{R2} - V_{R3} = 0$



4b) Convert equations to all voltages or all currents

- use Ohm's law to convert $I_R = \frac{V_R}{R}$ or $V_R = I_R R$

✓ node 1: $\frac{V_{R1}}{R_1} + \frac{V_{R2}}{R_2} = 0 \quad + 0 \cdot V_{R3}$

✓ node 2: $-\frac{V_{R2}}{R_2} + \frac{V_{R3}}{R_3} = 0$

Matrix Multiplication

matrix equation

$Ax = b$ ← constants

coefficients, unknowns

Solution is $A^{-1}b = x$ ← voltages or currents

$(\text{inv}(A)\,b = x)$

5) Put equations into matrix form and solve

$$\begin{array}{r} \text{loop 1} \\ \text{node 1} \\ \text{node 2} \end{array} \begin{bmatrix} 1 & -1 & -1 \\ 1/R_1 & 1/R_2 & 0 \\ 0 & -1/R_2 & 1/R_3 \end{bmatrix} \begin{bmatrix} V_{R1} \\ V_{R2} \\ V_{R3} \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \text{ source terms}$$

$A \cdot x = b$

$$\underset{\#\text{rows } m}{} \overset{\#\text{cols } n}{\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

A_{mn} — $m \times n$ matrix, $n \times 1$, $m \times 1$

row 1 of A × col 1 of x = row 1 of b

$b_1 = A_{11}x_1 + A_{12}x_2 + A_{13}x_3$

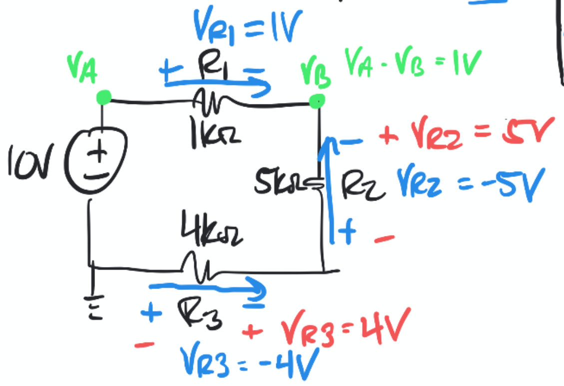
$\quad\;\; 1/R_1 \; V_{R1} \quad 1/R_2 \; V_{R2} \quad 0 \; V_{R3}$

Solve:

$$\begin{bmatrix} 1 & -1 & -1 \\ 1/1000 & 1/5000 & 0 \\ 0 & -1/5000 & 1/4000 \end{bmatrix} \begin{bmatrix} V_{R1} \\ V_{R2} \\ V_{R3} \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$A \times x = b$

$$A^{-1}b = x = \begin{bmatrix} V_{R1} \\ V_{R2} \\ V_{R3} \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -4 \end{bmatrix}$$

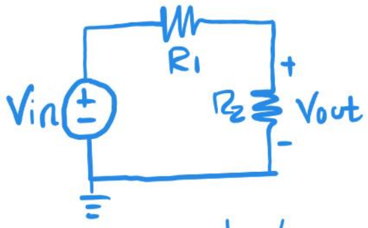


Problem #1: Create a voltage divider that outputs 0.6 of the input voltage: $V_{out} = 0.6 V_{in}$

The voltage divider equation is

$$V_{out} = V_{in} \frac{R_{out}}{\sum_{i=1}^{N} R_i}$$

The simplest voltage divider is a 2-resistor circuit



$$V_{out} = V_{in} \frac{R_2}{R_1 + R_2}$$

So, we must choose R_1 and R_2 such that $\frac{R_2}{R_1+R_2} = 0.6 \rightarrow \{ V_{out} = 0.6 V_{in} \}$

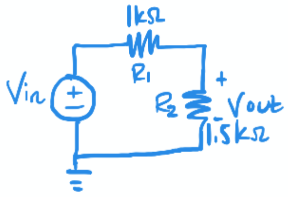
There are an infinite number of solutions, so I'll just choose $R_1 = 1k\Omega$

$$0.6 = \frac{R_2}{1k\Omega + R_2} \rightarrow 0.6k\Omega + 0.6R_2 = R_2$$

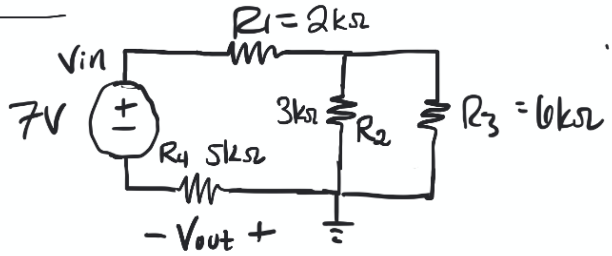
$$0.6k\Omega = 0.4R_2$$

$$R_2 = \frac{0.6k\Omega}{0.4} = 1.5k\Omega$$

So the solution is:



Problem #2:

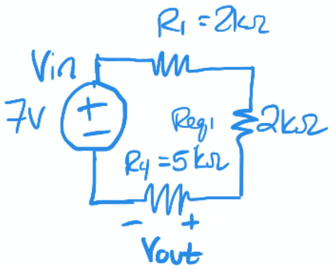


- Solve for $V_{R4} = V_{out}$ using
  - a) circuit reduction method
    AND
  - b) KVL/KCL/Ohm's Law method

a) circuit reduction

1) Combine R_2 and R_3: $R_2 \parallel R_3 = R_{eq1}$

$$R_{eq1} = \frac{1}{\frac{1}{3k\Omega} + \frac{1}{6k\Omega}} = \frac{1}{\frac{3}{6k\Omega}} = \frac{6k\Omega}{3} = 2k\Omega$$

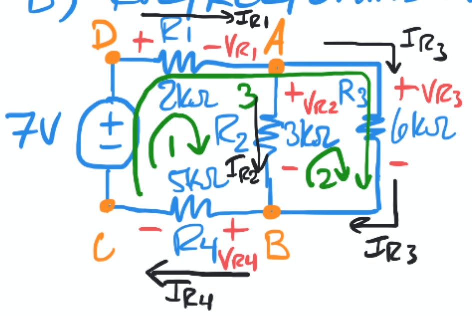


2) Use a voltage divider to find $V_{out} = V_{R4}$

$$V_{R4} = V_{out} = \frac{R_4}{R_1 + R_{eq1} + R_4} \cdot V_{in}$$

$$= \frac{5k\Omega}{2k\Omega + 2k\Omega + 5k\Omega} 7V = \frac{5}{9} \cdot 7V = \frac{35V}{9} = \underline{3.889V}$$

b) KVL/KCL/Ohm's Law



1) Place reference marks
2) Count number of unknowns
= number of resistors = 4
3) Count number of nodes and loops
4 3
4) Choose best nodes and loops, then write KCL and KVL equations in terms of V_R or I_R (use Ohm's Law where needed to convert $V_R \Longleftrightarrow I_R$)

- Best nodes: nodes NOT connected to a voltage source or ground: A & B
- Best loops: loops that do NOT contain any other loops: 1 & 2

convert to V_R via Ohm's Law

node A: $I_{R1} - I_{R2} - I_{R3} = 0 \longrightarrow \frac{V_{R1}}{R_1} - \frac{V_{R2}}{R_2} - \frac{V_{R3}}{R_3} = 0$ (A_{11}, A_{12}, A_{13}, $A_{14}=0$, b_1)

node B: $I_{R3} + I_{R2} - I_{R4} = 0 \longrightarrow \frac{V_{R3}}{R_3} + \frac{V_{R2}}{R_2} - \frac{V_{R4}}{R_4} = 0$ (b_2; A_{23}, A_{22}, A_{24}, $A_{21}=0$)

loop 1: $-7V + V_{R1} + V_{R2} + V_{R4} = 0$ ($-b_3$, A_{31}, A_{32}, A_{34})

loop 2: $-V_{R2} + V_{R3} = 0$ (A_{42}, A_{43}, b_4)

5) Put equations into matrix form:
Since we have 4 unknowns, our "A" matrix will now be 4x4

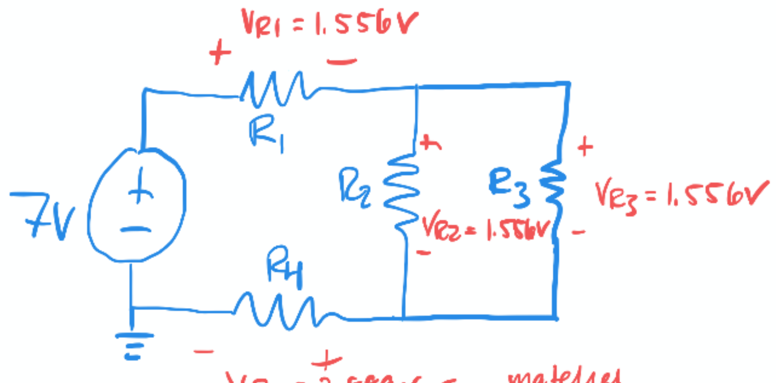
$$
\begin{array}{l}
\text{Equation} \\ \text{node A} \\ \text{node B} \\ \text{loop 1} \\ \text{loop 2}
\end{array}
\quad
\overset{A}{\begin{bmatrix} 1/R_1 & -1/R_2 & -1/R_3 & 0 \\ 0 & 1/R_2 & 1/R_3 & -1/R_4 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}}
\overset{x}{\begin{bmatrix} V_{R1} \\ V_{R2} \\ V_{R3} \\ V_{R4} \end{bmatrix}}
=
\overset{b}{\begin{bmatrix} 0 \\ 0 \\ 7 \\ 0 \end{bmatrix}}
$$

6) Solve using Matlab or calculator $A^{-1}b = x$

our V_R values

inv(A) in matlab

$$
\begin{bmatrix} 1/2000 & -1/3000 & -1/6000 & 0 \\ 0 & 1/3000 & 1/6000 & -1/5000 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}^{-1}
\begin{bmatrix} 0 \\ 0 \\ 7 \\ 0 \end{bmatrix}
=
\begin{bmatrix} 1.556 \\ 1.556 \\ 1.556 \\ 3.889 \end{bmatrix} V
$$



$V_{R4} = 3.889V$ ← matches circuit reduction solution

Quiz: · Combining R in series and parallel (circuit reduction)

- Voltage dividers
- KCL/KVL/Ohm's Law Circuit Analysis Method
- Coming to class and trying proof of skills
- Write matrix equations

- open book & open notes from course materials
- notes must be hand-written or printed out
- no electronic devices except a non-communicating calculator
- no discussion allowed during the quiz