

Intro to ECSE Class 7 Lecture Notes 11/31/23

Terminology & Basic Quantities

1) Charge: electrons have charge (think static electricity)

↳ units: Coulombs (C)

↳ variable: Q or q

↳ most fundamental physical quantity in a circuit

2) Current: moving charge (flow of charge)

↳ units: Amperes (A) = $\frac{\text{Coulombs}}{\text{s}}$ (charge per time)

↳ variable: I

↳ terminology:

• Current flows: "through" an element (like a resistor)
"into" or "out of" a location (i.e. node)

↳ Mathematical definition: $\frac{\Delta \text{charge}}{\Delta \text{time}} = \frac{dQ}{dt} = I$

• But how do get charge to move?

3) Voltage: what puts our system into action ~ force

↳ units: Volts (V)

↳ terminology:

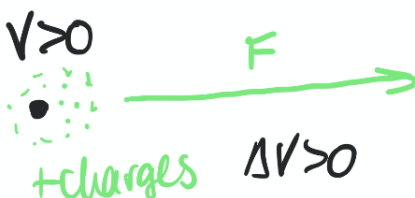
Voltage: "across" an element (i.e. resistor)
"at" a location (i.e. node)

↳ Note: Voltage must be measured between two different points: it is always a difference

↳ mathematical definition: $\frac{\Delta \text{potential energy}}{\Delta \text{charge}} = \frac{dU}{dQ} = V$

or $V \cdot \Delta \text{charge} = \Delta \text{potential energy}$

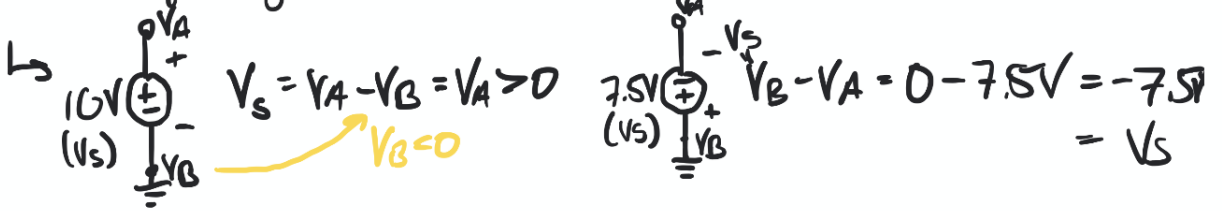
What does this mean?



4) Ground: our reference point for all voltages: $V_{\text{Gnd}} = 0$

↳ symbols: \perp \downarrow

↳ All voltages in a circuit are referenced to some other voltage \rightarrow ground is our standard reference



5) Power: the work done by a circuit

↳ units: Watts (W) = Joules/second

↳ terminology:

Power: "dissipated" by an element (i.e. resistor)
 "delivered" to an element

Note: always in reference to some circuit element

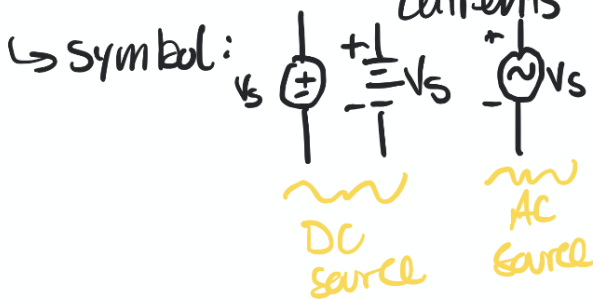
↳ mathematical definition: $P = \frac{\text{energy}}{\text{time}} = \frac{dE}{dt}$

c) Basic Circuit Elements

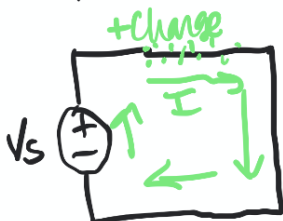
We have a wire with charge in or on it and we want to make it move: what do we need? a source



a) Voltage Source: provides a constant voltage at any/all currents



↳ use: provide a force to make charge move (current)



puts the system in motion!

$V = 5V$

$\Delta V > 0$

current

$V = 0V$

+ $+q$

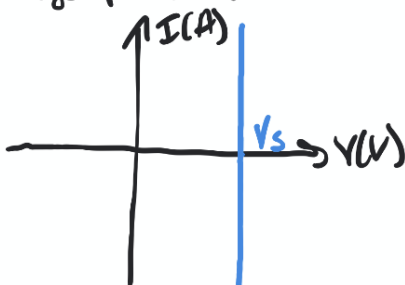
(charge)

$-q$ (charge)

\uparrow current implies (reality)


↳ I-V characteristic

"fingerprint" of circuit element

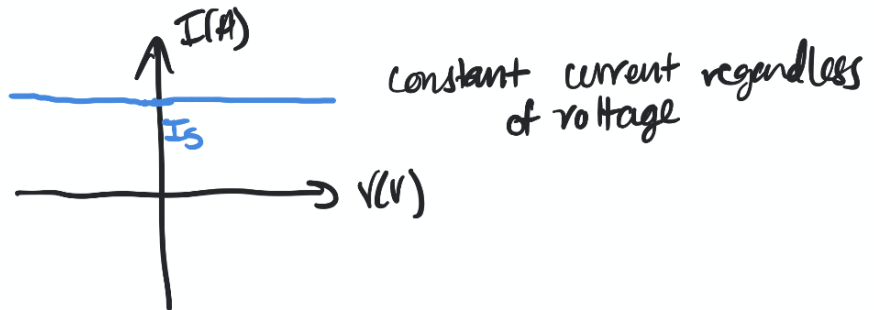


constant voltage regardless of current



b) Current Source : provides a constant current for any/all voltages

↳ symbol :  I_s arrow points in the direction of current flow

↳ I-V Characteristic

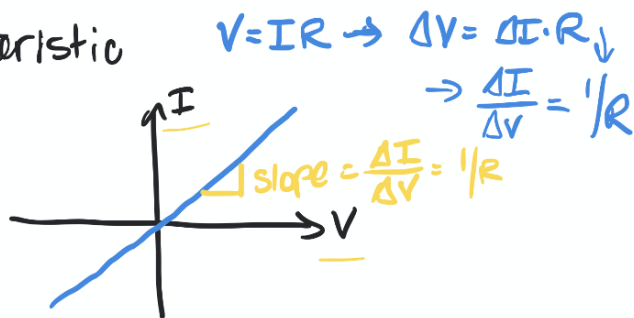


c) Resistor : allows us to control current for a given voltage or vice versa

↳ symbol:  R (USA)  R (Europe)

↳ units: Ohms (Ω) = $\frac{\text{Volts}}{\text{Amperes}}$

↳ I-V characteristic



7) Passive Sign Convention

• In electrical engineering current flows from + to - (Voltage)



8) KCL / KVL / Ohm's Law

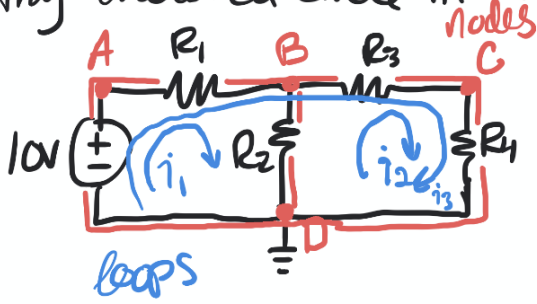
a) Nodes and loops

1) what is a node?

Any connection point between any number of elements

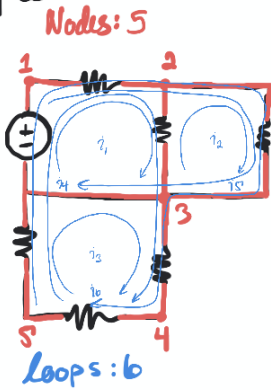
2) What is a loop?

Any enclosed circle in a circuit.



wires have $R=0$,
so ΔV across a wire is zero
- all points connected by only a wire have the same voltage

Example:

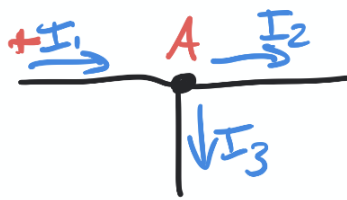


How many nodes? 5
How many loops? 6

b) Kirchoff's Current Law (KCL)

conservation of charge: all current entering and exiting a node must sum to zero

Define: current entering a node gets a "+" sign
current exiting a node gets a "-" sign



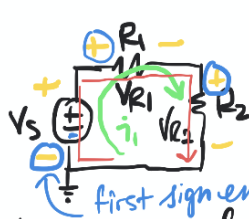
$$+I_1 - I_2 - I_3 = 0$$

$$I_1 = I_2 + I_3$$

Current entering (under I1)
Current leaving (under I2 and I3)

c) Kirchoff's Voltage Law (KVL)

conservation of energy: all voltages in a loop in a circuit must sum to zero



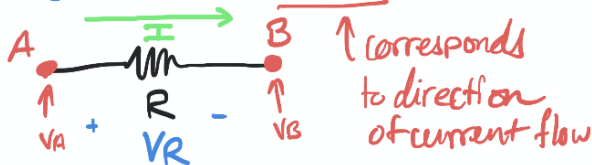
$$i_1: -V_s + V_{R1} + V_{R2} = 0$$

$$V_s = V_{R1} + V_{R2}$$

Voltages get the sign of the first "+" or "-" encountered moving in the direction of the loop in the circuit

d) Ohm's Law: Relates voltage, resistance, and current

$$\Delta V = IR = V_A - V_B = V_R$$

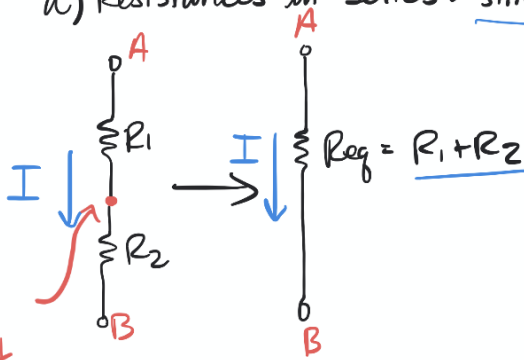


↑ corresponds to direction of current flow

End of lecture

9) Combining Resistances: have one common node with no other elements sharing it

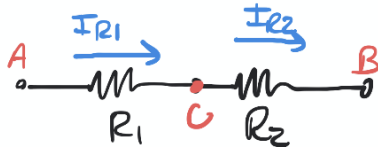
a) Resistances in Series: simply add



one common node not shared with other elements

Note: The current flowing through elements in series is the same

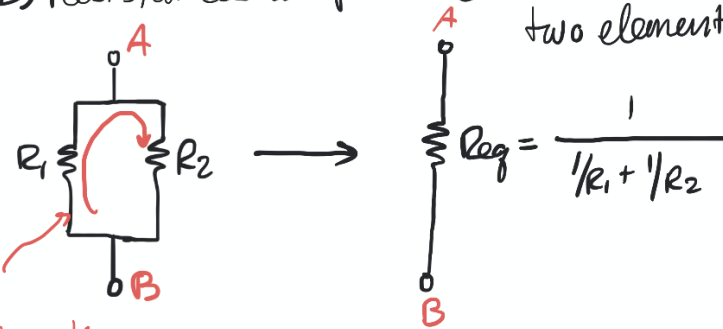
i.e.



due to KCL, at node C: $I_{R1} - I_{R2} = 0$,

So $I_{R1} = I_{R2}$

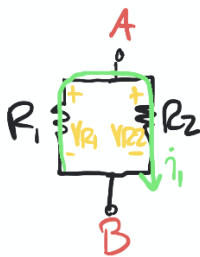
b) Resistances in parallel: form a loop with only two elements



reciprocal resistances add and then the reciprocal of the sum is taken

loop with only 2 elements

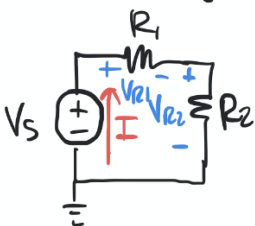
Note: the voltage across two elements in parallel is the same



KVL: $-V_{R1} + V_{R2} = 0$

So $V_{R1} = V_{R2}$

b) Voltage Divider: In a series circuit with a voltage source V_s , the voltage gets divided between the resistors proportional to the ratio of the resistor's resistance to the total series resistance



Let's find V_{R1} :

i) $V_{R1} = I R_1$ (Ohm's law)

ii) What is I ? In a series circuit, I is the same through all components,

So $I = \frac{V_s}{R_1 + R_2}$ (Ohm's law)

iii) Plug in I

$$V_{R_1} = IR_1 = \frac{V_s}{R_1 + R_2} R_1$$

Source voltage \downarrow

resistor whose voltage drop we're measuring \leftarrow

So $V_{R_1} = V_s \frac{R_1}{R_1 + R_2}$

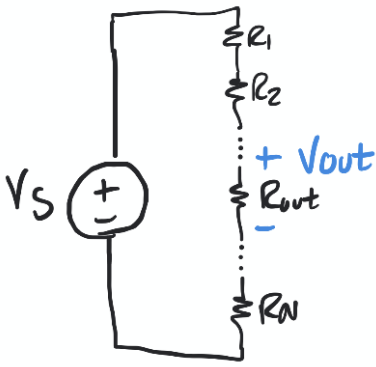
Voltage across R_1 \uparrow

total series resistance $\underbrace{R_1 + R_2}$

Voltage divider equation \leftarrow

$$\text{So } V_{\text{across } R_1} = \text{Source Voltage} \cdot \frac{R_1}{\text{total series resistance}}$$

If we have a series circuit with "N" resistors and we want to know the voltage across one of them:



$$V_{out} = V_s \frac{R_{out}}{\sum_{i=1}^N R_i}$$

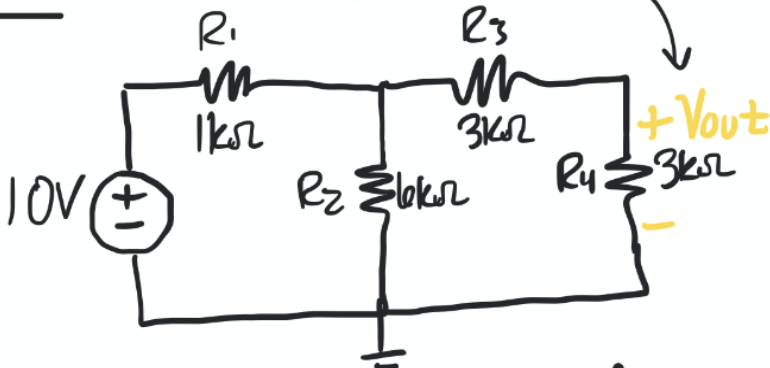
total series resistance $\underbrace{\sum_{i=1}^N R_i}$

11) Circuit Analysis Technique:

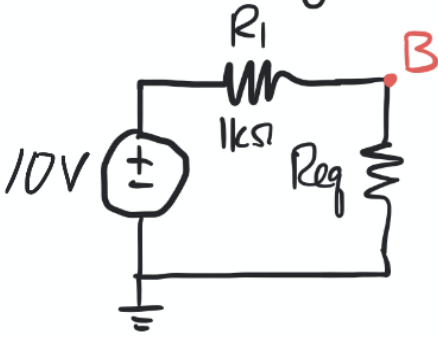
Circuit Reduction

→ Strategy: combine series and parallel resistances until we have a circuit that is solvable via a voltage divider or Ohm's law, then work backwards to find the desired quantity

Example: Find $V_{R4} = V_{out}$



goal: simplify to the following circuit via combining resistances



• then use voltage divider to find voltage at B,
then use Ohm's law or voltage dividers
as needed to find V_{R4} .

Try this before next class!